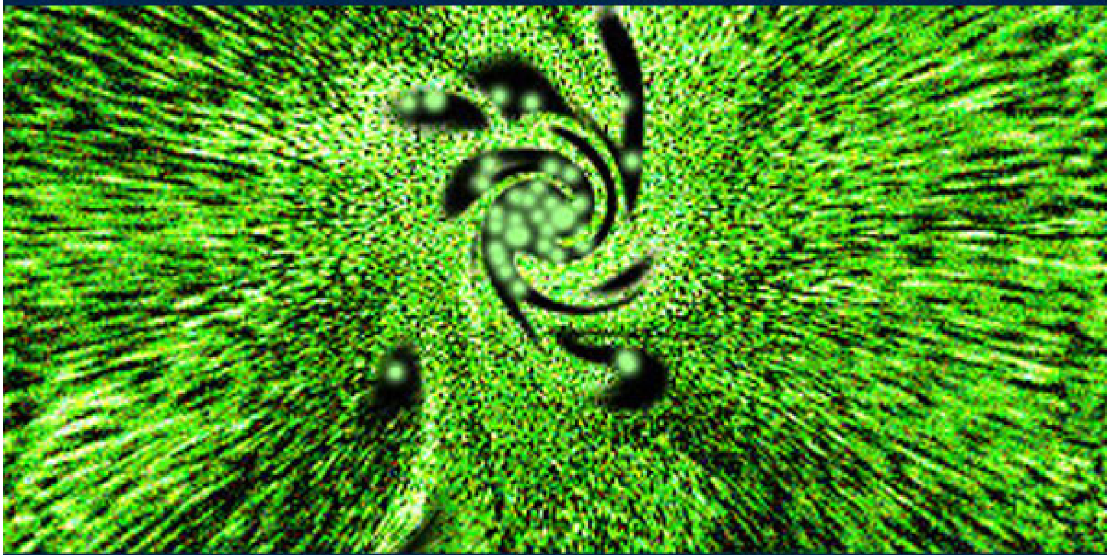
GCSE MATHS TUTOR

# Revision Guide



## PART THREE

# SHAPE & SPACE

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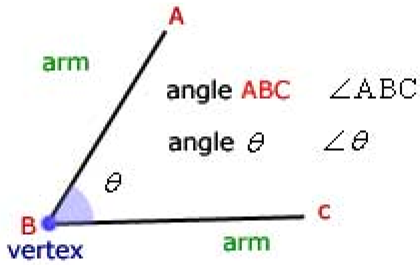
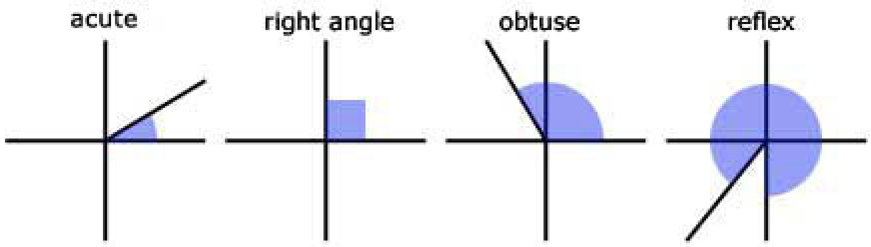
# Contents

# Angles

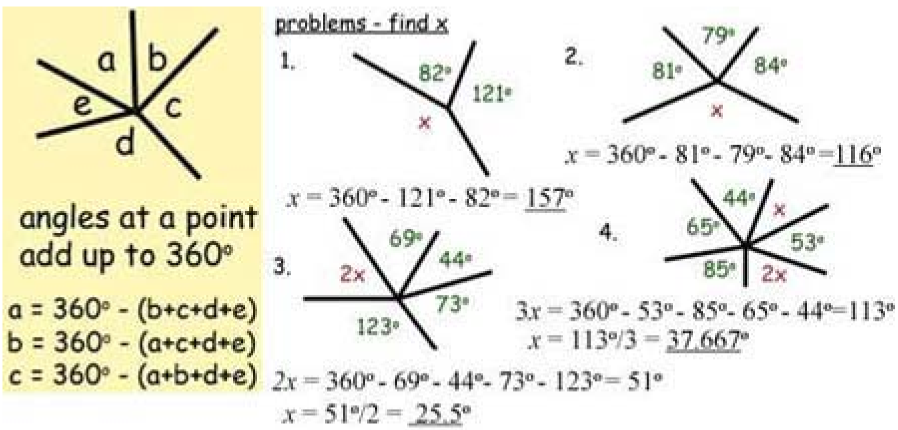
The parts of angles and how angles are named



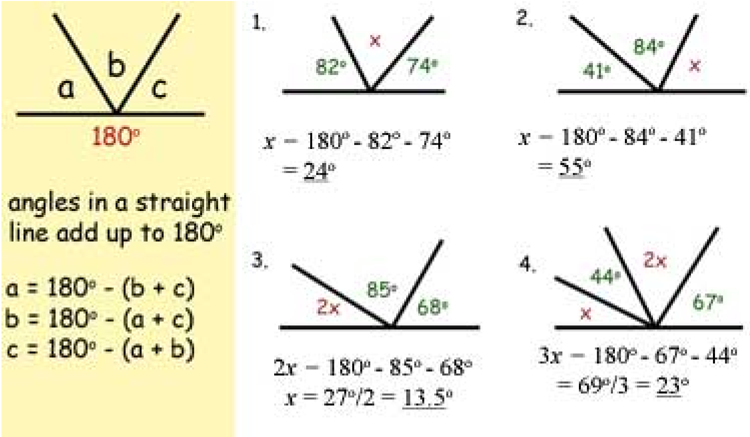

The names of angles



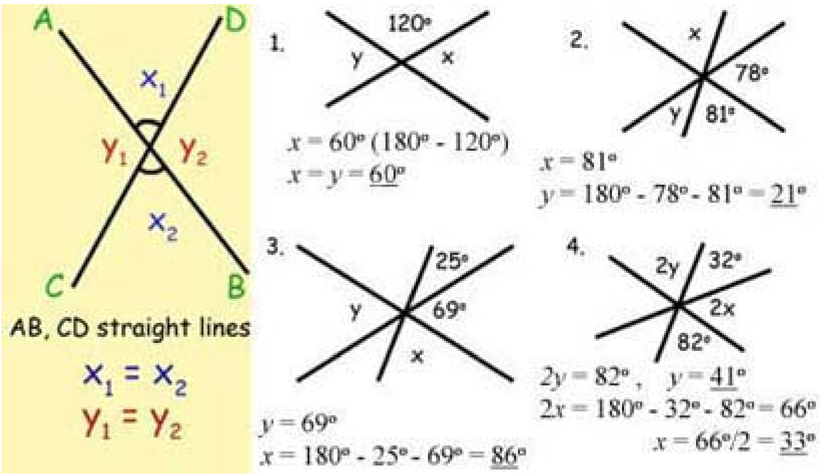

Angles around a point add up to 360 degrees.



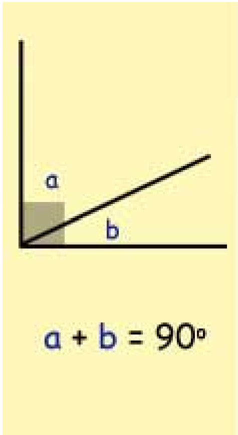
Angles in a straight line add up to 180 degrees (Supplementary angles)

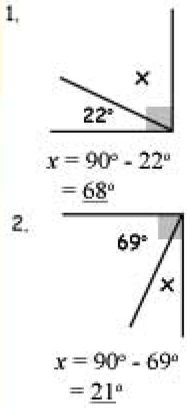



Vertically opposite angles are equal

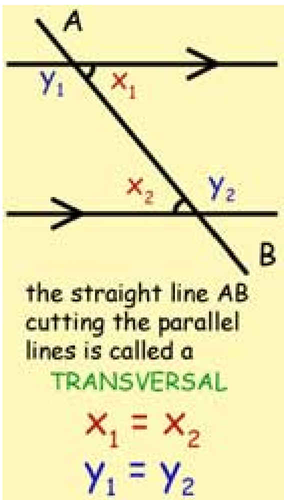


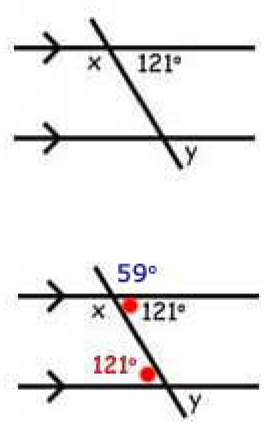
Complementary angles add up to 90 degrees.



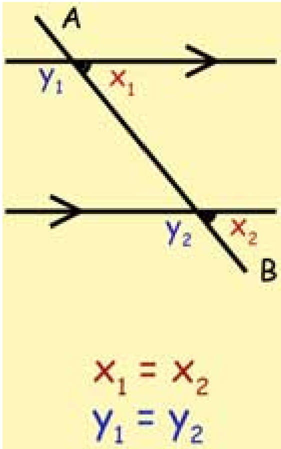



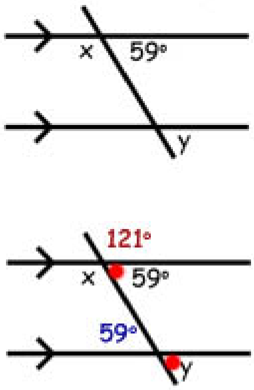

Alternate angles are equal



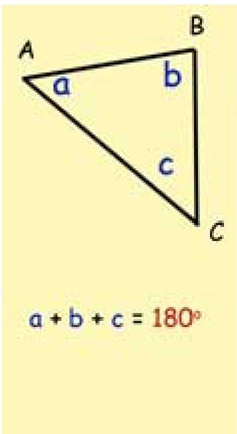



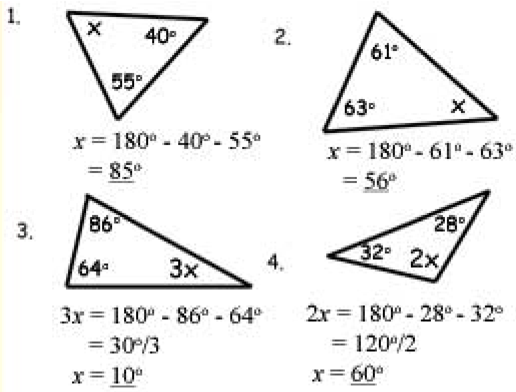
Corresponding angles



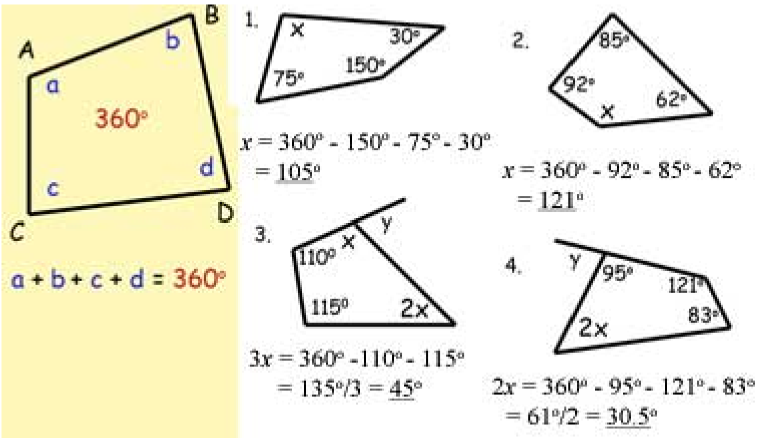




The angles in a triangle add up to 180 degrees.

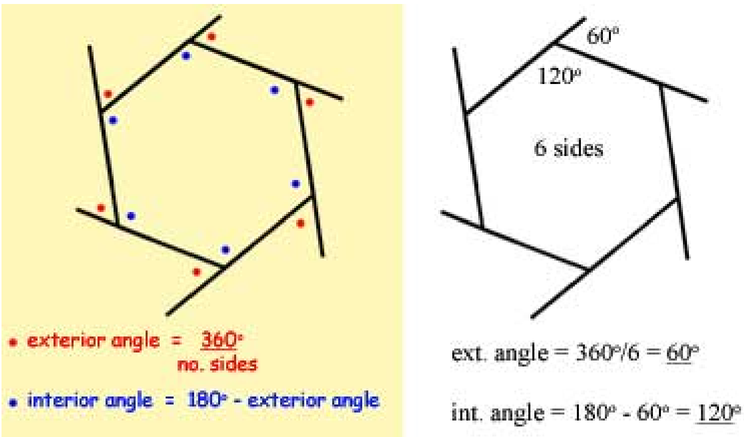


The angles in a quadrilateral add up to 360 degrees.




The angles in polygons

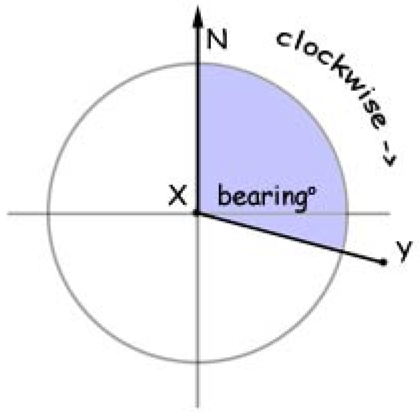


# Bearings

Bearings are a way of describing the position of places/objects by using clockwise angles relative to North.

e.g. (note a bearing has **3** digits)

- east is a bearing of 090 deg.
- south is a bearing of 180 deg.
- west is a bearing of 270 deg.
- north is a bearing of 000 deg.
- south west is a bearing of 225 deg.

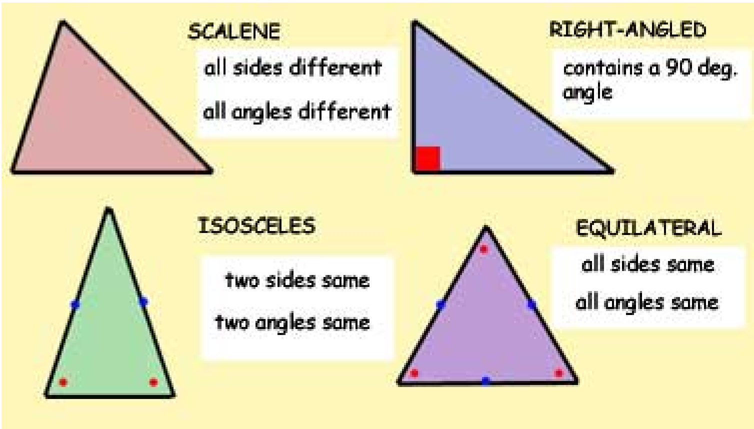



To find the bearing of a point Y from a point X:

- draw a line up the page from X to represent North
- draw a line between point X and point Y
- with your protractor on the North line(XN), measure angle NXY

# Triangle Similarity

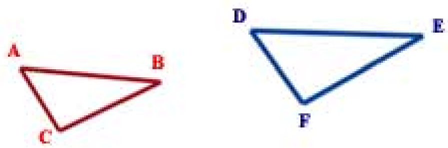
Triangle types




Definition of similarity

A triangle (or indeed any two dimensional shape) is deemed similar to another if it has **the same shape but a different size**.

Similarity and proportion - If triangle ABC is similar to triangle DEF then:




$$\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$$

<u>Triangle enlargement</u> - in the above example, the ratio of the sides(largest over smallest) gives the **Scale Factor**.

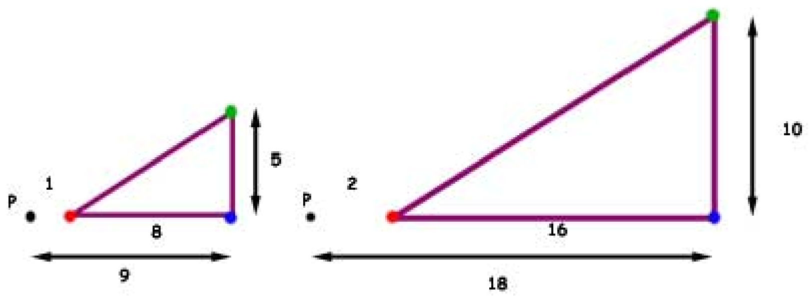
For enlargement the Scale Factor is greater than one( >1 ).

The **Centre of Enlargement** is a point used for constructing an enlargement from an original shape.

The Scale Factor is used to calculate where the new points on the enlarged shape are from the centre of enlargement.

<u>Example</u> - A right angled triangle positioned as shown, 2cm from a point P. Using the point P as reference, enlarge the triangle by a factor of 2.

The solution is to measure the distance between each of the points of the triangle to the point P, and then to multiply each distance by the scale factor 2.




<u>Triangle reduction</u> - here the Scale Factor is less than one( <1 ). The ratio is of the small sides of the first triangle to the large sides of the second triangle.

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

<u>Negative enlargement/reduction</u> - here the shape is either enlarged or reduced as before but is rotated through 180 degrees.

# Triangle Congruency

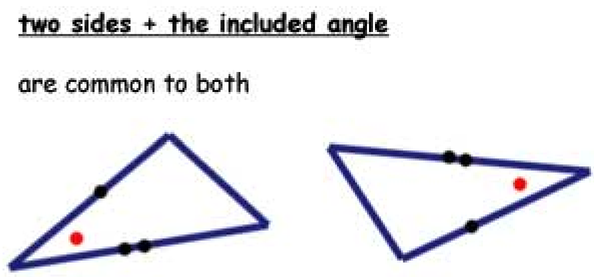
Definition of congruency

A triangle (or indeed any two dimensional shape) is deemed congruent to another if it has **the same shape and the same size**.

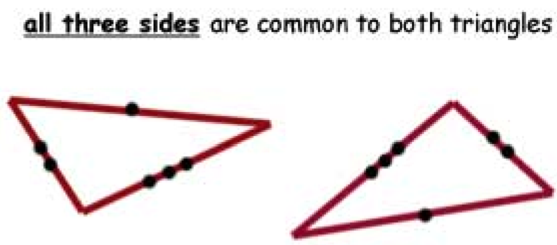
The second triangle can be rotated, reflected or translated in any way.

The first triangle must be able to fit over the second triangle exactly.

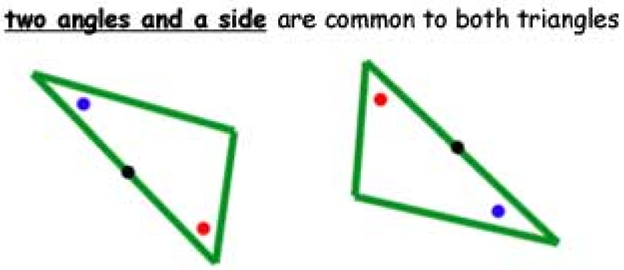
Tests for congruency - **S**ide **A**ngle **S**ide (SAS)



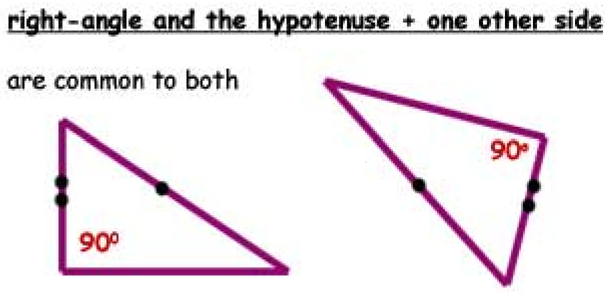

Tests for congruency - **S**ide **S**ide **S**ide (SSS)



Tests for congruency - **A**ngle **S**ide **A**ngle (ASA)

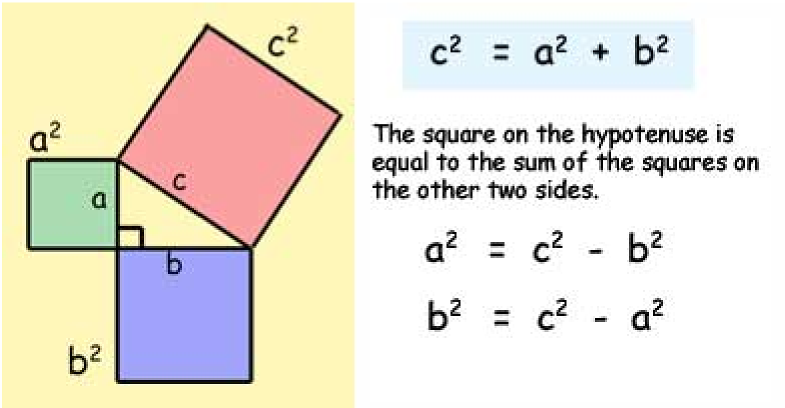



Tests for congruency - **R**ight angle **H**ypotenuse and one **S**ide (RHS)

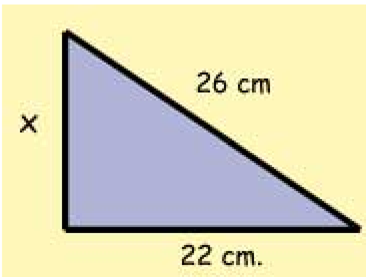


# Pythagoras' Theorem

Pythagoras' Theorem



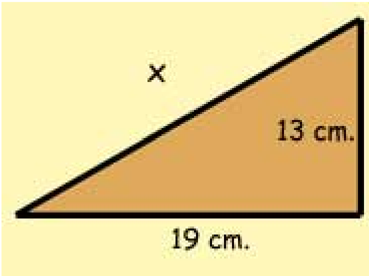

Example #1- given the hypotenuse & one side




$$
\begin{aligned}
22^2 + x^2 &= 26^2 \\
x^2 &= 26^2 - 22^2 \\
&= 676 - 484 \\
&= 192 \\
x &= \sqrt{192} = 13.8564 \\
&= \underline{13.86} \text{ (2 d.p.)}
\end{aligned}
$$

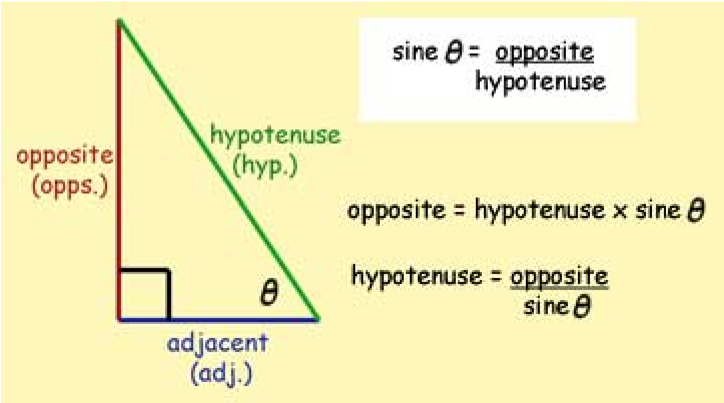
Example #2 - given two sides




$$x^2 = 13^2 + 19^2$$
$$= 169 + 361 = 530$$
$$x = \sqrt{530} = 23.0217$$
$$= \underline{23.02} \quad (2\ \text{d.p.})$$

# The Sine Ratio

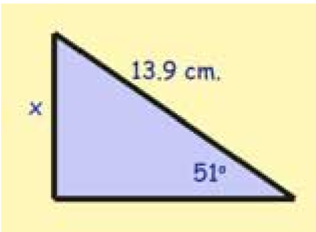
The Sine Ratio




Method for problems:

- write out the ratio putting in the values for the given sides and/or angle.
- put a '1' under the sine/cos/tan
- cross multiply(top left by bottom right = top right by bottom left)
- make the 'unknown' the subject of the equation

Example #1




$$\frac{\sin 51^\circ}{1} = \frac{x}{13.9}$$

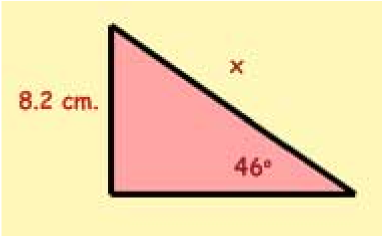
$$x = 13.9 \times \sin 51^\circ$$

$$= 13.9 \times 0.7771$$

$$= 10.8023$$

$$\underline{x = 10.80 \text{ cm}} \ (2 \text{ d.p.})$$

Example #2




$$\frac{\sin 46^\circ}{1} = \frac{8.2}{x}$$

$$8.2 = x \times \sin 46^\circ$$

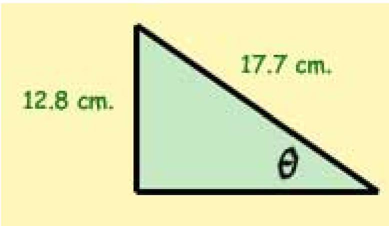
$$x = \frac{8.2}{\sin 46^\circ}$$

$$x = \frac{8.2}{0.7193}$$

$$= 11.3999$$

$$\underline{x = 11.40 \text{ cm}} \ (2 \text{ d.p.})$$

Example #3




$$\sin\theta = \frac{12.8}{17.7}$$

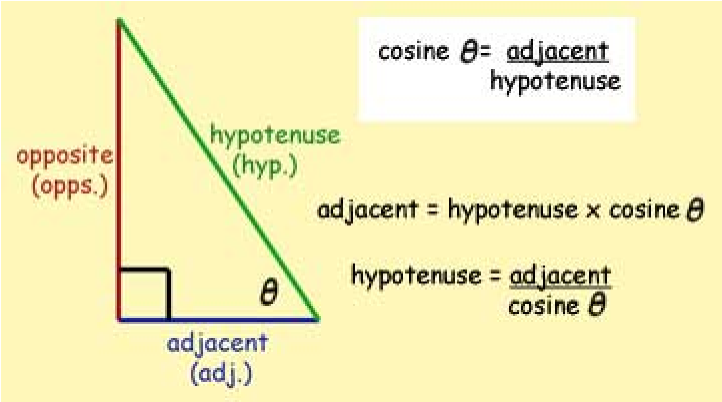
$$= 0.7232$$

$$\theta = 46.3193^\circ$$

$$= \underline{46.32^\circ}$$

## The Cosine Ratio

## The Cosine Ratio




Method for problems:

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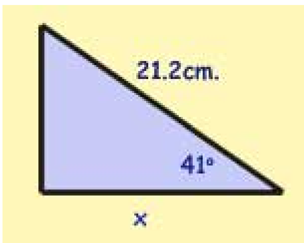
## Example #1




$$\frac{\cos 41^\circ}{1} = \frac{x}{21.2}$$

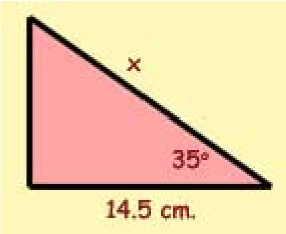
$$x = 21.2 \times \cos 41^\circ$$

$$= 21.2 \times 0.7547$$

$$= 15.9996$$

$$\underline{x = 16.00 \text{ cm}} \ (2 \text{ d.p.})$$

## Example #2




$$\frac{\cos 35^\circ}{1} = \frac{14.5}{x}$$

$$x\cos 35^\circ = 14.5$$

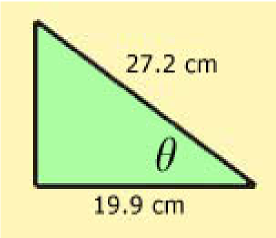
$$x = \frac{14.5}{\cos 35^\circ}$$

$$= \frac{14.5}{0.8192}$$

$$= 17.7001$$

$$\underline{x = 17.70} \quad (2\text{ d.p.})$$

Example #3




$$\frac{\cos\theta}{1} = \frac{19.9}{27.2}$$

$$= 0.7316$$

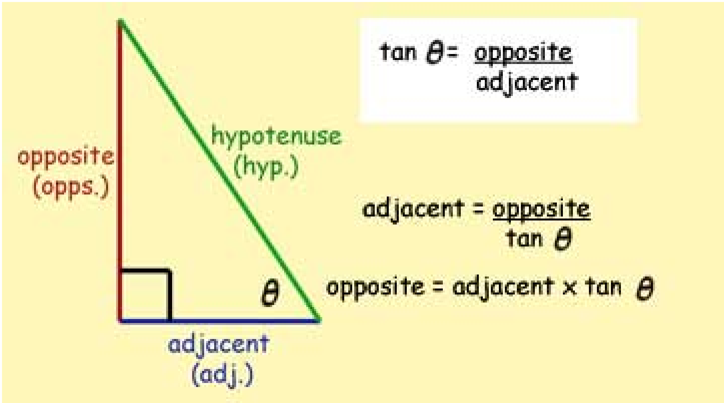
$$\theta = 0.7232$$

$$= 42.9793^\circ$$

$$= \underline{42.98^\circ} \ (2\text{ d.p.})$$

## The Tangent Ratio

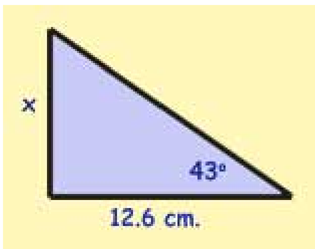
The Tangent Ratio




Method for problems:

- write out the ratio putting in the values for the given sides and/or angle.
- put a '1' under the sine/cos/tan
- cross multiply(top left by bottom right = top right by bottom left)
- make the 'unknown' the subject of the equation

Example #1




$$\frac{\tan\ 3^\circ}{1} = \frac{x}{12.6}$$

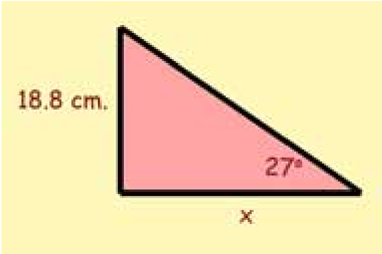
$$x = 12.6 \tan 43^\circ$$

$$= 12.6 \times 0.9325$$

$$= 11.7495$$

$$= \underline{11.75\text{ cm}}\ (2\text{ d.p.})$$

Example #2




$$\frac{\tan 27^\circ}{1} = \frac{18.8}{x}$$

$$x \tan 27^\circ = 18.8$$

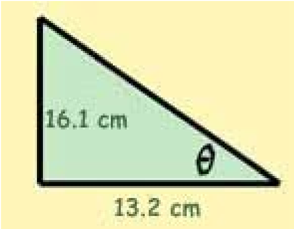
$$x = \frac{18.8}{\tan 27^\circ}$$

$$= \frac{18.8}{0.5095}$$

$$= 36.8989$$

$$\underline{x = 36.90 \text{ cm}} \quad (2 \text{ d.p.})$$

Example #3




$$\frac{\tan \theta}{1} = \frac{16.1}{13.2}$$

$$= 1.2197$$

$$\theta = 50.6525$$

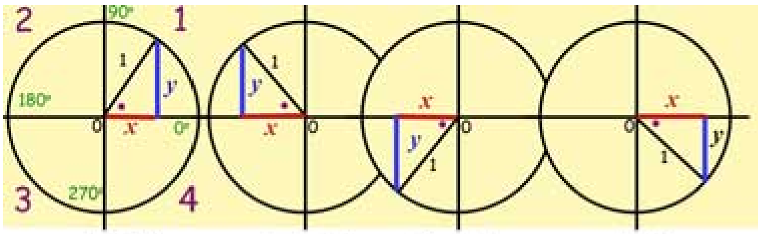
$$= \underline{50.65^\circ} \quad (2 \text{ d.p.})$$

## Sine, Cosine & Tangent Compared

Changing sign of ratios with increasing angle

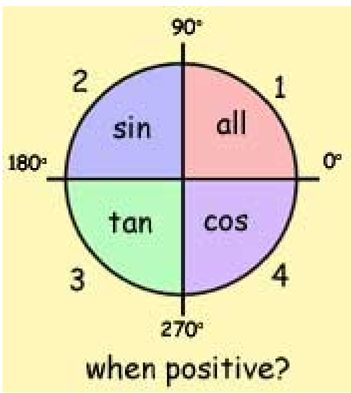
In turn, look at each ratio as the value of x and y changes with increasing angle.

$$\sin \theta = \frac{y}{1} \qquad \cos \theta = \frac{x}{1} \qquad \tan \theta = \frac{y}{x}$$

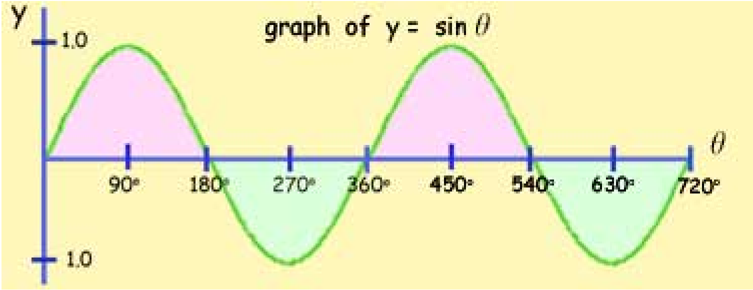



| | $x+\ y+$ | $x-\ y+$ | $x-\ y-$ | $x+\ y-$ |
|---|---|---|---|---|
| sin | + | + | - | - |
| cos | + | - | - | + |
| tan | + | - | + | - |

The results can be summarised in this diagram:



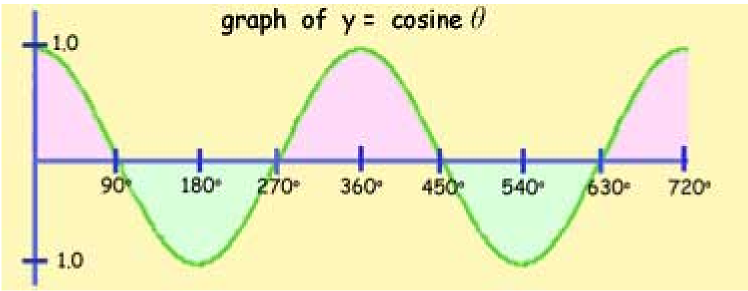

The Sine Curve




note:

- the sine graph starts at **zero**
- it repeats itself every **360** degrees
- y is never more than **1** or less than **-1**
- a sin graph 'leads' a cos graph by 90 degrees

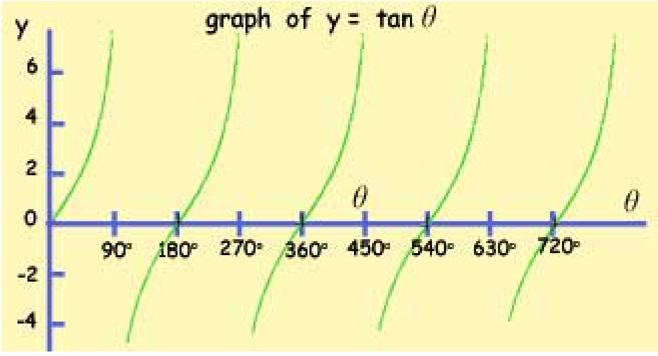
## The Cosine Curve




note:

- the cosine graph starts at **one**
- it repeats itself every **360** degrees
- y is never more than **1** or less than **-1**
- a cos graph 'lags' a sin graph by 90 degrees

## The Tangent Curve

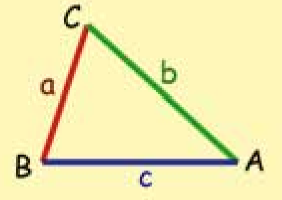



note:

- the tangent graph starts at **zero**
- it repeats itself every **180** degrees
- y can vary between numbers approaching infinity and minus infinity

## Sine Rule, Cosine Rule

The Sine Rule




$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- Use either the right, or left hand equation.
- You are given 3 quantities and required to work out the 4 th.

Manipulating the ratio - Take two ratios, cross multiply and rearrange to put the required quantity as the subject of the equation.

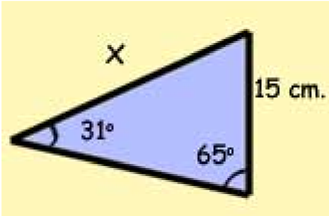
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$a \sin B = b \sin A$$

$$a = \frac{b \sin A}{\sin B} \qquad b = \frac{a \sin B}{\sin A}$$

$$\sin B = \frac{b \sin A}{a} \qquad \sin A = \frac{a \sin B}{b}$$

Example #1




$$\frac{15}{\sin 31^\circ} = \frac{x}{\sin 65^\circ}$$

$$x \sin 31^\circ = 15 \sin 65^\circ$$

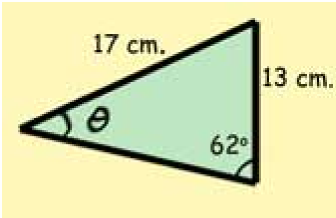
$$x = \frac{15 \sin 65^\circ}{\sin 31^\circ}$$

$$= \frac{15 \times 0.9063}{0.5150}$$

$$= 26.3971$$

$$\underline{x = 26.40 \text{ cm}} \text{ (2 d.p.)}$$

Example #2




$$\frac{17}{\sin 62^\circ} = \frac{13}{\sin \theta}$$

$$17 \sin \theta = 13 \sin 62^\circ$$

$$\sin \theta = \frac{13 \sin 62^\circ}{17}$$

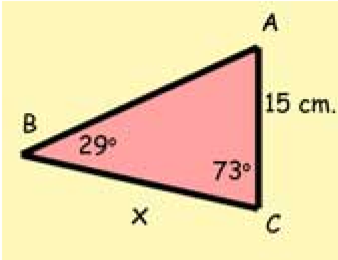
$$= \frac{13 \times 0.8829}{17}$$

$$= 0.6752$$

$$\theta = 42.4697^\circ$$

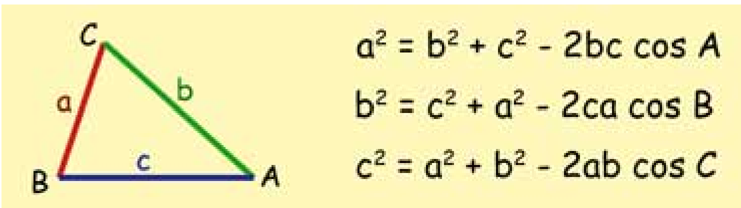
$$\underline{\theta = 42.47^\circ} \text{ (2 d.p.)}$$

Example #3




$$\text{angle A} = 180^\circ - 29^\circ - 73^\circ$$
$$= 78^\circ$$
$$\frac{x}{\sin 78^\circ} = \frac{15}{\sin 29^\circ}$$
$$x\sin 29^\circ = 15\sin 78^\circ$$
$$x = \frac{15\sin 78^\circ}{\sin 29^\circ}$$
$$= \frac{15 \times 0.9781}{0.4848}$$
$$= 30.2630$$
$$\underline{x = 30.26 \text{ cm}} \ (2 \text{ d.p.})$$

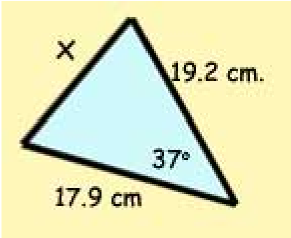
The Cosine Rule




There are two problem types:

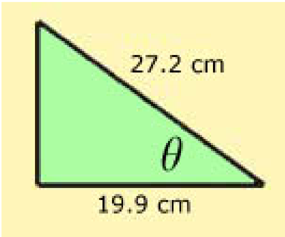
- You are given 2 sides + an included angle and required to work out the remaining side
- You are given all the sides and required to work out the angle.

Example #1




$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = (19.2)^2 + (17.90)^2 - \left[2 \times 19.2 \times 17.9 \times \cos 37^\circ\right]$$
$$= (368.64) + (320.4) - (687.36) \times (0.7986)$$
$$= 140.1143$$
$$x = 11.8369$$
$$= \underline{11.84 \text{ cm}} \ (2 \text{ d.p.})$$

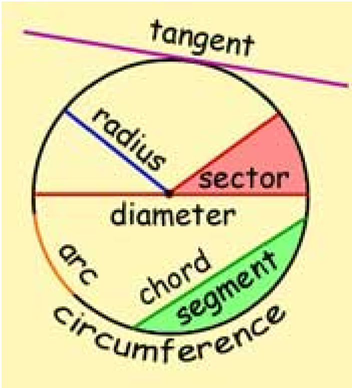
Example #2




$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos\theta = \frac{(23.6)^2 + (15.5)^2 - (21.8)^2}{2(23.6)(15.5)}$$
$$= \frac{(556.96) + (240.25) - (475.24)}{731.60}$$
$$= \frac{321.97}{731.60} = 0.4401$$
$$\theta = 63.8897^\circ$$
$$= \underline{63.89^\circ} \ (2 \text{ d.p.})$$

# Circles

The parts of a circle




**centre** - the point within the circle where the distance to points on the circumference is the same.

**radius** - the distance from the centre to any point on the circle. The diameter is twice the radius.

**circumference**(perimeter) - the distance around a circle.

**chord** is a straight line joining two points on the circumference.

**diameter** - a chord(of max. length) passing through the centre

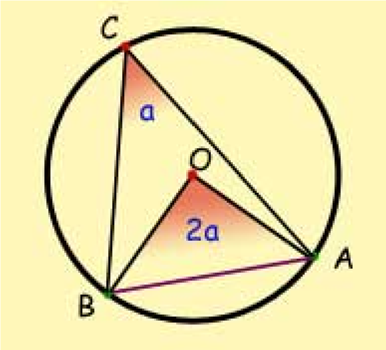
**sector** - a region enclosed by two radii and an arc.

**segment** - the region enclosed by a chord and an arc of the circle.

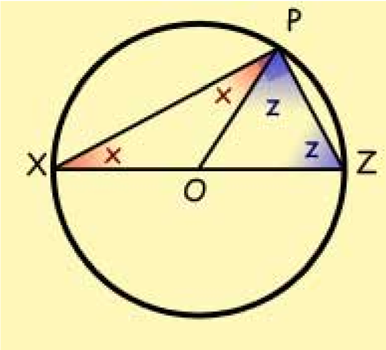
**tangent** - a straight line making contact at one point on the circumference, such that the radius from the centre is at right angles to the line.

Subtended angles

When a chord subtends an angle on the circumference of a circle, the angle subtended at the centre of the circle is twice the angle.

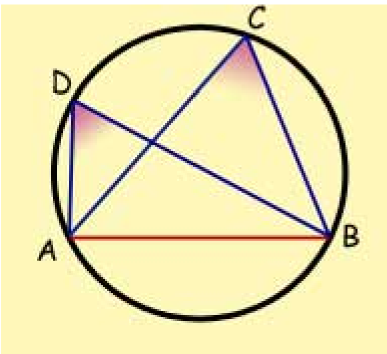



A diameter subtends a right-angle at the circumference



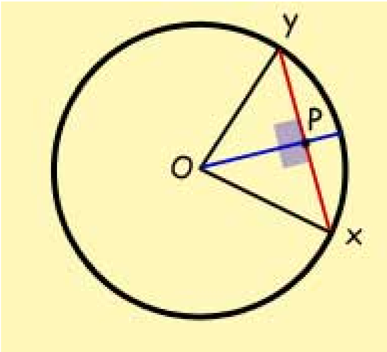

**angle XPZ = 90 deg.**

Angles subtended by a chord onto the circumference of a circle are equal.




**angle ADB = angle ACB**

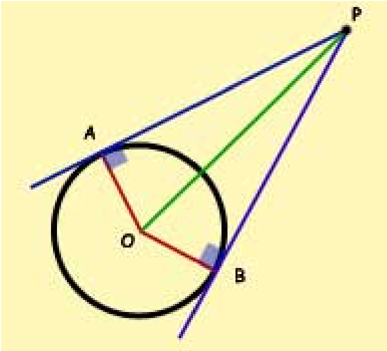
<u>Chords</u>




The line joining the centre of a circle and the mid-point of a chord is perpendicular to the chord. The chord is bisected into two equal halves.

**XP = PY**

Tangents




The tangents to a circle from a point are equal in length.

**AP = BP**

also,

the tangents subtend equal angles at the centre of the circle

**angle POA = angle POB**

and,

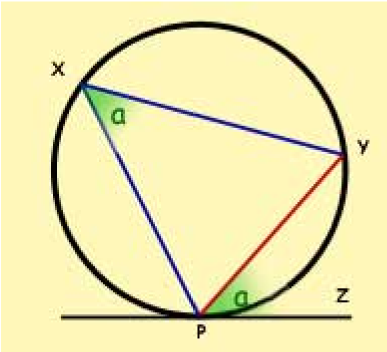
the angles between the tangents and the line joining the centre of the circle and the point are equal.

**angle APO = angle BPO**

**note** : Triangle APO and triangle BPO are congruent.

## The angle between a tangent and a chord

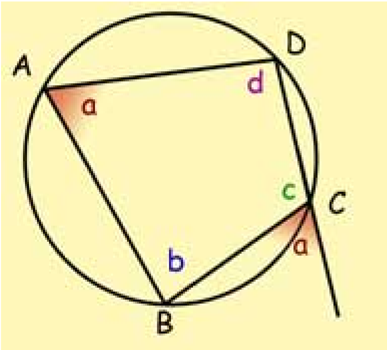
The angle between a tangent and a chord is equal to the angle subtended by the chord in the opposite segment.




angle ZPY = angle PXY

## Cyclic quadrilaterals

**Opposite** angles in a **cyclic** quadrilateral add up to **180 deg**.




As with all quadrilaterals, the **sum of the interior angles = 360 deg**.

Any **exterior angle** of a cyclic quadrilateral **equals the interior opposite angle**.

# Vectors

## Vectors & Scalars

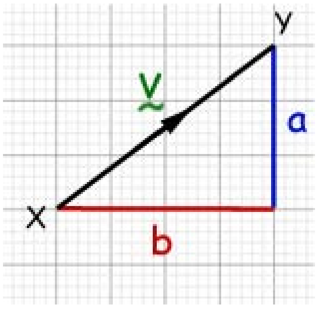
A **scalar** is a quantity that has magnitude only.

e.g. mass, length, temperature, speed

A **vector** is a quantity with both magnitude and direction.

e.g. force, displacement, acceleration, velocity, momentum

## Vector notation




The vector from X to Y may also be represented as **V** or –

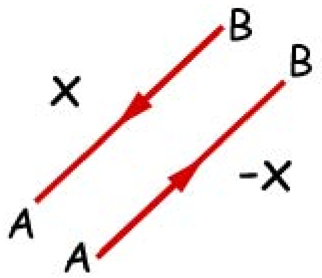
$$\overrightarrow{XY}\ ,\ \underline{V}\ ,\ \begin{pmatrix} b \\ a \end{pmatrix}$$

The magnitude of the vector(i.e. its number value) is expressed as:

$$\left|\overrightarrow{XY}\right| \text{ or } \left|\underline{V}\right|$$

Inverse vectors

An inverse vector is a vector of equal magnitude to the original but in the opposite direction.




$-\overrightarrow{AB}$ is the inverse of $\overrightarrow{AB}$

or

$\overrightarrow{BA}$ is the inverse of $\overrightarrow{AB}$

The Modulus(magnitude) of a vector

This modulus of a vector **X** is written I **X** I .

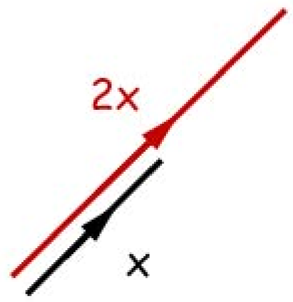
The modulus(length of the vector line) can be calculated using Pythagoras' Theorem.

This is dealt with in detail in the 'linear graphs section' here . However for completeness, the relevant formula is:

$$|\underline{X}| = \sqrt{(\text{y-step})^2 + (\text{x-step})^2}$$

Scalar multiplication

A scalar quantity(i.e. a number) can alter the magnitude of a vector but not its direction.




Example - In the diagram(above) the vector of magnitude X is multiplied by 2 to become magnitude 2X.

If the vector **X** starts at the origin and ends at the point (4,4), then the vector **2X** will end at (8,8).

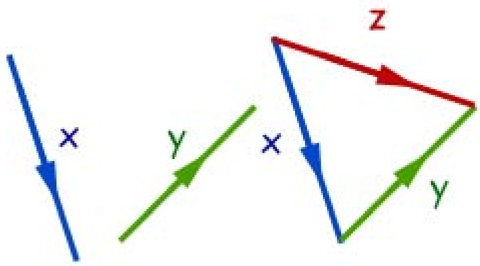
The scalar multiplication can be represented by column vectors:

$$\underline{X} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \qquad 2\underline{X} = 2\begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

The Triangle Law(Vector addition)

When adding vectors, remember they must run in the direction of the arrows(i.e head to tail).

A vector running against the arrowed direction is the resultant vector. That is, the **one** vector that would have the same effect as the others added together.



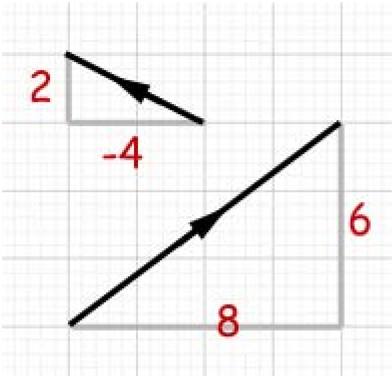

Example

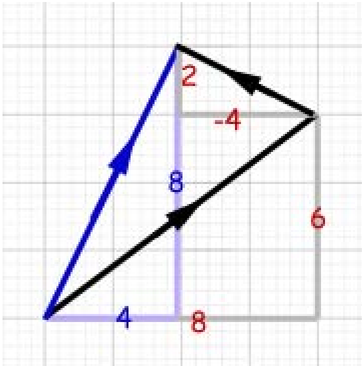
**A** and **B** are vectors, as shown below. Find the magnitude of their resultant **X**.

$$\underset{\sim}{A} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \qquad \underset{\sim}{B} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

First we must find the resultant vector. This is done by adding the column matrices representing the vectors.

$$\begin{pmatrix} 8 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$



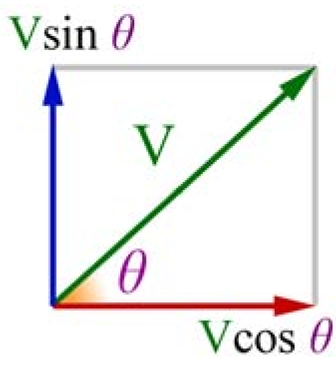




The magnitude of the resultant is given by using Pythagoras' Theorem:

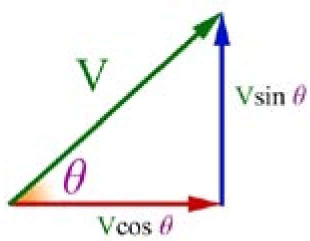
$$
\begin{aligned}
|\underline{X}| &= \sqrt{(\text{y-step})^2 + (\text{x-step})^2} \\
&= \sqrt{8^2 + 4^2} \\
&= \sqrt{64 + 16} \\
&= \sqrt{80} \\
&= 8.944 \\
&= \underline{8.94}\ (2\text{d.p.})
\end{aligned}
$$

Components

A single vector can be represented by two components set at 90 deg. to eachother. This arrangement is very useful in solving 'real world' problems.

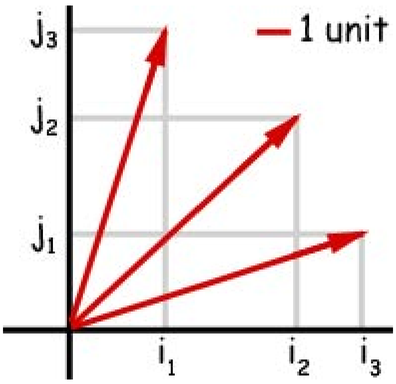



looking at the right angled triangle below you can see where this came from



## Unit vectors

A unit vector has unit length (1).




the **x-axis** coordinate is **i** and the **y-axis** coordinate is **j**.

Example of a unit vector : 5**i** + 2 **j** would be at coordinates (5 , 2).

## Unit vector addition (& subtraction):

In turn add **i** terms and then add **j** terms.

example:

5 **i** + 2 **j** plus 2 **i** + 5 **j** = 7 **i** + 7 **j**

in vector terms this can be expressed as:

$$\begin{pmatrix}5\\2\end{pmatrix}+\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}7\\7\end{pmatrix}$$

# Transformations

## Translation of points

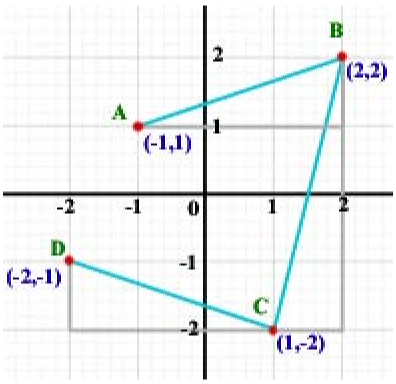
A point (x,y) can be moved to another position by applying a column matrix vector.

A column matrix vector is just two numbers, one above the other, surrounded by long brackets. The top number adds to the x-coordinate while the bottom number adds to the y-coordinate.

$$(x, y) \text{ transformed by the column vector } \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\text{gives the new point } (x+a, y+b)$$

## Example




move **right** or **left** = **x** more **positive** or more **negative**
move **up** or **down** = **y** more **positive** or more **negative**

**A**(-1,1) to **B**(2,2) by going to the **right 3** and **up 1** by applying vector $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

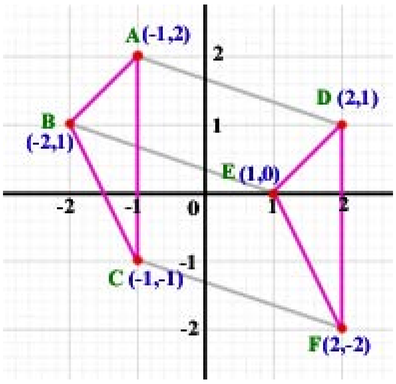
**B**(2,2) to **C**(1,-2) by going to the **left 1** and **down 4** by applying vector $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$

**C**(1,-2) to **D**(-2,-1) by going to the **left 3** and **up 1** by applying vector $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

Translation of shapes

This is similar to the translation of a point except that the vector column matrix is applied to each point of the shape in turn to move the whole shape to another position.

example Translate the triangle ABC by the vector column matrix $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$




Each x-coordinate is increased by 3 (moved right 3)

Each y-coordinate is reduced by 1 (moved down 1)

Reflections

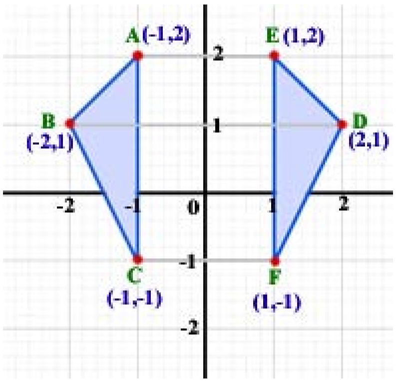
A reflection is a mirror image of the shape about an arbitrary line. This line can be for example x=0(y-axis), y=3, y=x(diagonal) etc.

Reflected points can be located by remembering that:

**each point is as far infront of the line as the image of the point is behind**

So measure the perpendicular distance of each point on the shape from the mirror line, then measure each distance the other side of the line to locate the points.

Example Draw the reflection of triangle ABC in the y-axis.




Point A is 1 unit infront of the y-axis mirror. Therefore the reflected point(E) is 1 unit the other side of the line.

Similarly, B is 2 units infront. D is 2 units behind. C is 1 unit infront. F is 1 unit behind.

Method for locating mirror images

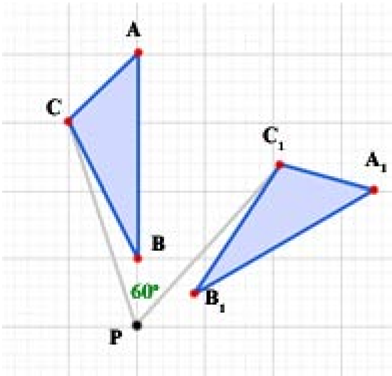
- Use your set-square to draw perpendiculars from the 'mirror line' through each point of the shape
- Measure the point-'mirror line' distance for each point.
- Produce each line **behind** the mirror line the **same distance** to locate the mirrored points.

Rotations

In order to rotate a shape, 3 pieces of information are required:

**the centre of rotation - the direction of rotation - the angle of rotation**

Example - Rotate triangle ABC through 60 deg. in a clockwise direction about point P.




method

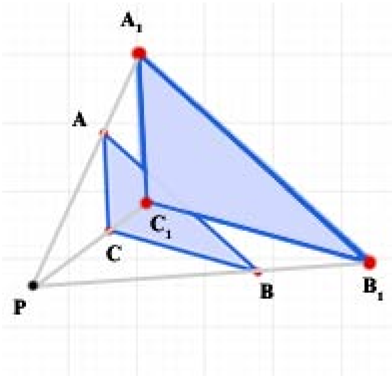
- draw a line PC between P and C
- measure the line PC
- draw a line 60 deg. clockwise from PC, centre P, the same length as PC
- repeat the method for **B** and **A** on the original shape
- finally join up the ends of the lines to make the original shape but rotated through 60 deg.

Enlargements

In order to enlarge a shape, 2 pieces of information are required:

**the centre of enlargement** - **the scale factor**

Example Enlarge triangle ABC by a scale factor of 1.5 form the point P.




method

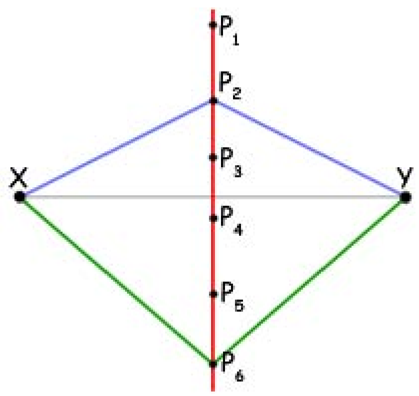
- Draw a line through A, measuring 1.5 times PA.
- Draw a line through B, measuring 1.5 times PB.
- Draw a line from P through C, measuring 1.5 times PC.
- Join up the ends of the lines.

# Loci

Equidistant line from two fixed points

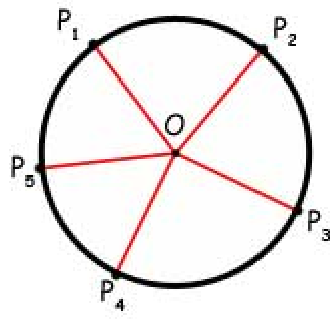
Each point P in turn on the red line is equidistant from points X and Y.

The red line represents the **locus of points** equidistant from X and Y.




Equidistant line from a fixed point(a circle)

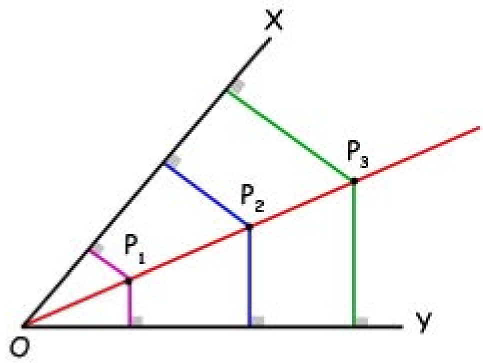
All the points P lie on a **locus of points** the same distance from O (i.e on the perimeter of the circle)



Equidistant line from two lines joined at a vertex

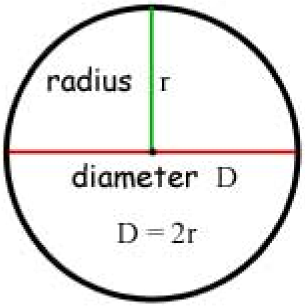
Each point P in turn on the red line is equidistant from lines OX and OY.

The red line represents the **locus of points** equidistant from lines OX and OY.



# Length & Area

## Area of a circle



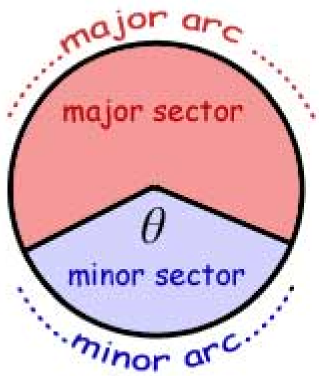

the area of a circle of radius 'r' = $\pi r^2$

the circumference of a circle of radius 'r' = $2\pi r$

since diameter $D = 2r$

the circumference of a circle is = $\pi D$

## Arcs




length ($L_a$) of an arc is given by:

$$L_a = \left(\frac{\text{subtended angle}}{\text{no. degrees in a circle}}\right) \times (\text{circumference of the circle})$$

$$= \frac{\theta}{360^\circ}(2\pi r)$$

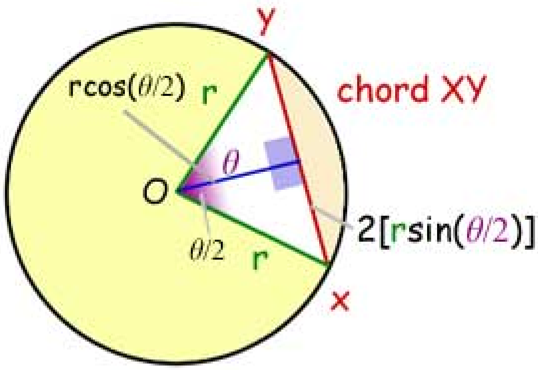
## Sectors

area ($A_s$) of a sector is given by:

$$A_s = \left(\frac{\text{subtended angle}}{\text{no. degrees in a circle}}\right) \times \text{(area of the circle)}$$

$$= \frac{\theta}{360^\circ}(\pi r^2)$$

## Segments



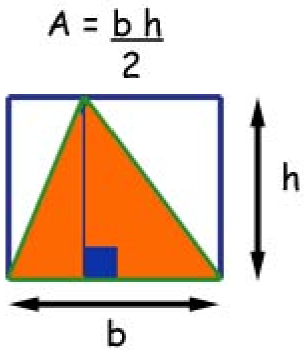

$$\text{area of minor segment XY} = \text{area of sector XOY - area of triangle XOY}$$

$$= \frac{\theta^\circ}{360^\circ} \times \text{circle area - } \frac{1}{2}\text{(base)} \times \text{(height)}$$

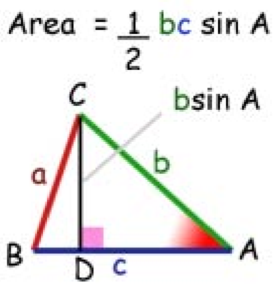
$$= \left(\frac{\theta^\circ}{360^\circ} \times \pi r^2\right) - \frac{1}{2}\left[2r\sin(\theta/2) \times r\cos(\theta/2)\right]$$

Triangles

When a triangle is incribed in a rectangle of height h and width b, the perpendicular divides the shape. It can be seen that each rectangle formed is composed of two triangles of equal area. Hence the area of the original triangle is half that of the rectangle.




The expression containing the sine is really the same as above.



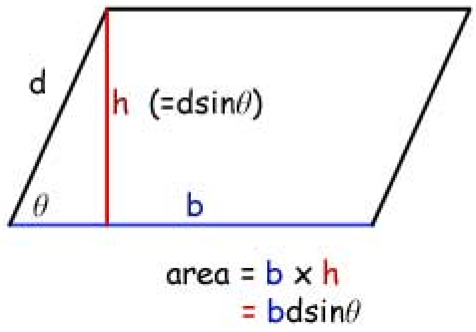

$$\text{area} = \frac{1}{2}(\text{base}) \times (\text{height})$$

$$= \frac{1}{2}(c) \times (b \sin A)$$

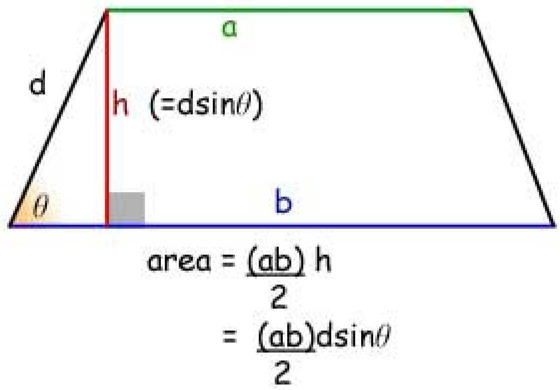
$$= \frac{1}{2} cb \sin A$$

$$= \frac{1}{2} bc \sin A$$

Parallelograms

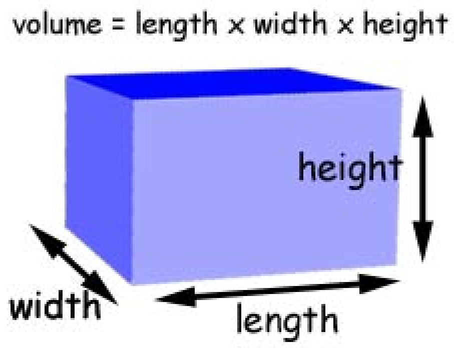



Trapeziums(Trapezoids)



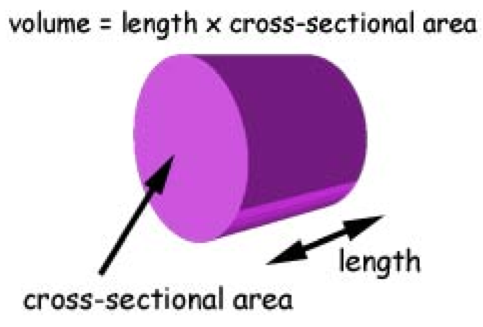
# Volume

Cuboid - The volume(capacity) of a cuboid is given by:

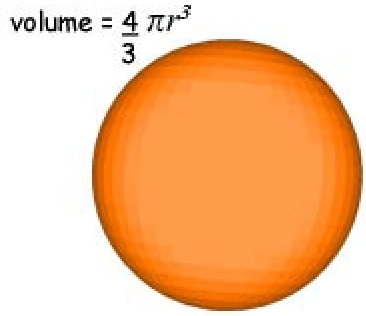



note: the sides of a cuboid are at right angles to eachother

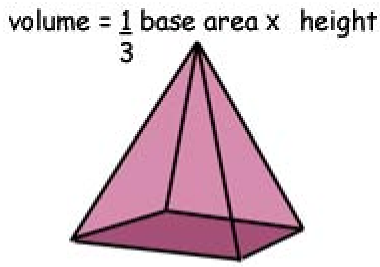
Prism - a shape with variable length and constant cross-sectional area



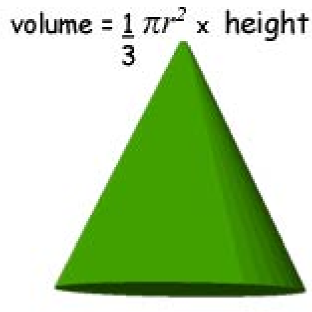
Sphere




Pyramid - The equation is independent of the number of sides of the pyramid. So the equation works equally for a tetrahedron(with a triangular base) and for other solids with differently shaped base areas.



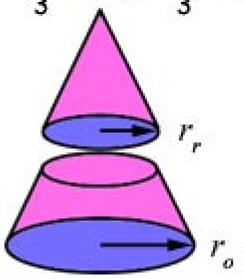

Cone - This is similar to the pyramid, the area of the circle being the base area.



<u>Frustrum</u> This is the part of a cone remaining when a top section of the cone, parallel to the base, is removed.

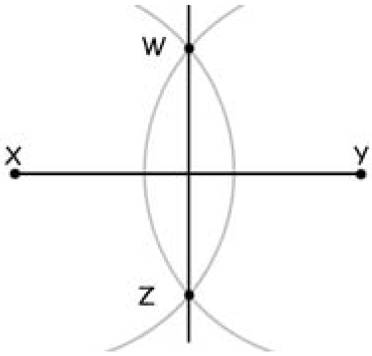
**volume of frustrum** = [**original vol. of cone**] - [**vol. of removed top section**]

$$\text{volume} = \frac{1}{3}\pi r_o^2 h_o - \frac{1}{3}\pi r_r^2 h_r$$



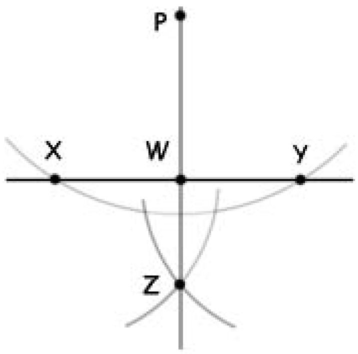
# Constructions

The perpendicular bisector of a line(also line equidistant from two points)



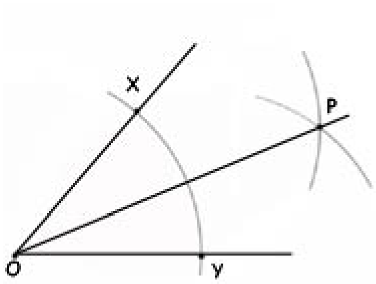

- set the radius of your compass to more than than half the length of XY(but less than XY)
- with centre X, draw an arc above and below the line XY
- with centre Y, draw an arc above and below the line XY intersecting the arcs from X
- the arcs intersect at points W and Z respectively above and below the line XY
- join the points W and Z
- the line WZ is the perpendicular bisector of XY

Perpendicular from a point to a line



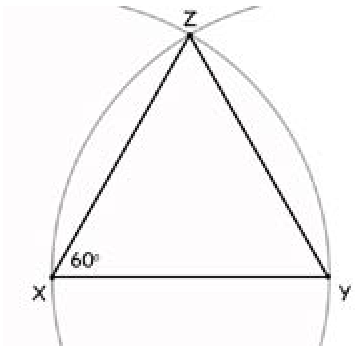

- set the radius of your compass so that an arc with centre P cuts the line at two points
- name these points of intersection X and Y
- with the radius greater than half XY and centre X draw an arc below the line XY
- repeat with centre Y
- where the arcs intersect call point Z
- the line joining Z to P is the perpendicular bisector of the line XY
- where this line meets XY call point W
- PW is the perpendicular from the point P to the line XY

Bisection of an angle



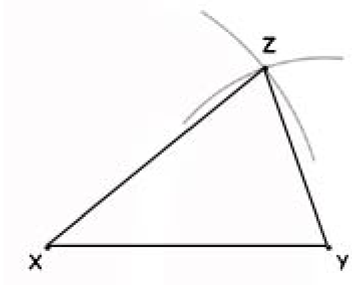

- with centre O draw arcs to cut the arms of the angle at X and Y
- using the same radius, from point X draw an arc between the arms of the angle
- repeat at point Y
- the two arcs intersect at point P
- draw a line between P and O
- PO is the bisector of the angle XOY

Construction of a 60 deg. angle (also of an equilateral triangle)




- draw a line XY
- with centre X and radius the length of the line, draw an arc above the line
- repeat from centre Y
- the point Z is where the arcs intersect
- join XZ
- join YZ
- angle ZXY is a 60 deg. angle

Construction of a triangle(sides different)




- draw a line XY of given length, as the base to the triangle
- with centre X and radius the length of the second side of the triangle, draw an arc above the line
- with centre Y and radius the length of the third side of the triangle, draw an arc above the line
- the point Z is where the arcs intersect
- join XZ
- join YZ

**notes**