## UNIT 8 (Extended) SHAPE AND SPACE

## Recommended Prior Knowledge

It is strongly recommended that candidates have a thorough knowledge and understanding of the topics in Unit 1, Unit 6 and Unit 7 .

## Context

This unit revises and develops mathematical concepts in Shape and Space. Candidates should use calculators where appropriate, however, it is recommended that regular non-calculator work is completed to strengthen candidates' mental arithmetic.

## Outline

The topics in this unit may be studied sequentially. There is some element of choice, however, and Centres may wish to teach topics in a different order. Fundamental properties of angles and geometrical shapes are studied together with solving trigonometrical problems in two and three dimensions. With all sections it is expected that candidates will be set questions of varying difficulty to complete for themselves. The unit gives candidates the opportunity to work investigatively and thus establish the skills needed for the submission of coursework.

| Learning Outcomes |  | Suggested Teaching Activities | Resources |
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| 26 | Use and interpret the geometrical terms: point, line, parallel, bearing, right angle, acute, obtuse and reflex angles, perpendicular, similarity, congruence; use and interpret vocabulary of triangles, quadrilaterals, circles and polygons. | This section is best covered as different topics are met. The geometrical terms can then be used in the context of solving problems. |  |
| 29 | Calculate unknown angles using the following geometrical properties: <br> (a) angles at a point, <br> (b) angles on a straight line and intersecting straight lines <br> (c) angles formed within parallel lines, <br> (d) angle properties of triangles and quadrilaterals, <br> (e) angle properties of regular polygons. <br> (f) angle properties of irregular polygons. | Revise basic angle properties by drawing simple diagrams which illustrate (a), (b) and (c). Define acute, obtuse and reflex angles; equilateral, isosceles and scalene triangles. <br> Define the terms (irregular) polygon and regular polygon. Use examples which include: triangles, quadrilaterals, pentagons, hexagons and octagons. <br> By dividing an $n$-sided polygon into a number of triangles show that the sum of the interior angles is 180( $n-2$ ) degrees. Show also that each exterior angle is $\frac{360^{\circ}}{n}$. Solve a variety of problems which use these formulae. Class activity: Draw a table of information for regular polygons. Use as headings: number of sides, name, exterior angle, sum of interior angles, interior angle. | Classifying angles at http://www.math.com/school/subject3/lessons/S3U1L4GL.html |
| 9 31 | Obtain appropriate upper and lower bounds to solutions of simple problems (e.g. the calculation of the perimeter or the area of a rectangle) given data to a specified accuracy. <br> Carry out calculations involving the perimeter and area of a | Extend the work on accuracy (Unit 6) to include calculating upper and lower bounds for various perimeters and areas, given lengths to a specified accuracy. <br> Revise, using straightforward examples, how to calculate | Calculating areas of parallelograms and trapeziums at http://www.bbc.co.uk/schools/gcsebitesize/maths/shapeih/areaan dvolumerev1.shtml |

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|  | rectangle and triangle, the circumference and area of a circle, the area of a parallelogram and a trapezium. <br> Solve problems involving the arc length and sector area as fractions of the circumference and area of a circle. | the circumference and area of a circle, and the perimeter and area of a rectangle and a triangle. Extend this to calculating the area of a parallelogram and a trapezium. <br> Class activity: Using isometric dot paper investigate the area of shapes that have a perimeter of $5,6,7, \ldots$. units. <br> Use straightforward examples to illustrate how to calculate arc length and the area of a sector. |  |
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| 32 | Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle ( angles will be quoted in, and answers required in, degrees and decimals to one decimal place). | Use simple examples involving the sine, cosine and tangent ratios to calculate the length of an unknown side of a rightangled triangle given an angle and the length of one side. <br> Use simple examples involving inverse ratios to calculate an unknown angle given the length of two sides of a rightangled triangle. <br> Re-state Pythagoras' theorem. <br> Class activity: Solve problems in context using Pythagoras' theorem and trigonometric ratios (include work with any shape that may be partitioned into right-angled triangles). <br> Class activity: Calculate the area of a segment of a circle given the radius and the sector angle. | Try the Degree Ceremony investigation at http://nrich.maths.org/public/leg.php |
| 32 | Interpret and use three-figure bearings measured clockwise from the north (i.e. $000^{\circ}-360^{\circ}$ ). | Discuss how bearings are measured and written. Use simple examples to show how to calculate bearings, e.g. calculate the bearing of $B$ from $A$ if you know the bearing of $A$ from $B$. <br> Class activity: Use a map to determine distance and direction between two places, etc. | Maps from around the world at http://www.theodora.com/maps/abc world maps.html |
| 32 | Solve trigonometrical problems in two dimensions involving angles of elevation and depression, extend sine and cosine functions to angles between $90^{\circ}$ and $180^{\circ}$, solve problems using the sine and cosine rules for any triangle and the formula area of triangle $=1 / 2 a b s i n C$; solve simple trigonometrical problems in three dimensions including angle between a line and a plane. | Draw a sine curve and discuss its properties. Use the curve to show, for example, $\sin 150^{\circ}=\sin 30^{\circ}$. Repeat for the cosine curve. <br> Define angles of elevation and depression. Use straightforward examples to illustrate how to solve problems using the sine and cosine rules. <br> Class activity: Solve two dimensional trigonometric problems in context. <br> Rearrange the formula for the area of a triangle $(1 / 2 b h)$ to the form $1 / 2 a b s i n C$. Illustrate its use with a few simple examples. <br> Introduce problems in three dimensions by finding the length of the diagonal of a cuboid and determining the angle it makes with the base. Extend by using more complex figures, e.g. a pyramid. | Various problems at http://nrich.maths.org/public/leg.php <br> Try the investigation at http://nrich.maths.org/public/leg.php |

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Use and interpret vocabulary of simple solid figures
Use the relationships between areas of similar triangles, with corresponding results for similar figures and extension to volumes and surface areas of similar solids

31 Carry out calculations involving the volume of a cuboid, prism and cylinder and the surface area of a cuboid and a cylinder.
Solve problems involving the surface area and volume of a sphere, pyramid and cone (given formulae for the sphere, pyramid and cone).

Illustrate common solids, e.g. cube, cuboid, tetrahedron cylinder, cone, sphere, prism, pyramid, etc. Define the terms vertex, edge and face.

Starting with simple examples draw the nets of various solids. Show, for example, that the net of a cube can be drawn in different ways.
Class activity: Draw nets on card and make various geometrical shapes.

Use nets to illustrate how to calculate the surface area of a cuboid, a triangular prism, a cylinder, a pyramid and a cone Show how to obtain the formula $\pi r(r+1)$ for the surface area of a cone. Calculate the surface area of a sphere using the formula $4 \pi r^{2}$.

Use straightforward examples to illustrate how to calculate the volume of various prisms (cross-sectional area $x$ length). Calculate the volume of a pyramid (including cone) using the formula $\frac{1}{3} x$ area of base $x$ perpendicular height. Calculate the volume of a sphere using the formula $\frac{4}{3} \pi r^{3}$.
Class activity: Find the surface area and volume of various composite shapes.

Class activity: An A4 sheet of paper can be rolled into a cylinder in two ways. Which gives the biggest volume? If the area of paper remains constant but the length and width can vary investigate what width and length gives the maximum cylinder volume.

Explore geometric solids and their properties a http://www.illuminations.nctm.org

## Try the dipstick investigation a

 http://www.ex.ac.uk/cimt/resource/dipstick.htm