

3 Sets H , M and P are defined by

$$\begin{aligned}H &= \{\text{students studying history}\}, \\M &= \{\text{students studying mathematics}\}, \\P &= \{\text{students studying physics}\}.\end{aligned}$$

Express the following statements in set notation.

- (i) No student studies both history and physics.
- (ii) All physics students also study mathematics.

Describe in words which students belong to the set

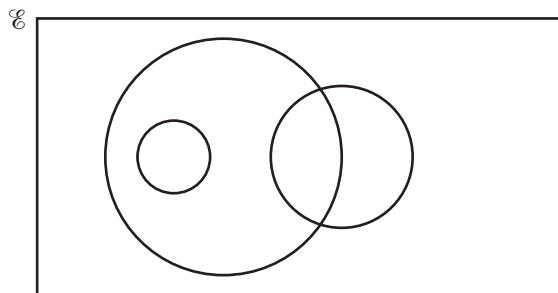
- (iii) $H' \cap M \cap P'$,
- (iv) $(H \cup M) \cap P'$.

[5]

8 The universal set \mathcal{E} and the sets O , P and S are given by

$$\begin{aligned}\mathcal{E} &= \{x : x \text{ is an integer such that } 3 \leq x \leq 100\}, \\O &= \{x : x \text{ is an odd number}\}, \\P &= \{x : x \text{ is a prime number}\}, \\S &= \{x : x \text{ is a perfect square}\}.\end{aligned}$$

In the Venn diagram below, each of the sets O , P and S is represented by a circle.



- (i) Copy the Venn diagram and label each circle with the appropriate letter. [2]
- (ii) Place each of the numbers 34, 35, 36 and 37 in the appropriate part of your diagram. [2]
- (iii) State the value of $n(O \cap S)$ and of $n(O \cup S)$. [3]

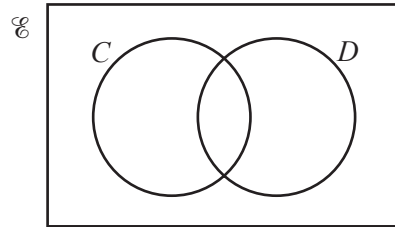
3 Given that $\mathcal{E} = \{\text{students in a college}\}$,
 $A = \{\text{students who are over 180 cm tall}\}$,
 $B = \{\text{students who are vegetarian}\}$,
 $C = \{\text{students who are cyclists}\}$,

express in words each of the following

- (i) $A \cap B \neq \emptyset$, (ii) $A \subset C'$. [2]

Express in set notation the statement

- (iii) all students who are both vegetarians and cyclists are not over 180 cm tall. [2]



The Venn diagram above represents the sets

$$\mathcal{E} = \{\text{homes in a certain town}\},$$

$$C = \{\text{homes with a computer}\},$$

$$D = \{\text{homes with a dishwasher}\}.$$

It is given that

$$n(C \cap D) = k,$$

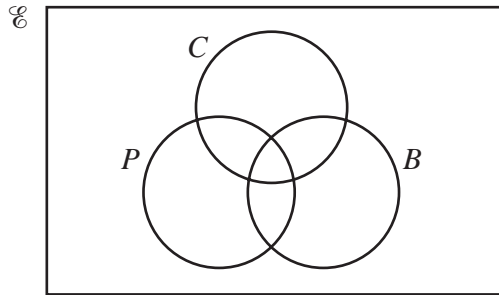
$$n(C) = 7 \times n(C \cap D),$$

$$n(D) = 4 \times n(C \cap D),$$

and $n(\mathcal{E}) = 6 \times n(C' \cap D').$

- (i) Copy the Venn diagram above and insert, in each of its four regions, the number, in terms of k , of homes represented by that region. [5]
- (ii) Given that there are 165 000 homes which do not have both a computer and a dishwasher, calculate the number of homes in the town. [2]

6 (a)



The Venn diagram above represents the universal set \mathcal{U} of all teachers in a college. The sets C , B and P represent teachers who teach Chemistry, Biology and Physics respectively. Sketch the diagram twice.

(i) On the first diagram shade the region which represents those teachers who teach Physics and Chemistry but not Biology. [1]

(ii) On the second diagram shade the region which represents those teachers who teach either Biology or Chemistry or both, but not Physics. [1]

(b) In a group of 20 language teachers, F is the set of teachers who teach French and S is the set of teachers who teach Spanish. Given that $n(F) = 16$ and $n(S) = 10$, state the maximum and minimum possible values of

(i) $n(F \cap S)$,

(ii) $n(F \cup S)$.

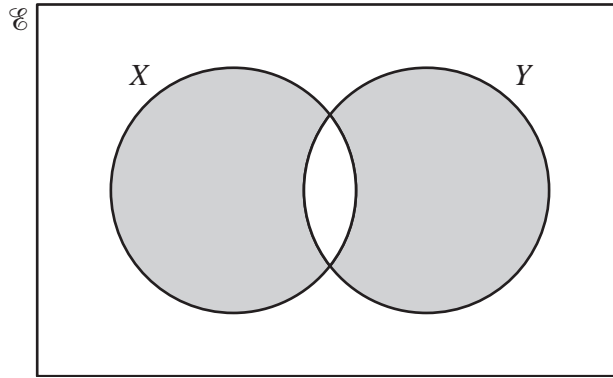
[4]

2 (a) Illustrate the following statements using a separate Venn diagram for each.

(i) $A \cap B = \emptyset$, (ii) $(C \cup D) \subset E$.

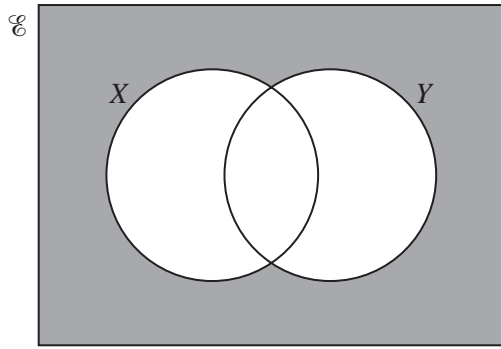
[2]

(b)



Express, in set notation, the set represented by the shaded region.

[2]



Express, in set notation, the set represented by the shaded region. [1]

(b) In a class of 30 students, 17 are studying politics, 14 are studying economics and 10 are studying both of these subjects.

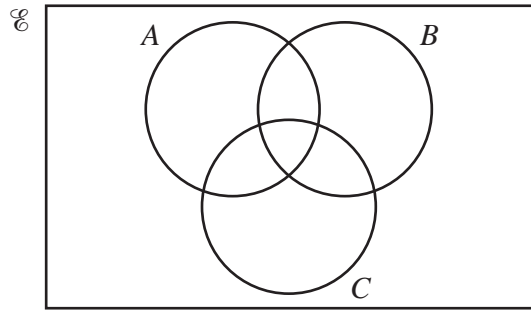
(i) Illustrate this information using a Venn diagram. [1]

Find the number of students studying

(ii) neither of these subjects, [1]

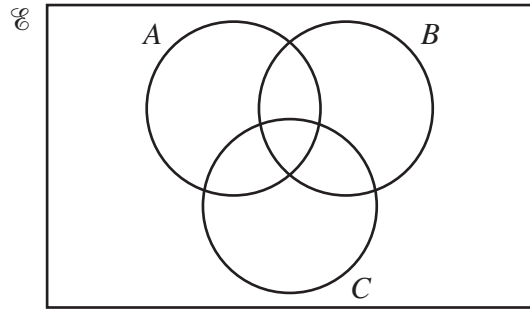
(iii) exactly one of these subjects. [1]

4 (a)



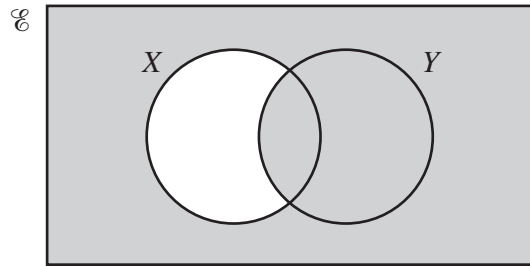
- (i) Copy the Venn diagram above and shade the region that represents $(A \cap B) \cup C$. [1]
- (ii) Copy the Venn diagram above and shade the region that represents $A' \cap B'$. [1]
- (iii) Copy the Venn diagram above and shade the region that represents $(A \cup B) \cap C$. [1]
- (b) It is given that the universal set $\mathcal{U} = \{x : 2 \leq x \leq 20, x \text{ is an integer}\}$,
 $X = \{x : 4 < x < 15, x \text{ is an integer}\}$,
 $Y = \{x : x \geq 9, x \text{ is an integer}\}$,
 $Z = \{x : x \text{ is a multiple of } 5\}$.
- (i) List the elements of $X \cap Y$. [1]
- (ii) List the elements of $X \cup Y$. [1]
- (iii) Find $(X \cup Y)' \cap Z$. [1]

2 (a)



Copy the diagram and shade the region which represents the set $A \cup (B \cap C')$. [1]

(b)



Express, in set notation, the set represented by the shaded region. [1]

(c) The universal set \mathcal{E} and the sets P and Q are such that $n(\mathcal{E}) = 30$, $n(P) = 18$ and $n(Q) = 16$. Given that $n(P \cup Q)' = 2$, find $n(P \cap Q)$. [2]

3 The volume $V\text{cm}^3$ of a spherical ball of radius $r\text{cm}$ is given by $V = \frac{4}{3}\pi r^3$. Given that the radius is increasing at a constant rate of $\frac{1}{\pi}\text{ cm s}^{-1}$, find the rate at which the volume is increasing when $V = 288\pi$. [4]

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.