QUESTION BANK

2002 – 2011

ADDITONAL MATHEMATICS

Compiled & Edited
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1 Find the coordinates of the points at which the straight line $y + 2x = 7$ intersects the curve $y^2 = xy - 1$. [4]

2 A curve has gradient $e^{4x} + e^{-x}$ at the point (x, y). Given that the curve passes through the point $(0, 3)$, find the equation of the curve. [4]

3 A rectangle has sides of length $(2 + \sqrt{18})$ metres and $(5 - \frac{4}{\sqrt{2}})$ metres. Express, in the form $a + b\sqrt{2}$, where a and b are integers,

(i) the value of A, where A square metres is the area of the rectangle,

(ii) the value of D^2, where D metres is the length of the diagonal of the rectangle.

[5]

4 The points P, Q and R are such that $\overrightarrow{QR} = 4\overrightarrow{PQ}$. Given that the position vectors of P and Q relative to an origin O are $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ 20 \end{pmatrix}$ respectively, find the unit vector parallel to $\overrightarrow{OR}$. [5]

5 **(i)** Sketch, on the same diagram and for $0 \leqslant x \leqslant 2\pi$, the graphs of $y = \frac{1}{4} + \sin x$ and $y = \frac{1}{2}\cos 2x$. [4]

(ii) The x-coordinates of the points of intersection of the two graphs referred to in part **(i)** satisfy the equation $2\cos 2x - k \sin x = 1$. Find the value of k. [2]

6 **(a)** Calculate the number of different 6-digit numbers which can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition and assuming that a number cannot begin with 0. [2]

(b) A committee of 4 people is to be chosen from 4 women and 5 men. The committee must contain at least 1 woman. Calculate the number of different committees that can be formed. [4]

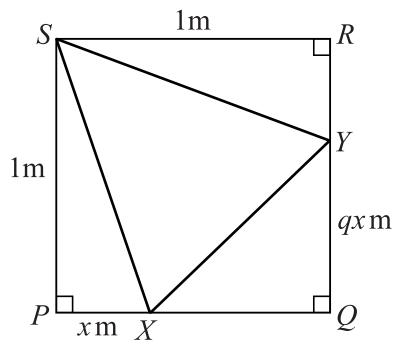
7 Obtain

(i) the expansion, in ascending powers of x, of $(2 - x^2)^5$,

(ii) the coefficient of x^6 in the expansion of $(1 + x^2)^2(2 - x^2)^5$.

[6]

8



The diagram shows a square $PQRS$ of side 1 m. The points X and Y lie on PQ and QR respectively such that $PX = x$ m and $QY = qx$ m, where q is a constant such that $q > 1$.

(i) Given that the area of triangle SXY is A m^2, show that

$$A = \tfrac{1}{2}(1 - x + qx^2).$$ [3]

(ii) Given that x can vary, show that $QY = YR$ when A is a minimum and express the minimum value of A in terms of q. [4]

9 Given that $y = (x - 5)\sqrt{2x+5}$,

(i) show that $\frac{dy}{dx}$ can be written in the form $\frac{kx}{\sqrt{2x+5}}$ and state the value of k, [4]

(ii) find the approximate change in y as x decreases from 10 to $10 - p$, where p is small, [2]

(iii) find the rate of change of x when $x = 10$, if y is changing at the rate of 3 units per second at this instant. [2]

10 A toothpaste firm supplies tubes of toothpaste to 5 different stores. The number of tubes of toothpaste supplied per delivery to each store, the sizes and sale prices of the tubes, together with the number of deliveries made to each store over a 3-month period are shown in the table below.

		Number of tubes per delivery			Number of deliveries over 3 months
Size of tube		50 ml	75 ml	100 ml	
Name of store	Alwin	400	300	400	13
	Bestbuy	–	–	600	7
	Costless	400	–	600	10
	Dealwise	500	300	–	5
	Econ	600	600	400	8
Sale price per tube		$2.10	$3.00	$3.75	

(i) Write down two matrices such that the elements of their product under matrix multiplication would give the volume of toothpaste supplied to each store per delivery.

(ii) Write down two matrices such that the elements of their product under matrix multiplication would give the number of tubes of toothpaste of each size supplied by the firm over the 3-month period. Find this product.

(iii) Using the matrix product found in part **(ii)** and a further matrix, find the total amount of money which would be obtained from the sale of all the tubes of toothpaste delivered over the 3-month period.

[8]

11 Express $2x^2 - 8x + 5$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 8x + 5$ for the domain $0 \leqslant x \leqslant 5$.

(i) Find the range of f.

(ii) Explain why f does not have an inverse.

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 8x + 5$ for the domain $x \geqslant k$.

(iii) Find the smallest value of k for which g has an inverse.

(iv) For this value of k, find an expression for g^{-1}.

[10]

[Turn over

12 Answer only **one** of the following two alternatives.

EITHER

(a) The curve $y = ax^n$, where a and n are constants, passes through the points (2.25, 27), (4, 64) and (6.25, p). Calculate the value of a, of n and of p. [5]

(b) The mass, m grams, of a radioactive substance is given by the formula $m = m_0 e^{-kt}$, where t is the time in days after the mass was first recorded and m_0 and k are constants.

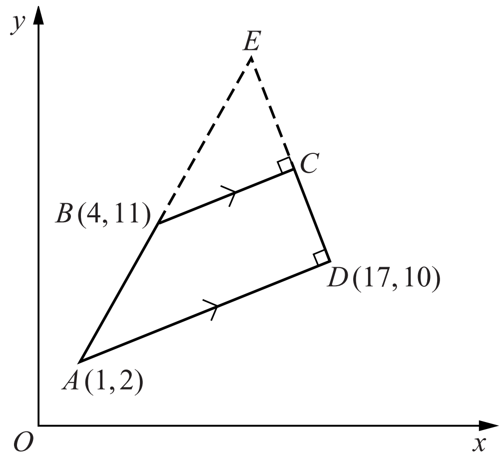
The table below gives experimental values of t and m.

t (days)	10	20	30	40	50
m (grams)	40.2	27.0	18.0	12.2	8.1

Plot ln m against t and use your graph to estimate the value of m_0 and of k. [6]

OR

Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a trapezium $ABCD$ in which BC is parallel to AD. The side AD is perpendicular to DC. Point A is (1, 2), B is (4, 11) and D is (17, 10). Find

(i) the coordinates of C.

The lines AB and DC are extended to meet at E. Find

(ii) the coordinates of E,

(iii) the ratio of the area of triangle EBC to the area of trapezium $ABCD$.

[11]

1 It is given that $\mathbf{A} = \begin{pmatrix} 5 & 7 \\ 4 & 5 \end{pmatrix}$ and that $\mathbf{A} - 3\mathbf{A}^{-1} - k\mathbf{I} = \mathbf{0}$, where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ is the zero matrix. Evaluate k. [4]

2 **(i)** Sketch, on the same diagram, the graphs of $y = | x | + 1$ and $y = | 2x - 3 |$.

(ii) State the number of solutions of the equation $| 2x - 3 | = | x | + 1$.

[4]

3 Sets H, M and P are defined by

H = {students studying history},
M = {students studying mathematics},
P = {students studying physics}.

Express the following statements in set notation.

(i) No student studies both history and physics.

(ii) All physics students also study mathematics.

Describe in words which students belong to the set

(iii) $H' \cap M \cap P'$,

(iv) $(H \cup M) \cap P'$.

[5]

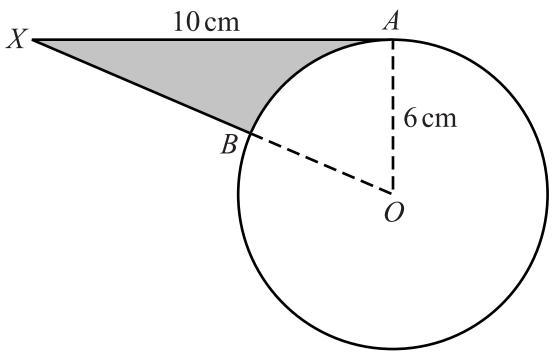
4 Solve the equation $x^3 - 4x^2 - 11x + 2 = 0$, expressing non-integer solutions in the form $a \pm b\sqrt{2}$, where a and b are integers. [6]

5 A plane flies from an airport A to an airport B. The position vector of B relative to A is $(1200\mathbf{i} + 240\mathbf{j})$ km, where $\mathbf{i}$ is a unit vector due east and $\mathbf{j}$ is a unit vector due north. Because of the constant wind which is blowing, the flight takes 4 hours. The velocity in still air of the plane is $(250\mathbf{i} + 160\mathbf{j})$ kmh^{-1}. Find the speed of the wind and the bearing of the direction from which the wind is blowing. [6]

6 **(i)** Show that $\dfrac{d}{dx}\left(\dfrac{\cos x}{1 - \sin x}\right)$ can be written in the form $\dfrac{k}{1 - \sin x}$ and state the value of k. [4]

(ii) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sqrt{2}}{1 - \sin x}\,dx$. [3]

7



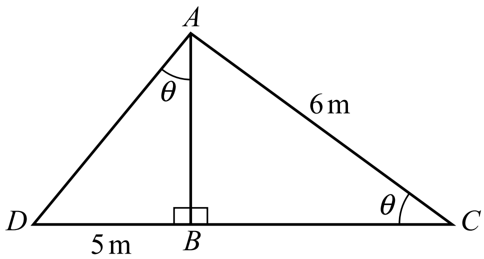
The diagram shows a circle, centre O and radius 6 cm. The tangent from X touches the circle at A and $XA = 10$ cm. The line from X to O cuts the circle at B.

(i) Show that angle AOB is approximately 1.03 radians. [1]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

8



In the diagram, angle ABC = angle $ABD = 90°$, $AC = 6$ m, $BD = 5$ m and angle ACB = angle $DAB = \theta$.

(i) Use each of the triangles ABC and ABD to express AB in terms of θ. [2]

(ii) Hence evaluate θ. [5]

9 **(a)** Find the value of k for which the line $y = x + k$ is a tangent to the curve $y^2 = 4x + 8$. [4]

(b) Find the value of a and of b for which the solution set of the quadratic inequality $x^2 + ax > b$ is $\{x : x > 2\} \cup \{x : x < -4\}$. [3]

10 Solve the equation

(i) $\lg(2x) - \lg(x - 3) = 1$, [3]

(ii) $\log_3 y + 4\log_y 3 = 4$. [4]

11 Functions f and g are defined, for $x \in \mathbb{R}$, by

$$\mathrm{f} : x \mapsto 3x - 7,$$

$$\mathrm{g} : x \mapsto \frac{12}{x-2}, \quad x \neq 2.$$

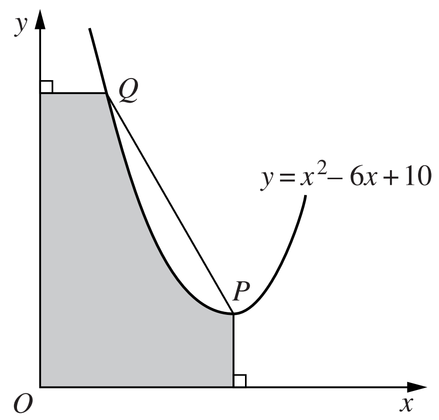
(i) Find f^{-1} and g^{-1} in terms of x, stating the value of x for which g^{-1} is not defined. [3]

(ii) Find the values of x for which $\mathrm{fg}(x) = x$. [3]

(iii) Sketch the graphs of f and f^{-1} on the same diagram, giving the coordinates of the points of intersection of each graph with the axes. [3]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows part of the curve $y = x^2 - 6x + 10$ passing through the points P and Q. The curve has a minimum point at P and the gradient of the line PQ is –2. Calculate the area of the shaded region. [11]

OR

A particle travels in a straight line, starting from rest at point A, passing through point B and coming to rest again at point C. The particle takes 5 s to travel from A to B with constant acceleration. The motion of the particle from B to C is such that its speed, v ms^{-1}, t seconds after leaving A, is given by

$$v = \frac{1}{225}(20 - t)^3 \quad \text{for } 5 \leqslant t \leqslant T.$$

(i) Find the speed of the particle at B and the value of T.

(ii) Find the acceleration of the particle when $t = 14$.

(iii) Sketch the velocity-time curve for $0 \leqslant t \leqslant T$.

(iv) Calculate the distance AC.

[11]

1 Find the values of k for which the line $y = kx - 2$ meets the curve $y^2 = 4x - x^2$. [4]

2 The area of a rectangle is $(1 + \sqrt{6})\text{m}^2$. The length of one side is $(\sqrt{3} + \sqrt{2})\text{m}$. Find, without using a calculator, the length of the other side in the form $\sqrt{a} - \sqrt{b}$, where a and b are integers. [4]

3 **(i)** Find the first 3 terms in the expansion, in ascending powers of x, of $(2 - x)^5$. [3]

(ii) Hence find the value of the constant k for which the coefficient of x in the expansion of $(k + x)(2 - x)^5$ is -8. [2]

4 An ocean liner is travelling at $36\ \text{km h}^{-1}$ on a bearing of 090°. At 0600 hours the liner, which is 90 km from a lifeboat and on a bearing of 315° from the lifeboat, sends a message for assistance. The lifeboat sets off immediately and travels in a straight line at constant speed, intercepting the liner at 0730 hours. Find the speed at which the lifeboat travels. [5]

5 Find the distance between the points of intersection of the curve $y = 3 + \frac{4}{x}$ and the line $y = 4x + 9$. [6]

6 Given that $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$, find $\mathbf{B}$ such that $4\mathbf{A}^{-1} + \mathbf{B} = \mathbf{A}^2$. [6]

7 The function f is defined, for $0° \leqslant x \leqslant 360°$, by $\text{f}(x) = 4 - \cos 2x$.

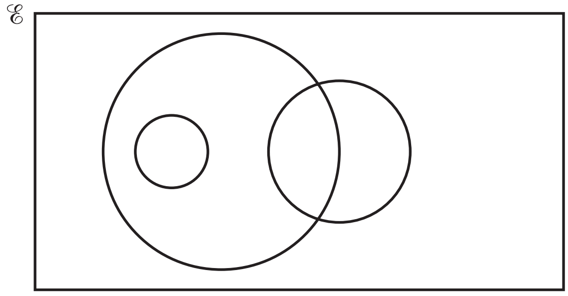
(i) State the amplitude and period of f. [2]

(ii) Sketch the graph of f, stating the coordinates of the maximum points. [4]

8 The universal set $\mathscr{E}$ and the sets O, P and S are given by

$\mathscr{E} = \{x : x \text{ is an integer such that } 3 \leqslant x \leqslant 100\}$,
$O = \{x : x \text{ is an odd number}\}$,
$P = \{x : x \text{ is a prime number}\}$,
$S = \{x : x \text{ is a perfect square}\}$.

In the Venn diagram below, each of the sets O, P and S is represented by a circle.



(i) Copy the Venn diagram and label each circle with the appropriate letter. [2]

(ii) Place each of the numbers 34, 35, 36 and 37 in the appropriate part of your diagram. [2]

(iii) State the value of $\text{n}(O \cap S)$ and of $\text{n}(O \cup S)$. [3]

9 Solve

(i) $\log_4 2 + \log_9 (2x + 5) = \log_8 64$, [4]

(ii) $9^y + 5(3^y - 10) = 0$. [4]

10

x	2	3	4	5	6
y	9.2	8.8	9.4	10.4	11.6

The table above shows experimental values of the variables x and y. On graph paper draw the graph of xy against x^2. [3]

Hence

(i) express y in terms of x, [4]

(ii) find the value of x for which $x = \dfrac{45}{y}$. [2]

11 A curve has the equation $y = xe^{2x}$.

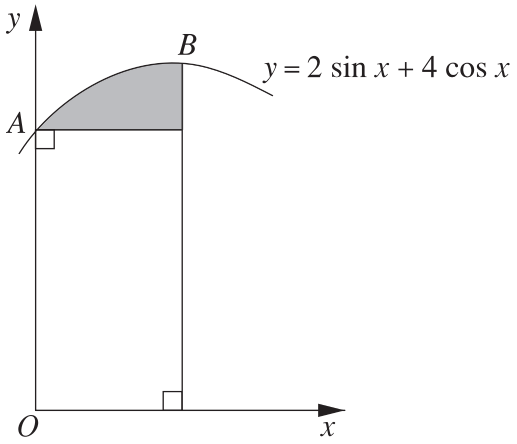
(i) Find the x-coordinate of the turning point of the curve. [4]

(ii) Find the value of k for which $\dfrac{d^2y}{dx^2} = ke^{2x}(1 + x)$. [3]

(iii) Determine whether the turning point is a maximum or a minimum. [2]

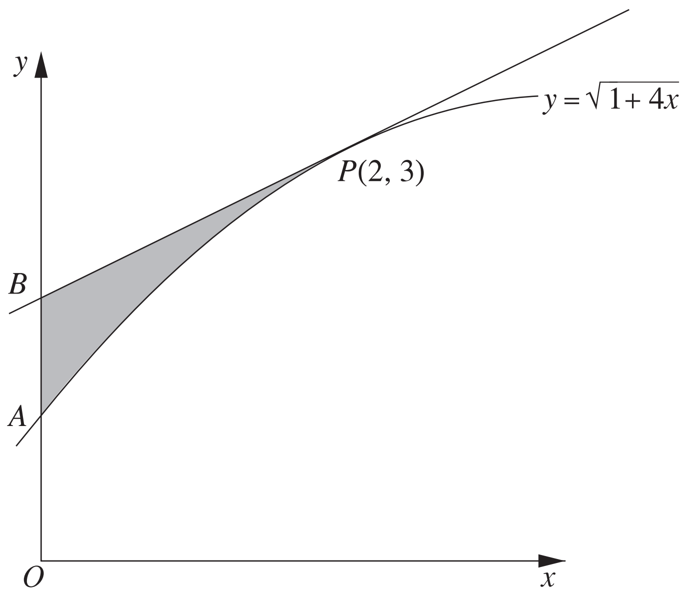
12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows part of the curve $y = 2 \sin x + 4 \cos x$, intersecting the y-axis at A and with its maximum point at B. A line is drawn from A parallel to the x-axis and a line is drawn from B parallel to the y-axis. Find the area of the shaded region. [11]

OR



The diagram shows part of the curve $y = \sqrt{1 + 4x}$, intersecting the y-axis at A. The tangent to the curve at the point $P(2, 3)$ intersects the y-axis at B. Find the area of the shaded region ABP. [11]

1 Given that $4x^4 - 12x^3 - b^2x^2 - 7bx - 2$ is exactly divisible by $2x + b$,

(i) show that $3b^3 + 7b^2 - 4 = 0$, [2]

(ii) find the possible values of b. [5]

2 The position vectors of points A and B, relative to an origin O, are $6\mathbf{i} - 3\mathbf{j}$ and $15\mathbf{i} + 9\mathbf{j}$ respectively.

(i) Find the unit vector parallel to $\overrightarrow{AB}$. [3]

The point C lies on AB such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

(ii) Find the position vector of C. [3]

3 Express $\displaystyle\int_1^8 \left(3\sqrt{x} + \frac{2}{\sqrt{x}}\right) \mathrm{d}x$ in the form $a + b\sqrt{2}$, where a and b are integers. [6]

4 Simplify $\dfrac{16^{x+1} + 20(4^{2x})}{2^{x-3}8^{x+2}}$. [4]

5 A function f is defined by $\mathrm{f} : x \mapsto \dfrac{\mathrm{e}^x + 1}{4}$ for the domain $x \geqslant 0$.

(i) Evaluate $\mathrm{f}^2(0)$. [3]

(ii) Obtain an expression for f^{-1}. [2]

(iii) State the domain and the range of f^{-1}. [2]

6 Find the solution set of the quadratic inequality

(i) $x^2 - 8x + 12 > 0$, [3]

(ii) $x^2 - 8x < 0$. [2]

Hence find the solution set of the inequality $|x^2 - 8x + 6| < 6$. [2]

7 **(i)** Find the number of different arrangements of the letters of the word MEXICO.

Find the number of these arrangements

(ii) which begin with M,

(iii) which have the letter X at one end and the letter C at the other end. [5]

Four of the letters of the word MEXICO are selected at random. Find the number of different combinations if

(iv) there is no restriction on the letters selected,

(v) the letter M must be selected. [3]

8 **(a)** Find all the angles between 0° and 360° which satisfy the equation

$$3(\sin x - \cos x) = 2(\sin x + \cos x).$$ [4]

(b) Find all the angles between 0 and 3 radians which satisfy the equation

$$1 + 3\cos^2 y = 4\sin y.$$ [4]

9 A motorcyclist travels on a straight road so that, t seconds after leaving a fixed point, his velocity, $v\,\mathrm{ms}^{-1}$, is given by $v = 12t - t^2$. On reaching his maximum speed at $t = 6$, the motorcyclist continues at this speed for another 6 seconds and then comes to rest with a constant deceleration of $4\,\mathrm{ms}^{-2}$.

(i) Find the total distance travelled. [6]

(ii) Sketch the velocity-time graph for the whole of the motion. [2]

10 A curve has the equation $y = \dfrac{2x + 4}{x - 2}$.

(i) Find the value of k for which $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k}{(x-2)^2}$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

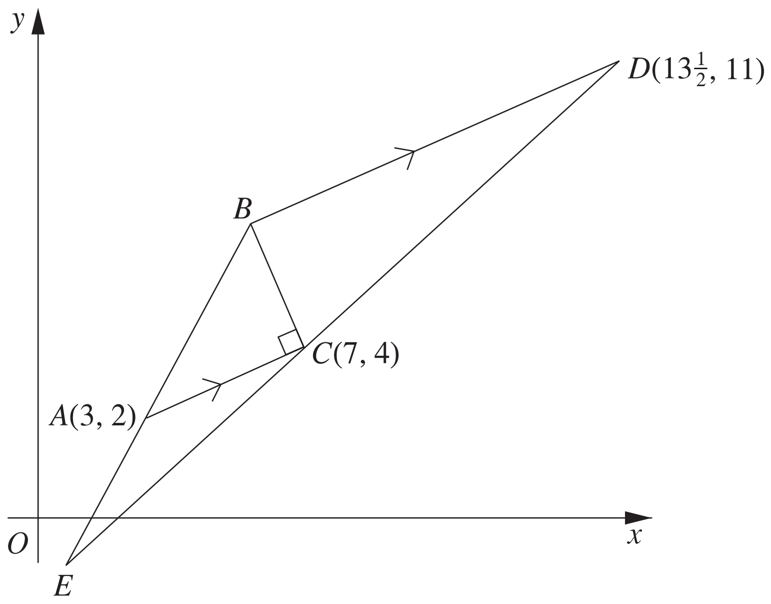
A point (x, y) moves along the curve in such a way that the x-coordinate of the point is increasing at a constant rate of 0.05 units per second.

(iii) Find the corresponding rate of change of the y-coordinate at the instant that $y = 6$. [3]

11 Answer only **one** of the following two alternatives.

EITHER

Solutions to this question by accurate drawing will not be accepted.

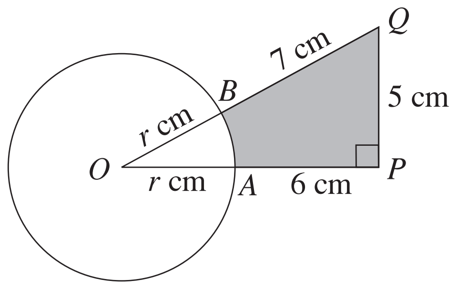


The diagram shows a triangle ABC in which A is the point $(3, 2)$, C is the point $(7, 4)$ and angle $ACB = 90°$. The line BD is parallel to AC and D is the point $(13\frac{1}{2}, 11)$. The lines BA and DC are extended to meet at E. Find

(i) the coordinates of B, [7]

(ii) the ratio of the area of the quadrilateral $ABDC$ to the area of the triangle EBD. [3]

OR



The diagram shows a right-angled triangle OPQ and a circle, centre O and radius r cm, which cuts OP and OQ at A and B respectively. Given that $AP = 6$ cm, $PQ = 5$ cm, $QB = 7$ cm and angle $OPQ = 90°$, find

(i) the length of the arc AB, [6]

(ii) the area of the shaded region. [4]

1 Given that $y = \dfrac{3x - 2}{x^2 + 5}$, find

(i) an expression for $\dfrac{dy}{dx}$,

(ii) the x-coordinates of the stationary points. [4]

2 Find the x-coordinates of the three points of intersection of the curve $y = x^3$ with the line $y = 5x - 2$, expressing non-integer values in the form $a \pm \sqrt{b}$, where a and b are integers. [5]

3 **(i)** Sketch on the same diagram the graphs of $y = |2x + 3|$ and $y = 1 - x$. [3]

(ii) Find the values of x for which $x + |2x + 3| = 1$. [3]

4 The function f is defined, for $0° \leqslant x \leqslant 360°$, by

$$f(x) = a \sin(bx) + c,$$

where a, b and c are positive integers. Given that the amplitude of f is 2 and the period of f is 120°,

(i) state the value of a and of b. [2]

Given further that the minimum value of f is -1,

(ii) state the value of c, [1]

(iii) sketch the graph of f. [3]

5 The straight line $5y + 2x = 1$ meets the curve $xy + 24 = 0$ at the points A and B. Find the length of AB, correct to one decimal place. [6]

6 The table below shows

the daily production, in kilograms, of two types, S_1 and S_2, of sweets from a small company,

the percentages of the ingredients A, B and C required to produce S_1 and S_2.

	Percentage			Daily production (kg)
	A	B	C	
Type S_1	60	30	10	300
Type S_2	50	40	10	240

Given that the costs, in dollars per kilogram, of A, B and C are 4, 6 and 8 respectively, use matrix multiplication to calculate the total cost of daily production. [6]

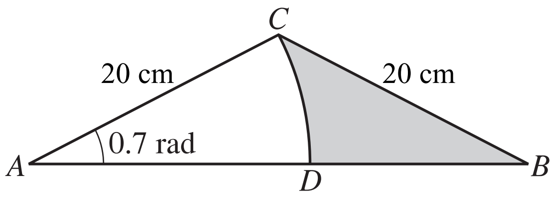
7 To a cyclist travelling due south on a straight horizontal road at $7\,\text{ms}^{-1}$, the wind appears to be blowing from the north-east. Given that the wind has a constant speed of $12\,\text{ms}^{-1}$, find the direction from which the wind is blowing. [5]

8 A curve has the equation $y = (ax + 3)\ln x$, where $x > 0$ and a is a positive constant. The normal to the curve at the point where the curve crosses the x-axis is parallel to the line $5y + x = 2$. Find the value of a. [7]

9 **(a)** Calculate the term independent of x in the binomial expansion of $\left(x - \dfrac{1}{2x^5}\right)^{18}$. [3]

(b) In the binomial expansion of $(1 + kx)^n$, where $n \geqslant 3$ and k is a constant, the coefficients of x^2 and x^3 are equal. Express k in terms of n. [4]

10

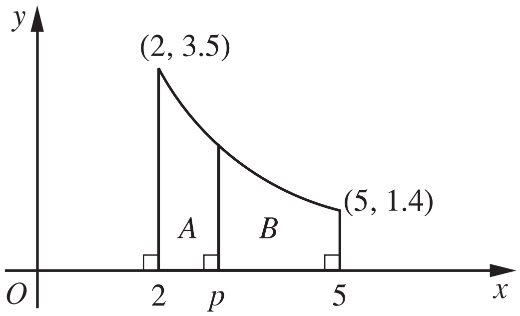


The diagram shows an isosceles triangle ABC in which $BC = AC = 20$ cm, and angle $BAC = 0.7$ radians. DC is an arc of a circle, centre A. Find, correct to 1 decimal place,

(i) the area of the shaded region, [4]

(ii) the perimeter of the shaded region. [4]

11



The diagram shows part of a curve, passing through the points (2, 3.5) and (5, 1.4). The gradient of the curve at any point (x, y) is $-\dfrac{a}{x^3}$, where a is a positive constant.

(i) Show that $a = 20$ and obtain the equation of the curve. [5]

The diagram also shows lines perpendicular to the x-axis at $x = 2$, $x = p$ and $x = 5$. Given that the areas of the regions A and B are equal,

(ii) find the value of p. [5]

12 Answer only **one** of the following two alternatives.

EITHER

(a) An examination paper contains 12 different questions of which 3 are on trigonometry, 4 are on algebra and 5 are on calculus. Candidates are asked to answer 8 questions. Calculate

(i) the number of different ways in which a candidate can select 8 questions if there is no restriction,

(ii) the number of these selections which contain questions on only 2 of the 3 topics, trigonometry, algebra and calculus.

[4]

(b) A fashion magazine runs a competition, in which 8 photographs of dresses are shown, lettered A, B, C, D, E, F, G and H. Competitors are asked to submit an arrangement of 5 letters showing their choice of dresses in descending order of merit. The winner is picked at random from those competitors whose arrangement of letters agrees with that chosen by a panel of experts.

(i) Calculate the number of possible arrangements of 5 letters chosen from the 8.

Calculate the number of these arrangements

(ii) in which A is placed first,

(iii) which contain A.

[6]

OR

The table shows experimental values of the variables x and y which are related by the equation $y = Ab^x$, where A and b are constants.

x	2	4	6	8	10
y	9.8	19.4	37.4	74.0	144.4

(i) Use the data above in order to draw, on graph paper, the straight line graph of lg y against x, using 1 cm for 1 unit of x and 10 cm for 1 unit of lg y. [2]

(ii) Use your graph to estimate the value of A and of b. [5]

(iii) On the same diagram, draw the straight line representing $y = 2^x$ and hence find the value of x for which $Ab^x = 2^x$. [3]

1 The position vectors of the points A and B, relative to an origin O, are $\mathbf{i} - 7\mathbf{j}$ and $4\mathbf{i} + k\mathbf{j}$ respectively, where k is a scalar. The unit vector in the direction of $\overrightarrow{AB}$ is $0.6\mathbf{i} + 0.8\mathbf{j}$. Find the value of k. [4]

2 Given that x is measured in radians and $x > 10$, find the smallest value of x such that

$$10\cos\left(\frac{x+1}{2}\right) = 3.$$ [4]

3 Given that $\mathscr{E}$ = {students in a college},
A = {students who are over 180 cm tall},
B = {students who are vegetarian},
C = {students who are cyclists},
express in words each of the following

(i) $A \cap B \neq \varnothing$, **(ii)** $A \subset C'$. [2]

Express in set notation the statement

(iii) all students who are both vegetarians and cyclists are not over 180 cm tall. [2]

4 Prove the identity $(1 + \sec\theta)(\operatorname{cosec}\theta - \cot\theta) \equiv \tan\theta$. [4]

5 The roots of the quadratic equation $x^2 - \sqrt{20}x + 2 = 0$ are c and d. Without using a calculator, show that $\frac{1}{c} + \frac{1}{d} = \sqrt{5}$. [5]

6 **(a)** Find the values of x for which $2x^2 > 3x + 14$. [3]

(b) Find the values of k for which the line $y + kx = 8$ is a tangent to the curve $x^2 + 4y = 20$. [3]

7 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \mathrm{e}^x,$$

$$\mathrm{g} : x \mapsto 2x - 3.$$

(i) Solve the equation $\mathrm{fg}(x) = 7$. [2]

Function h is defined as gf.

(ii) Express h in terms of x and state its range. [2]

(iii) Express h^{-1} in terms of x. [2]

8 Solve

(i) $\log_3(2x + 1) = 2 + \log_3(3x - 11)$, [4]

(ii) $\log_4 y + \log_2 y = 9$. [4]

9 Express $6 + 4x - x^2$ in the form $a - (x + b)^2$, where a and b are integers. [2]

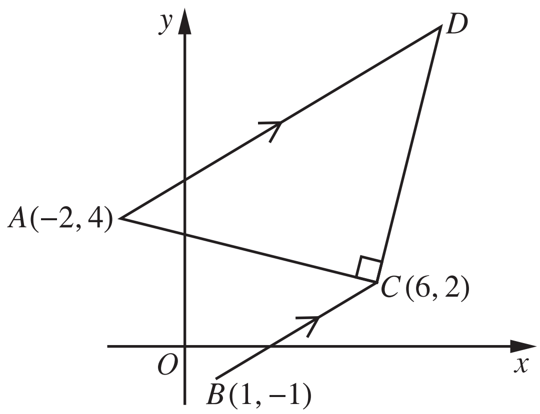
(i) Find the coordinates of the turning point of the curve $y = 6 + 4x - x^2$ and determine the nature of this turning point. [3]

The function f is defined by $\mathrm{f} : x \mapsto 6 + 4x - x^2$ for the domain $0 \leqslant x \leqslant 5$.

(ii) Find the range of f. [2]

(iii) State, giving a reason, whether or not f has an inverse. [1]

10 Solutions to this question by accurate drawing will not be accepted.



In the diagram the points A, B and C have coordinates $(-2, 4)$, $(1, -1)$ and $(6, 2)$ respectively. The line AD is parallel to BC and angle $ACD = 90°$.

(i) Find the equations of AD and CD. [6]

(ii) Find the coordinates of D. [2]

(iii) Show that triangle ACD is isosceles. [2]

11 It is given that $y = (x + 1)(2x - 3)^{3/2}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $kx\sqrt{2x - 3}$ and state the value of k. [4]

Hence

(ii) find, in terms of p, an approximate value of y when $x = 6 + p$, where p is small, [3]

(iii) evaluate $\displaystyle\int_2^6 x\sqrt{2x - 3}\,\mathrm{d}x$. [3]

12 Answer only **one** of the following two alternatives.

EITHER

A particle moves in a straight line so that, t s after leaving a fixed point O, its velocity, $v\ \mathrm{ms}^{-1}$, is given by $v = 10(1 - e^{-\frac{1}{2}t})$.

(i) Find the acceleration of the particle when $v = 8$. [4]

(ii) Calculate, to the nearest metre, the displacement of the particle from O when $t = 6$. [4]

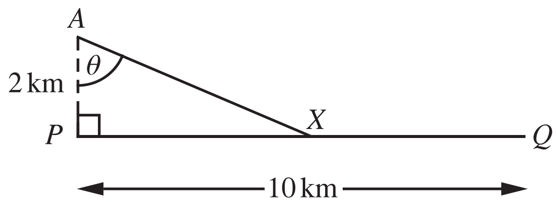
(iii) State the value which v approaches as t becomes very large. [1]

(iv) Sketch the velocity-time graph for the motion of the particle. [2]

OR

(i) By considering $\sec\theta$ as $(\cos\theta)^{-1}$ show that $\dfrac{d}{d\theta}(\sec\theta) = \dfrac{\sin\theta}{\cos^2\theta}$. [2]

(ii) The diagram shows a straight road joining two points, P and Q, 10 km apart. A man is at point A, where AP is perpendicular to PQ and AP is 2 km. The man wishes to reach Q as quickly as possible and travels across country in a straight line to meet the road at point X, where angle $PAX = \theta$ radians.



The man travels across country along AX at 3 km h^{-1} but on reaching the road he travels at 5 km h^{-1} along XQ. Given that he takes T hours to travel from A to Q, show that

$$T = \frac{2\sec\theta}{3} + 2 - \frac{2\tan\theta}{5}.$$ [4]

(iii) Given that θ can vary, show that T has a stationary value when $PX = 1.5$ km. [5]

1 Given that $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$, find $(\mathbf{A}^2)^{-1}$. [4]

2 A student has a collection of 9 CDs, of which 4 are by the Beatles, 3 are by Abba and 2 are by the Rolling Stones. She selects 4 of the CDs from her collection. Calculate the number of ways in which she can make her selection if

(i) her selection must contain her favourite Beatles CD, [2]

(ii) her selection must contain 2 CDs by one group and 2 CDs by another. [3]

3 Given that θ is acute and that $\sin\theta = \frac{1}{\sqrt{3}}$, express, without using a calculator, $\frac{\sin\theta}{\cos\theta - \sin\theta}$ in the form $a + \sqrt{b}$, where a and b are integers. [5]

4 The position vectors of points A and B relative to an origin O are $-3\mathbf{i} - \mathbf{j}$ and $\mathbf{i} + 2\mathbf{j}$ respectively. The point C lies on AB and is such that $\overrightarrow{AC} = \frac{3}{5}\overrightarrow{AB}$. Find the position vector of C and show that it is a unit vector. [6]

5 The function f is defined, for $0° \leqslant x \leqslant 180°$, by

$$\mathrm{f}(x) = A + 5\cos Bx,$$

where A and B are constants.

(i) Given that the maximum value of f is 3, state the value of A. [1]

(ii) State the amplitude of f. [1]

(iii) Given that the period of f is 120°, state the value of B. [1]

(iv) Sketch the graph of f. [3]

6 Given that each of the following functions is defined for the domain $-2 \leqslant x \leqslant 3$, find the range of

(i) $\mathrm{f} : x \mapsto 2 - 3x$, [1]

(ii) $\mathrm{g} : x \mapsto |2 - 3x|$, [2]

(iii) $\mathrm{h} : x \mapsto 2 - |3x|$. [2]

State which of the functions f, g and h has an inverse. [2]

7 **(a)** Variables l and t are related by the equation $l = l_0(1 + \alpha)^t$ where l_0 and α are constants.

Given that $l_0 = 0.64$ and $\alpha = 2.5 \times 10^{-3}$, find the value of t for which $l = 0.66$. [3]

(b) Solve the equation $1 + \lg(8 - x) = \lg(3x + 2)$. [4]

8

x	10	100	1000	10 000
y	1900	250	31	4

The table above shows experimental values of the variables x and y which are related by an equation of the form $y = kx^n$, where k and n are constants.

(i) Using graph paper, draw the graph of $\lg y$ against $\lg x$. [3]

(ii) Use your graph to estimate the value of k and of n. [4]

9 **(i)** Determine the set of values of k for which the equation

$$x^2 + 2x + k = 3kx - 1$$

has no real roots. [5]

(ii) Hence state, giving a reason, what can be deduced about the curve $y = (x + 1)^2$ and the line $y = 3x - 1$. [2]

10 The remainder when $2x^3 + 2x^2 - 13x + 12$ is divided by $x + a$ is three times the remainder when it is divided by $x - a$.

(i) Show that $2a^3 + a^2 - 13a + 6 = 0$. [3]

(ii) Solve this equation completely. [5]

11 A particle travels in a straight line so that, t seconds after passing a fixed point A on the line, its acceleration, a ms^{-2}, is given by $a = -2 - 2t$. It comes to rest at a point B when $t = 4$.

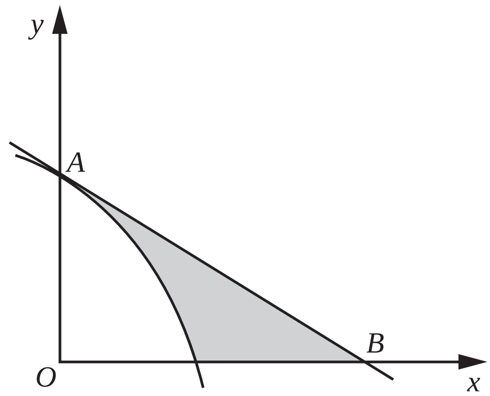
(i) Find the velocity of the particle at A. [4]

(ii) Find the distance AB. [3]

(iii) Sketch the velocity-time graph for the motion from A to B. [1]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram, which is not drawn to scale, shows part of the graph of $y = 8 - e^{2x}$, crossing the y-axis at A. The tangent to the curve at A crosses the x-axis at B. Find the area of the shaded region bounded by the curve, the tangent and the x-axis. [10]

OR

A piece of wire, of length 2 m, is divided into two pieces. One piece is bent to form a square of side x m and the other is bent to form a circle of radius r m.

(i) Express r in terms of x and show that the total area, $A\,\text{m}^2$, of the two shapes is given by

$$A = \frac{(\pi + 4)x^2 - 4x + 1}{\pi}.$$

[4]

Given that x can vary, find

(ii) the stationary value of A, [4]

(iii) the nature of this stationary value. [2]

1 A curve has the equation $y = \dfrac{8}{2x - 1}$.

(i) Find an expression for $\dfrac{dy}{dx}$. [3]

(ii) Given that y is increasing at a rate of 0.2 units per second when $x = -0.5$, find the corresponding rate of change of x. [2]

2 A flower show is held over a three-day period – Thursday, Friday and Saturday. The table below shows the entry price per day for an adult and for a child, and the number of adults and children attending on each day.

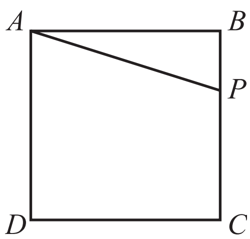
	Thursday	Friday	Saturday
Price (\$) – Adult	12	10	10
Price (\$) – Child	5	4	4
Number of adults	300	180	400
Number of children	40	40	150

(i) Write down two matrices such that their product will give the amount of entry money paid on Thursday and hence calculate this product. [2]

(ii) Write down two matrices such that the elements of their product give the amount of entry money paid for each of Friday and Saturday and hence calculate this product. [2]

(iii) Calculate the total amount of entry money paid over the three-day period. [1]

3



The diagram shows a square $ABCD$ of area $60\,\text{m}^2$. The point P lies on BC and the sum of the lengths of AP and BP is 12 m. Given that the lengths of AP and BP are x m and y m respectively, form two equations in x and y and hence find the length of BP. [5]

4 The functions f and g are defined by

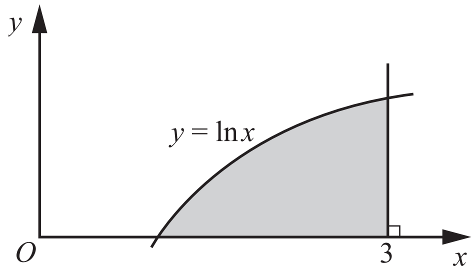
$$\mathrm{f} : x \mapsto \sin x, \quad 0 \leqslant x \leqslant \frac{\pi}{2},$$

$$\mathrm{g} : x \mapsto 2x - 3, \quad x \in \mathbb{R}.$$

Solve the equation $\mathrm{g}^{-1}\mathrm{f}(x) = \mathrm{g}^2(2.75)$. [5]

5 **(i)** Differentiate $x\ln x - x$ with respect to x. [2]

(ii)



The diagram shows part of the graph of $y = \ln x$. Use your result from part **(i)** to evaluate the area of the shaded region bounded by the curve, the line $x = 3$ and the x –axis. [4]

6 A curve has the equation $y = \dfrac{\mathrm{e}^{2x}}{\sin x}$, for $0 < x < \pi$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that the x-coordinate of the stationary point satisfies $2\sin x - \cos x = 0$. [4]

(ii) Find the x-coordinate of the stationary point. [2]

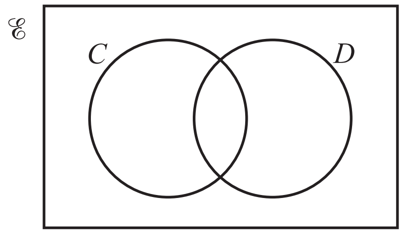
7 Solve, for x and y, the simultaneous equations

$$125^x = 25(5^y),$$

$$7^x \div 49^y = 1.$$

[6]

8



The Venn diagram above represents the sets

$\mathscr{E}$ = {homes in a certain town},

C = {homes with a computer},

D = {homes with a dishwasher}.

It is given that

$$\begin{aligned} &\mathrm{n}(C \cap D) &&= k, \\ &\mathrm{n}(C) &&= 7 \times \mathrm{n}(C \cap D), \\ &\mathrm{n}(D) &&= 4 \times \mathrm{n}(C \cap D), \\ \text{and}\quad &\mathrm{n}(\mathscr{E}) &&= 6 \times \mathrm{n}(C' \cap D'). \end{aligned}$$

(i) Copy the Venn diagram above and insert, in each of its four regions, the number, in terms of k, of homes represented by that region. [5]

(ii) Given that there are 165 000 homes which do not have both a computer and a dishwasher, calculate the number of homes in the town. [2]

9 A plane, whose speed in still air is 300 km h^{-1}, flies directly from X to Y. Given that Y is 720 km from X on a bearing of 150° and that there is a constant wind of 120 km h^{-1} blowing towards the west, find the time taken for the flight. [7]

10 **(a)** Solve, for $0° < x < 360°$,

$$4\tan^2 x + 15\sec x = 0.$$ [4]

(b) Given that $y > 3$, find the smallest value of y such that

$$\tan(3y - 2) = -5.$$ [4]

11 **(a)** **(i)** Expand $(2 + x)^5$. [3]

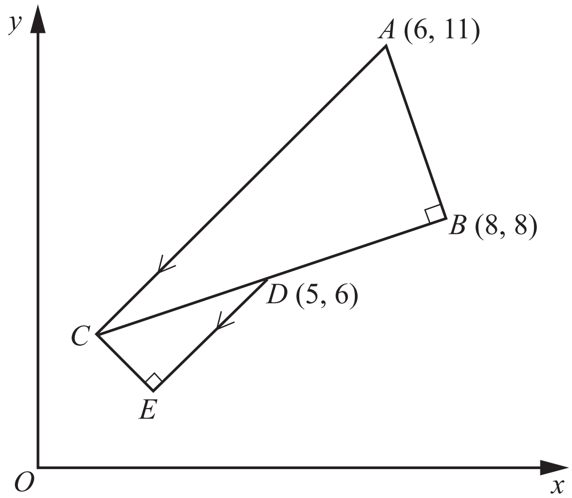
(ii) Use your answer to part **(i)** to find the integers a and b for which $(2 + \sqrt{3})^5$ can be expressed in the form $a + b\sqrt{3}$. [3]

(b) Find the coefficient of x in the expansion of $\left(x - \frac{4}{x}\right)^7$. [3]

12 Answer only **one** of the following two alternatives.

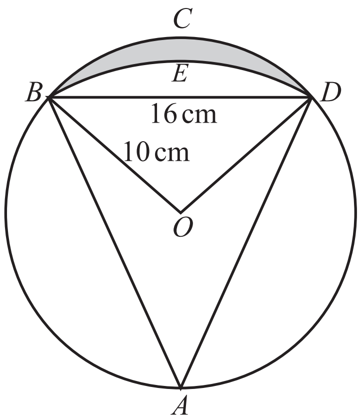
EITHER

Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a right-angled triangle *ABC*, where *A* is the point (6, 11) and *B* is the point (8, 8).
The point *D* (5, 6) is the mid-point of *BC*. The line *DE* is parallel to *AC* and angle *DEC* is a right-angle.
Find the area of the entire figure *ABDECA*. [11]

OR



The diagram, which is not drawn to scale, shows a circle *ABCDA*, centre *O* and radius 10 cm.
The chord *BD* is 16 cm long. *BED* is an arc of a circle, centre *A*.

(i) Show that the length of *AB* is approximately 17.9 cm.

For the shaded region enclosed by the arcs *BCD* and *BED*, find

(ii) its perimeter, **(iii)** its area.

[11]

1 A curve has the equation $y = (x - 1)(2x - 3)^8$. Find the gradient of the curve at the point where $x = 2$. [4]

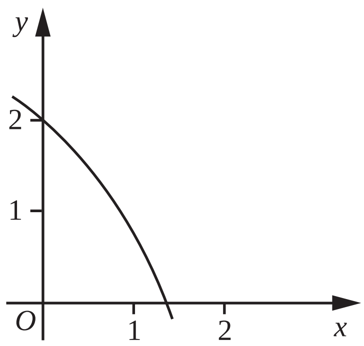
2 The line $y + 4x = 23$ intersects the curve $xy + x = 20$ at two points, A and B. Find the equation of the perpendicular bisector of the line AB. [6]

3 A plane flies due north from A to B, a distance of 1000 km, in a time of 2 hours. During this time a steady wind, with a speed of $150\,\text{km}\,\text{h}^{-1}$, is blowing from the south-east. Find

(i) the speed of the plane in still air, [4]

(ii) the direction in which the plane must be headed. [2]

4



The diagram shows part of the curve $y = \text{f}(x)$, where $\text{f}(x) = p - \text{e}^x$ and p is a constant.
The curve crosses the y-axis at (0, 2).

(i) Find the value of p. [2]

(ii) Find the coordinates of the point where the curve crosses the x-axis. [2]

(iii) Copy the diagram above and on it sketch the graph of $y = \text{f}^{-1}(x)$. [2]

5 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} -2 & -1 \\ 6 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ 4 & 3 \end{pmatrix}$. Find matrices **P** and **Q** such that

(i) $\mathbf{P} = \mathbf{B}^2 - 2\mathbf{A}$, [3]

(ii) $\mathbf{Q} = \mathbf{B}(\mathbf{A}^{-1})$. [4]

6 The cubic polynomial $\text{f}(x)$ is such that the coefficient of x^3 is 1 and the roots of $\text{f}(x) = 0$ are -2, $1 + \sqrt{3}$ and $1 - \sqrt{3}$.

(i) Express $\text{f}(x)$ as a cubic polynomial in x with integer coefficients. [3]

(ii) Find the remainder when $\text{f}(x)$ is divided by $x - 3$. [2]

(iii) Solve the equation $\text{f}(-x) = 0$. [2]

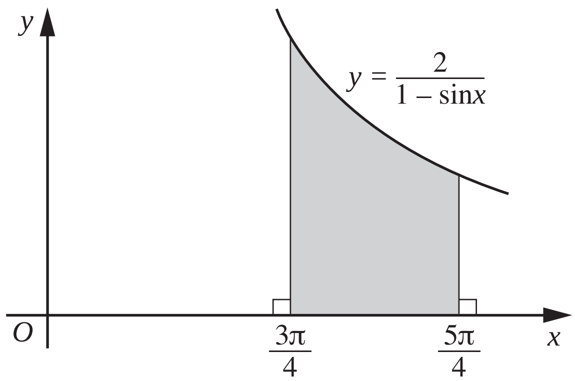
7 A particle moves in a straight line, so that, t s after leaving a fixed point O, its velocity, $v\,\mathrm{m\,s^{-1}}$, is given by

$$v = pt^2 + qt + 4,$$

where p and q are constants. When $t = 1$ the acceleration of the particle is $8\,\mathrm{m\,s^{-2}}$. When $t = 2$ the displacement of the particle from O is 22 m. Find the value of p and of q. [7]

8 **(i)** Given that $y = \dfrac{1 + \sin x}{\cos x}$, show that $\dfrac{dy}{dx} = \dfrac{1}{1 - \sin x}$. [5]

(ii)



The diagram shows part of the curve $y = \dfrac{2}{1 - \sin x}$. Using the result given in part **(i)**, find the area of the shaded region bounded by the curve, the x-axis and the lines $x = \dfrac{3\pi}{4}$ and $x = \dfrac{5\pi}{4}$. [3]

9 **(a)** Given that $u = \log_4 x$, find, in simplest form in terms of u,

(i) x,

(ii) $\log_4\left(\dfrac{16}{x}\right)$,

(iii) $\log_x 8$.

[5]

(b) Solve the equation $(\log_3 y)^2 + \log_3(y^2) = 8$. [4]

10 The function f is defined, for $0° \leqslant x \leqslant 180°$, by

$$\mathrm{f}(x) = 3\cos 4x - 1.$$

(i) Solve the equation $\mathrm{f}(x) = 0$. [3]

(ii) State the amplitude of f. [1]

(iii) State the period of f. [1]

(iv) State the maximum and minimum values of f. [2]

(v) Sketch the graph of $y = \mathrm{f}(x)$. [3]

11 Answer only **one** of the following two alternatives.

EITHER

The table below shows values of the variables x and y which are related by the equation $y = \dfrac{a}{x+b}$, where a and b are constants.

x	0.1	0.4	1.0	2.0	3.0
y	8.0	6.0	4.0	2.6	1.9

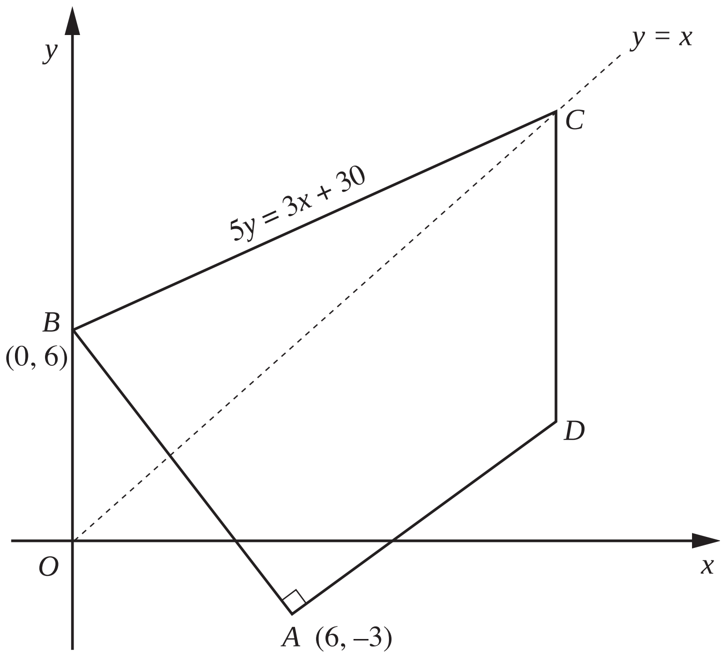
(i) Using graph paper, plot y against xy and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

An alternative method for obtaining a straight line graph for the equation $y = \dfrac{a}{x+b}$ is to plot x on the vertical axis and $\dfrac{1}{y}$ on the horizontal axis.

(iii) Without drawing a second graph, use your values of a and b to estimate the gradient and the intercept on the vertical axis of the graph of x plotted against $\dfrac{1}{y}$. [3]

OR



The diagram, which is not drawn to scale, shows a quadrilateral $ABCD$ in which A is $(6, -3)$, B is $(0, 6)$ and angle BAD is 90°. The equation of the line BC is $5y = 3x + 30$ and C lies on the line $y = x$. The line CD is parallel to the y-axis.

(i) Find the coordinates of C and of D. [6]

(ii) Show that triangle BAD is isosceles and find its area. [4]

1 Variables x and y are connected by the equation $y = (3x - 1)\ln x$. Given that x is increasing at the rate of 3 units per second, find the rate of increase of y when $x = 1$. [4]

2 The table shows the number of games played and the results of five teams in a football league.

	Played	Won	Drawn	Lost
Parrots	8	5	3	0
Quails	7	4	1	2
Robins	8	4	0	4
Swallows	7	2	1	4
Terns	8	1	1	6

A win earns 3 points, a draw 1 point and a loss 0 points. Write down two matrices which on multiplication display in their product the total number of points earned by each team and hence calculate these totals. [4]

3 The points A and B are such that the unit vector in the direction of $\overrightarrow{AB}$ is $0.28\mathbf{i} + p\mathbf{j}$, where p is a positive constant.

(i) Find the value of p. [2]

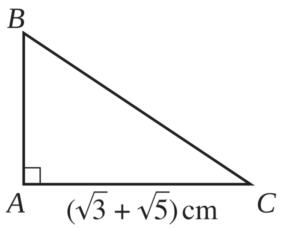
The position vectors of A and B, relative to an origin O, are $q\mathbf{i} - 7\mathbf{j}$ and $12\mathbf{i} + 17\mathbf{j}$ respectively.

(ii) Find the value of the constant q. [3]

4 **(a)** Differentiate $e^{\tan x}$ with respect to x. [2]

(b) Evaluate $\int_0^{\frac{1}{2}} e^{1-2x}\,dx$. [4]

5

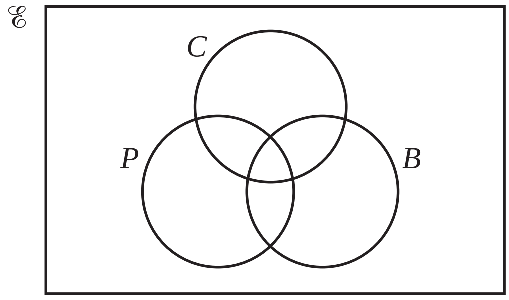


The diagram shows a right-angled triangle ABC in which the length of AC is $(\sqrt{3} + \sqrt{5})$ cm. The area of triangle ABC is $(1 + \sqrt{15})$ cm^2.

(i) Find the length of AB in the form $(a\sqrt{3} + b\sqrt{5})$ cm, where a and b are integers. [3]

(ii) Express $(BC)^2$ in the form $(c + d\sqrt{15})$ cm^2, where c and d are integers. [3]

6 **(a)**



The Venn diagram above represents the universal set ℰ of all teachers in a college. The sets C, B and P represent teachers who teach Chemistry, Biology and Physics respectively. Sketch the diagram twice.

(i) On the first diagram shade the region which represents those teachers who teach Physics and Chemistry but not Biology. [1]

(ii) On the second diagram shade the region which represents those teachers who teach either Biology or Chemistry or both, but not Physics. [1]

(b) In a group of 20 language teachers, F is the set of teachers who teach French and S is the set of teachers who teach Spanish. Given that $n(F) = 16$ and $n(S) = 10$, state the maximum and minimum possible values of

(i) $n(F \cap S)$,

(ii) $n(F \cup S)$.

[4]

7 **(a)** 7 boys are to be seated in a row. Calculate the number of different ways in which this can be done if 2 particular boys, Andrew and Brian, have exactly 3 of the other boys between them. [4]

(b) A box contains sweets of 6 different flavours. There are at least 2 sweets of each flavour. A girl selects 3 sweets from the box. Given that these 3 sweets are not all of the same flavour, calculate the number of different ways she can select her 3 sweets. [3]

8 **(i)** In the binomial expansion of $\left(x + \frac{k}{x^3}\right)^8$, where k is a positive constant, the term independent of x is 252.

Evaluate k. [4]

(ii) Using your value of k, find the coefficient of x^4 in the expansion of $\left(1 - \frac{x^4}{4}\right)\left(x + \frac{k}{x^3}\right)^8$. [3]

9 A cuboid has a total surface area of $120\,\text{cm}^2$. Its base measures x cm by $2x$ cm and its height is h cm.

(i) Obtain an expression for h in terms of x. [2]

Given that the volume of the cuboid is $V\,\text{cm}^3$,

(ii) show that $V = 40x - \dfrac{4x^3}{3}$. [1]

Given that x can vary,

(iii) show that V has a stationary value when $h = \dfrac{4x}{3}$. [4]

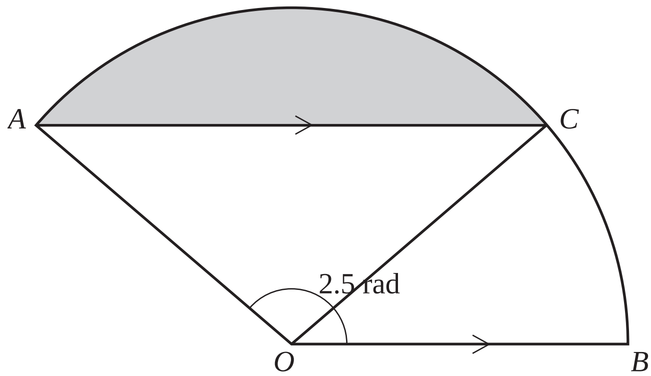
10 **(a)** Given that $a = \sec x + \operatorname{cosec} x$ and $b = \sec x - \operatorname{cosec} x$, show that

$$a^2 + b^2 \equiv 2\sec^2 x \operatorname{cosec}^2 x.$$ [4]

(b) Find, correct to 2 decimal places, the values of y between 0 and 6 radians which satisfy the equation

$$2\cot y = 3\sin y.$$ [5]

11



The diagram shows a sector $OACB$ of a circle, centre O, in which angle $AOB = 2.5$ radians. The line AC is parallel to OB.

(i) Show that angle $AOC = (5 - \pi)$ radians. [3]

Given that the radius of the circle is 12 cm, find

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [3]

12 Answer only **one** of the following two alternatives.

EITHER

(i) Express $2x^2 - 8x + 3$ in the form $a(x + b)^2 + c$, where a, b and c are integers. [2]

A function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 8x + 3$, $x \in \mathbb{R}$.

(ii) Find the coordinates of the stationary point on the graph of $y = \mathrm{f}(x)$. [2]

(iii) Find the value of $\mathrm{f}^2(0)$. [2]

A function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 8x + 3$, $x \in \mathbb{R}$, where $x \leqslant N$.

(iv) State the greatest value of N for which g has an inverse. [1]

(v) Using the result obtained in part **(i)**, find an expression for g^{-1}. [3]

OR

The equation of a curve is $y = 10 - x^2 + 6x$.

(i) Find the set of values of x for which $y \geqslant 15$. [3]

(ii) Express y in the form $a - (x + b)^2$, where a and b are integers. [2]

(iii) Hence, or otherwise, find the coordinates of the stationary point on the curve. [2]

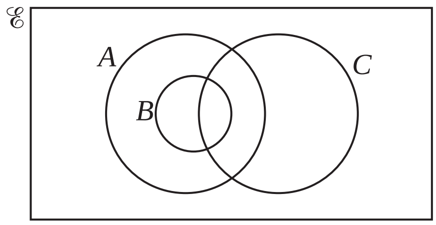
Functions f and g are defined, for $x \in \mathbb{R}$, by

$\mathrm{f} : x \mapsto 10 - x^2 + 6x$,

$\mathrm{g} : x \mapsto 2x - k$, where k is a constant.

(iv) Find the value of k for which the equation $\mathrm{gf}(x) = 0$ has two equal roots. [3]

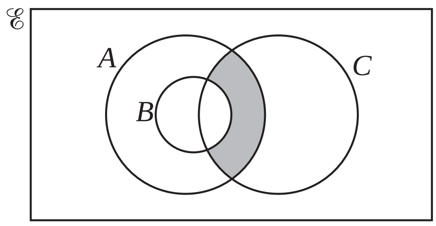
1 **(a)**



The diagram above shows a universal set ℰ and the three sets A, B and C.

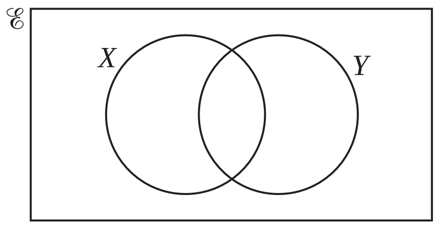
(i) Copy the above diagram and shade the region representing $(A \cap C') \cup B$. [1]

(ii)



Express, in set notation, the set represented by the shaded region in the diagram above. [1]

(b)



The diagram shows a universal set ℰ and the sets X and Y. Show, by means of two diagrams, that the set $(X \cup Y)'$ is not the same as the set $X' \cup Y'$. [2]

2 Find the equation of the normal to the curve $y = \dfrac{2x + 4}{x - 2}$ at the point where $x = 4$. [5]

3 The straight line $3x = 2y + 18$ intersects the curve $2x^2 - 23x + 2y + 50 = 0$ at the points A and B. Given that A lies below the x-axis and that the point P lies on AB such that $AP : PB = 1 : 2$, find the coordinates of P. [6]

4 **(i)** Find the first three terms, in ascending powers of u, in the expansion of $(2 + u)^5$. [2]

(ii) By replacing u with $2x - 5x^2$, find the coefficient of x^2 in the expansion of $(2 + 2x - 5x^2)^5$. [4]

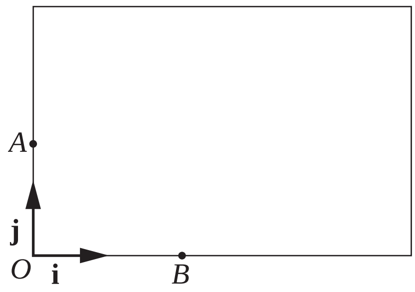
5 A curve has the equation $y = \sqrt{x} + \dfrac{9}{\sqrt{x}}$.

(i) Find expressions for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [4]

(ii) Show that the curve has a stationary value when $x = 9$. [1]

(iii) Find the nature of this stationary value. [2]

6



The diagram shows a large rectangular television screen in which one corner is taken as the origin O and $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along two of the edges. In a game, an alien spacecraft appears at the point A with position vector $12\mathbf{j}$ cm and moves across the screen with velocity $(40\mathbf{i} + 15\mathbf{j})$ cm per second. A player fires a missile from a point B; the missile is fired 0.5 seconds after the spacecraft appears on the screen. The point B has position vector $46\mathbf{i}$ cm and the velocity of the missile is $(k\mathbf{i} + 30\mathbf{j})$ cm per second, where k is a constant. Given that the missile hits the spacecraft,

(i) show that the spacecraft moved across the screen for 1.8 seconds before impact, [4]

(ii) find the value of k. [3]

7 **(a)** Use the substitution $u = 5^x$ to solve the equation $5^{x+1} = 8 + 4(5^{-x})$. [5]

(b) Given that $\log(p - q) = \log p - \log q$, express p in terms of q. [3]

8 **(a)** Solve, for $0 \leqslant x \leqslant 2$, the equation $1 + 5\cos 3x = 0$, giving your answer in radians correct to 2 decimal places. [3]

(b) Find all the angles between 0° and 360° such that

$$\sec y + 5\tan y = 3\cos y.$$ [5]

9

x	0.100	0.125	0.160	0.200	0.400
y	0.050	0.064	0.085	0.111	0.286

The table above shows experimental values of the variables x and y.

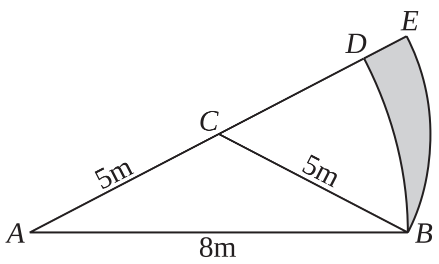
(i) On graph paper draw the graph of $\frac{1}{y}$ against $\frac{1}{x}$. [3]

Hence,

(ii) express y in terms of x, [4]

(iii) find the value of x for which $y = 0.15$. [2]

10



The diagram shows an isosceles triangle ABC in which $AB = 8$ m, $BC = CA = 5$ m. $ABDA$ is a sector of the circle, centre A and radius 8 m. $CBEC$ is a sector of the circle, centre C and radius 5 m.

(i) Show that angle BCE is 1.287 radians correct to 3 decimal places. [2]

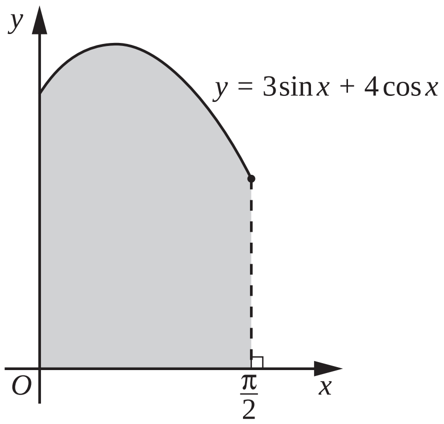
(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [4]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

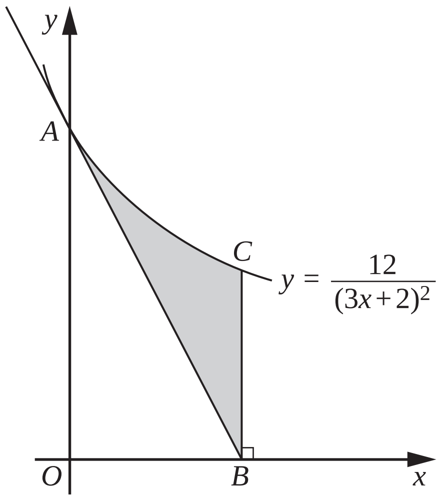


The graph shows part of the curve $y = 3\sin x + 4\cos x$ for $0 \leqslant x \leqslant \frac{\pi}{2}$ radians.

(i) Find the coordinates of the maximum point of the curve. [5]

(ii) Find the area of the shaded region. [5]

OR



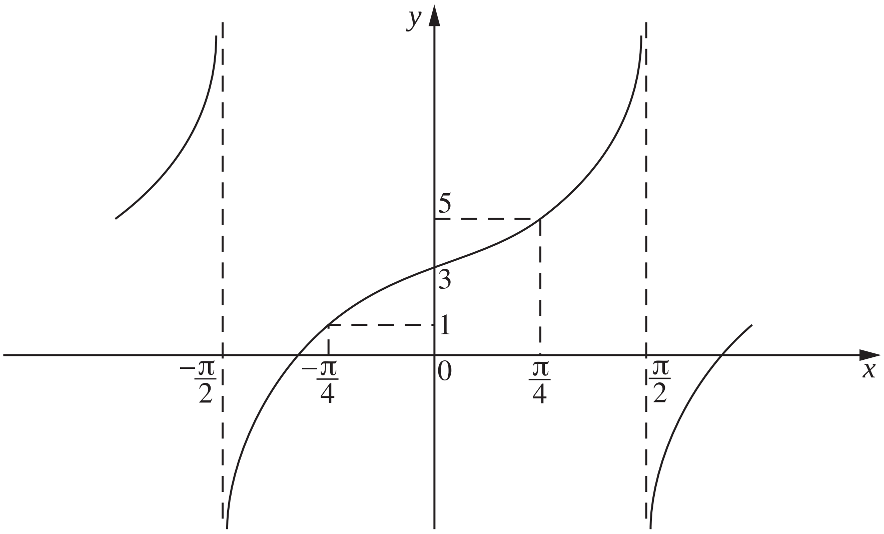
The diagram, which is not drawn to scale, shows part of the curve $y = \dfrac{12}{(3x+2)^2}$, intersecting the y-axis at A. The tangent to the curve at A meets the x-axis at B. The point C lies on the curve and BC is parallel to the y-axis.

(i) Find the x-coordinate of B. [4]

(ii) Find the area of the shaded region. [6]

1 A triangle has a base of length $(13 - 2x)$ m and a perpendicular height of x m. Calculate the range of values of x for which the area of the triangle is greater than $3\,\text{m}^2$. [3]

2



The diagram shows part of the graph of $y = a \tan(bx) + c$. Find the value of

(i) c, **(ii)** b, **(iii)** a. [3]

3 The roots of the equation $x^2 - \sqrt{28}x + 2 = 0$ are p and q, where $p > q$. Without using a calculator, express $\frac{p}{q}$ in the form $m + \sqrt{n}$, where m and n are integers. [5]

4 An artist has 6 watercolour paintings and 4 oil paintings. She wishes to select 4 of these 10 paintings for an exhibition.

(i) Find the number of different selections she can make. [2]

(ii) In how many of these selections will there be more watercolour paintings than oil paintings? [3]

5 **(i)** Express $\frac{1}{\sqrt{32}}$ as a power of 2. [1]

(ii) Express $(64)^{\frac{1}{x}}$ as a power of 2. [1]

(iii) Hence solve the equation $\frac{(64)^{\frac{1}{x}}}{2^x} = \frac{1}{\sqrt{32}}$. [3]

6 **(i)** Differentiate $x^2 \ln x$ with respect to x. [2]

(ii) Use your result to show that $\int_1^e 4x \ln x \, dx = e^2 + 1$. [4]

7 Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 8 & 10 \\ -4 & 2 \end{pmatrix}$, find the matrices **X** and **Y** such that

(i) $\mathbf{X} = \mathbf{A}^2 + 2\mathbf{B}$, [3]

(ii) $\mathbf{YA} = \mathbf{B}$. [4]

8 The equation of the curve C is $2y = x^2 + 4$. The equation of the line L is $y = 3x - k$, where k is an integer.

(i) Find the largest value of the integer k for which L intersects C. [4]

(ii) In the case where $k = -2$, show that the line joining the points of intersection of L and C is bisected by the line $y = 2x + 5$. [4]

9 The position vectors, relative to an origin O, of three points P, Q and R are $\mathbf{i} + 3\mathbf{j}$, $5\mathbf{i} + 11\mathbf{j}$ and $9\mathbf{i} + 9\mathbf{j}$ respectively.

(i) By finding the magnitude of the vectors $\overrightarrow{PR}$, $\overrightarrow{RQ}$ and $\overrightarrow{QP}$, show that angle PQR is 90°. [4]

(ii) Find the unit vector parallel to $\overrightarrow{PR}$. [2]

(iii) Given that $\overrightarrow{OQ} = m\overrightarrow{OP} + n\overrightarrow{OR}$, where m and n are constants, find the value of m and of n. [3]

10 The functions f and g are defined, for $x \in \mathbb{R}$, by

$$\mathrm{f} : x \mapsto 3x - 2,$$

$$\mathrm{g} : x \mapsto \frac{7x - a}{x + 1}, \text{ where } x \neq -1 \text{ and } a \text{ is a positive constant.}$$

(i) Obtain expressions for f^{-1} and g^{-1}. [3]

(ii) Determine the value of a for which $\mathrm{f}^{-1}\mathrm{g}(4) = 2$. [3]

(iii) If $a = 9$, show that there is only one value of x for which $\mathrm{g}(x) = \mathrm{g}^{-1}(x)$. [3]

11 A particle, moving in a straight line, passes through a fixed point O with velocity $14\,\mathrm{ms}^{-1}$. The acceleration, $a\,\mathrm{ms}^{-2}$, of the particle, t seconds after passing through O, is given by $a = 2t - 9$. The particle subsequently comes to instantaneous rest, firstly at A and later at B. Find

(i) the acceleration of the particle at A and at B, [4]

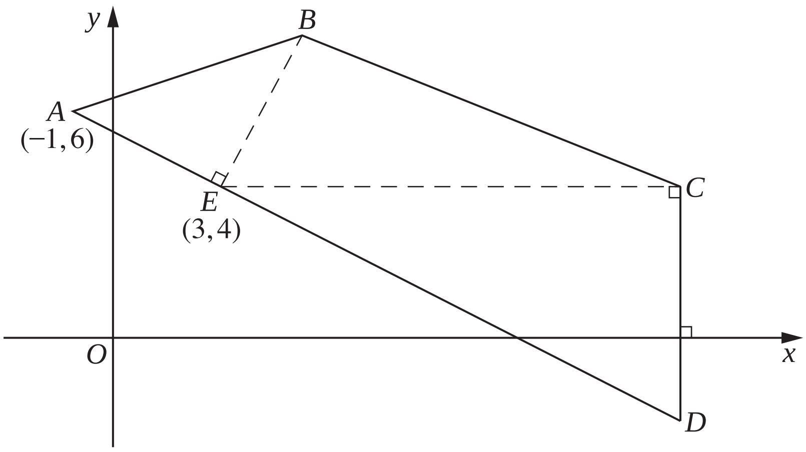
(ii) the greatest speed of the particle as it travels from A to B, [2]

(iii) the distance AB. [4]

12 Answer only **one** of the following two alternatives.

EITHER

Solutions to this question by accurate drawing will not be accepted.



The diagram shows a quadrilateral *ABCD*. The point *E* lies on *AD* such that angle *AEB* = 90°. The line *EC* is parallel to the *x*-axis and the line *CD* is parallel to the *y*-axis. The points *A* and *E* are (– 1, 6) and (3, 4) respectively. Given that the gradient of *AB* is $\frac{1}{3}$,

(i) find the coordinates of *B*. [5]

Given also that the area of triangle *EBC* is 24 units^2,

(ii) find the coordinates of *C*, [3]

(iii) find the coordinates of *D*. [2]

OR

(a) The expression $f(x) = x^3 + ax^2 + bx + c$ leaves the same remainder, *R*, when it is divided by $x + 2$ and when it is divided by $x – 2$.

(i) Evaluate *b*. [2]

f(*x*) also leaves the same remainder, *R*, when divided by $x – 1$.

(ii) Evaluate *a*. [2]

f(*x*) leaves a remainder of 4 when divided by $x – 3$.

(iii) Evaluate *c*. [1]

(b) Solve the equation $x^3 + 3x^2 = 2$, giving your answers to 2 decimal places where necessary. [5]

1 Express $\dfrac{8-3\sqrt{2}}{4+3\sqrt{2}}$ in the form $a+b\sqrt{2}$, where a and b are integers. [3]

2 A committee of 5 people is to be selected from 6 men and 4 women. Find

(i) the number of different ways in which the committee can be selected, [1]

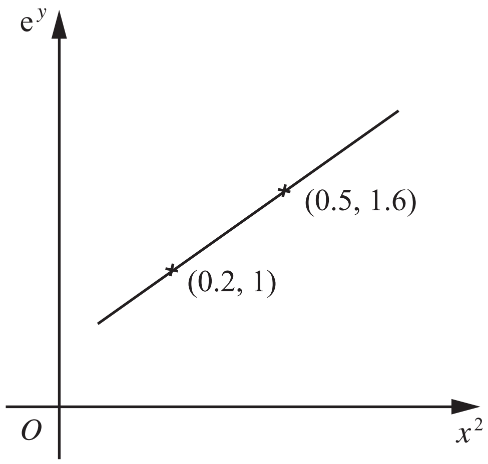
(ii) the number of these selections with more women than men. [4]

3 The line $y = 3x + k$ is a tangent to the curve $x^2 + xy + 16 = 0$.

(i) Find the possible values of k. [3]

(ii) For each of these values of k, find the coordinates of the point of contact of the tangent with the curve. [2]

4 Variables x and y are such that, when e^y is plotted against x^2, a straight line graph passing through the points (0.2, 1) and (0.5, 1.6) is obtained.



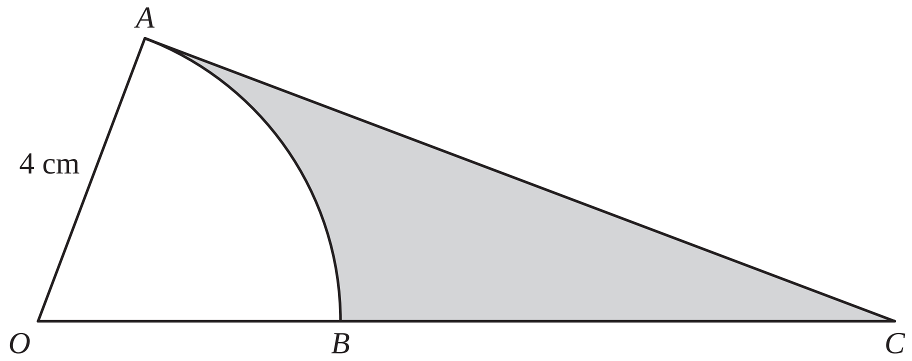
(i) Find the value of e^y when $x = 0$. [2]

(ii) Express y in terms of x. [3]

5 Variables x and y are connected by the equation $y = \dfrac{x}{\tan x}$. Given that x is increasing at the rate of 2 units per second, find the rate of increase of y when $x = \dfrac{\pi}{4}$. [5]

6 Solve the equation $x^2(2x + 3) = 17x - 12$. [6]

7



The diagram shows a sector *OAB* of a circle, centre *O*, radius 4 cm. The tangent to the circle at *A* meets the line *OB* extended at *C*. Given that the area of the sector *OAB* is 10 cm^2, calculate

(i) the angle *AOB* in radians, [2]

(ii) the perimeter of the shaded region. [4]

8 **(i)** Given that $\log_9 x = a\log_3 x$, find a. [1]

(ii) Given that $\log_{27} y = b\log_3 y$, find b. [1]

(iii) Hence solve, for x and y, the simultaneous equations

$$6\log_9 x + 3\log_{27} y = 8,$$
$$\log_3 x + 2\log_9 y = 2.$$

[4]

9 A curve is such that $\frac{dy}{dx} = 2\cos\left(2x - \frac{\pi}{2}\right)$. The curve passes through the point $\left(\frac{\pi}{2}, 3\right)$.

(i) Find the equation of the curve. [4]

(ii) Find the equation of the normal to the curve at the point where $x = \frac{3\pi}{4}$. [4]

10 In this question, **i** is a unit vector due east and **j** is a unit vector due north.

At 0900 hours a ship sails from the point P with position vector $(2\mathbf{i} + 3\mathbf{j})$ km relative to an origin O. The ship sails north-east with a speed of $15\sqrt{2}$ km h^{-1}.

(i) Find, in terms of **i** and **j**, the velocity of the ship. [2]

(ii) Show that the ship will be at the point with position vector $(24.5\mathbf{i} + 25.5\mathbf{j})$ km at 1030 hours. [1]

(iii) Find, in terms of **i**, **j** and t, the position of the ship t hours after leaving P. [2]

At the same time as the ship leaves P, a submarine leaves the point Q with position vector $(47\mathbf{i} - 27\mathbf{j})$ km. The submarine proceeds with a speed of 25 km h^{-1} due north to meet the ship.

(iv) Find, in terms of **i** and **j**, the velocity of the ship relative to the submarine. [2]

(v) Find the position vector of the point where the submarine meets the ship. [2]

11 Solve the equation

(i) $3\sin x + 5\cos x = 0$ for $0° < x < 360°$, [3]

(ii) $3\tan^2 y - \sec y - 1 = 0$ for $0° < y < 360°$, [5]

(iii) $\sin(2z - 0.6) = 0.8$ for $0 < z < 3$ radians. [4]

[Question 12 is printed on the next page.]

12 Answer only **one** of the following two alternatives.

EITHER

A curve has equation $y = (x^2 - 3)e^{-x}$.

(i) Find the coordinates of the points of intersection of the curve with the x-axis. [2]

(ii) Find the coordinates of the stationary points of the curve. [5]

(iii) Determine the nature of these stationary points. [3]

OR

A particle moves in a straight line such that its displacement, s m, from a fixed point O at a time t s, is given by

$$s = \ln(t + 1) \quad \text{for} \quad 0 \leqslant t \leqslant 3,$$

$$s = \tfrac{1}{2}\ln(t - 2) - \ln(t + 1) + \ln 16 \quad \text{for } t > 3.$$

Find

(i) the initial velocity of the particle, [2]

(ii) the velocity of the particle when $t = 4$, [2]

(iii) the acceleration of the particle when $t = 4$, [2]

(iv) the value of t when the particle is instantaneously at rest, [2]

(v) the distance travelled by the particle in the 4th second. [2]

1 The equation of a curve is given by $y = x^2 + ax + 3$, where a is a constant. Given that this equation can also be written as $y = (x + 4)^2 + b$, find

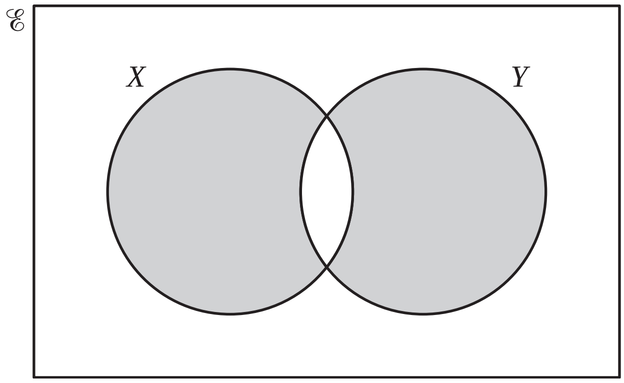
(i) the value of a and of b, [2]

(ii) the coordinates of the turning point of the curve. [1]

2 (a) Illustrate the following statements using a separate Venn diagram for each.

(i) $A \cap B = \varnothing$, (ii) $(C \cup D) \subset E$. [2]

(b)



Express, in set notation, the set represented by the shaded region. [2]

3 Find the coordinates of the points where the straight line $y = 2x - 3$ intersects the curve $x^2 + y^2 + xy + x = 30$. [5]

4 (i) Sketch, on the same diagram, the graphs of $y = x - 3$ and $y = |2x - 9|$. [3]

(ii) Solve the equation $|2x - 9| = x - 3$. [2]

5 Find the coefficient of x^3 in the expansion of

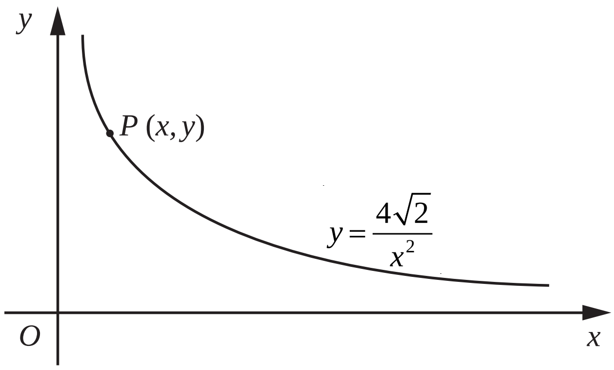
(i) $(1 + 3x)^8$, [2]

(ii) $(1 - 4x)(1 + 3x)^8$. [3]

6 (a) Given that $\sin x = p$, find an expression, in terms of p, for $\sec^2 x$. [2]

(b) Prove that $\sec A \operatorname{cosec} A - \cot A \equiv \tan A$. [4]

7



The diagram shows part of the curve $y = \dfrac{4\sqrt{2}}{x^2}$. The point $P\ (x, y)$ lies on this curve.

(i) Write down an expression, in terms of x, for $(OP)^2$. [1]

(ii) Denoting $(OP)^2$ by S, find an expression for $\dfrac{dS}{dx}$. [2]

(iii) Find the value of x for which S has a stationary value and the corresponding value of OP. [3]

8 Solve the equation

(i) $2^{2x+1} = 20$, [3]

(ii) $\dfrac{5^{4y-1}}{25^{y}} = \dfrac{125^{y+3}}{25^{2-y}}$. [4]

9 Given that $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -5 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, calculate

(i) **AB**, [2]

(ii) **BC**, [2]

(iii) the matrix **X** such that **AX** = **B**. [4]

10 **(a)** Find

(i) $\displaystyle\int \frac{12}{(2x-1)^4}\, dx$, [2]

(ii) $\displaystyle\int x(x-1)^2 dx$. [3]

(b) **(i)** Given that $y = 2(x-5)\sqrt{x+4}$, show that $\dfrac{dy}{dx} = \dfrac{3(x+1)}{\sqrt{x+4}}$. [3]

(ii) Hence find $\displaystyle\int \frac{(x+1)}{\sqrt{x+4}}\, dx$. [2]

11 The function f is defined by

$$f(x) = (x + 1)^2 + 2 \quad \text{for} \quad x \geqslant -1.$$

Find

(i) the range of f, [1]

(ii) $f^2(1)$, [1]

(iii) an expression for $f^{-1}(x)$. [3]

The function g is defined by

$$g(x) = \frac{20}{x + 1} \quad \text{for} \quad x \geqslant 0.$$

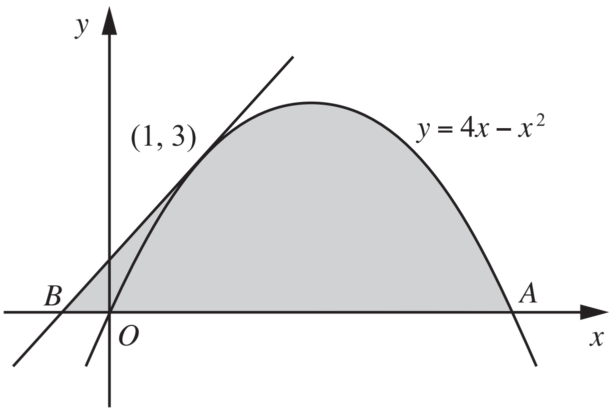
Find

(iv) $g^{-1}(2)$, [2]

(v) the value of x for which $fg(x) = 38$. [4]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows the curve $y = 4x - x^2$, which crosses the x-axis at the origin O and the point A. The tangent to the curve at the point (1, 3) crosses the x-axis at the point B.

(i) Find the coordinates of A and of B. [5]

(ii) Find the area of the shaded region. [5]

OR

Solutions to this question by accurate drawing will not be accepted.

The points $A(-2, 2)$, $B(4, 4)$ and $C(5, 2)$ are the vertices of a triangle. The perpendicular bisector of AB and the line through A parallel to BC intersect at the point D. Find the area of the quadrilateral $ABCD$. [10]

1 Differentiate with respect to x

(i) $\sqrt{1 + x^3}$, [2]

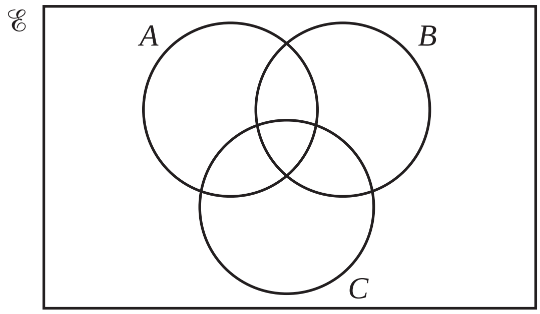
(ii) $x^2 \cos 2x$. [3]

2 **(i)** Find the first 3 terms of the expansion, in ascending powers of x, of $(1 + 3x)^6$. [2]

(ii) Hence find the coefficient of x^2 in the expansion of $(1 + 3x)^6 (1 - 3x - 5x^2)$. [3]

3 Find the set of values of k for which the equation $x^2 + (k - 2)x + (2k - 4) = 0$ has real roots. [5]

4 **(a)**



(i) Copy the Venn diagram above and shade the region that represents $(A \cap B) \cup C$. [1]

(ii) Copy the Venn diagram above and shade the region that represents $A' \cap B'$. [1]

(iii) Copy the Venn diagram above and shade the region that represents $(A \cup B) \cap C$. [1]

(b) It is given that the universal set $\mathscr{E} = \{x : 2 \leqslant x \leqslant 20, x \text{ is an integer}\}$,
$X = \{x : 4 < x < 15, x \text{ is an integer}\}$,
$Y = \{x : x \geqslant 9, x \text{ is an integer}\}$,
$Z = \{x : x \text{ is a multiple of } 5\}$.

(i) List the elements of $X \cap Y$. [1]

(ii) List the elements of $X \cup Y$. [1]

(iii) Find $(X \cup Y)' \cap Z$. [1]

5 Solve the equation $3x(x^2 + 6) = 8 - 17x^2$. [6]

6 Given that $\log_8 p = x$ and $\log_8 q = y$, express in terms of x and/or y

(i) $\log_8 \sqrt{p} + \log_8 q^2$, [2]

(ii) $\log_8 \left(\dfrac{q}{8}\right)$, [2]

(iii) $\log_2 (64p)$. [3]

7 The function f is defined by

$$f(x) = (2x + 1)^2 - 3 \quad \text{for } x \geqslant -\frac{1}{2}.$$

Find

(i) the range of f, [1]

(ii) an expression for $f^{-1}(x)$. [3]

The function g is defined by

$$g(x) = \frac{3}{1 + x} \quad \text{for } x > -1.$$

(iii) Find the value of x for which $fg(x) = 13$. [4]

8 **(a)** Solve the equation $(2^{3-4x})(4^{x+4}) = 2$. [3]

(b) **(i)** Simplify $\sqrt{108} - \dfrac{12}{\sqrt{3}}$, giving your answer in the form $k\sqrt{3}$, where k is an integer. [2]

(ii) Simplify $\dfrac{\sqrt{5}+3}{\sqrt{5}-2}$, giving your answer in the form $a\sqrt{5} + b$, where a and b are integers. [3]

9 **(a)** Variables x and y are related by the equation $y = 5x + 2 - 4e^{-x}$.

(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Hence find the approximate change in y when x increases from 0 to p, where p is small. [2]

(b) A square of area A cm^2 has a side of length x cm. Given that the area is increasing at a constant rate of 0.5 cm^2 s^{-1}, find the rate of increase of x when $A = 9$. [4]

10 Solve

(i) $4\sin x = \cos x$ for $0° < x < 360°$, [3]

(ii) $3 + \sin y = 3\cos^2 y$ for $0° < y < 360°$, [5]

(iii) $\sec\left(\frac{z}{3}\right) = 4$ for $0 < z < 5$ radians. [3]

11 Answer only **one** of the following two alternatives.

EITHER

A curve has equation $y = \frac{\ln x}{x^2}$, where $x > 0$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

(ii) Show that $\frac{d^2y}{dx^2}$ can be written in the form $\frac{a\ln x + b}{x^4}$, where a and b are integers. [3]

(iii) Hence, or otherwise, determine the nature of the stationary point of the curve. [2]

OR

A curve is such that $\frac{dy}{dx} = 6\cos\left(2x + \frac{\pi}{2}\right)$ for $-\frac{\pi}{4} \leqslant x \leqslant \frac{5\pi}{4}$. The curve passes through the point $\left(\frac{\pi}{4}, 5\right)$.

Find

(i) the equation of the curve, [4]

(ii) the x-coordinates of the stationary points of the curve, [3]

(iii) the equation of the normal to the curve at the point on the curve where $x = \frac{3\pi}{4}$. [4]

1 Find the coordinates of the points of intersection of the curve $y^2 + y = 10x - 8x^2$ and the straight line $y + 4x + 1 = 0$. [5]

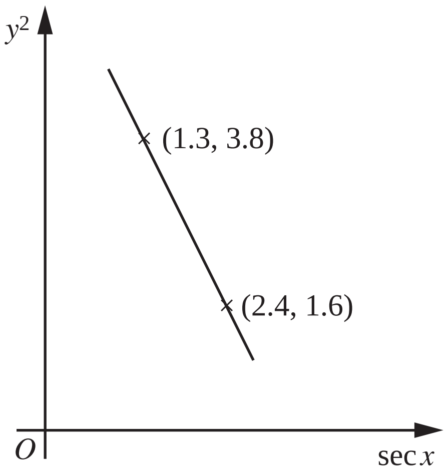
2 The expression $6x^3 + ax^2 - (a + 1)x + b$ has a remainder of 15 when divided by $x + 2$ and a remainder of 24 when divided by $x + 1$. Show that $a = 8$ and find the value of b. [5]

3 Given that $\overrightarrow{OA} = \begin{pmatrix} -17 \\ 25 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, find

(i) the unit vector parallel to $\overrightarrow{AB}$, [3]

(ii) the vector $\overrightarrow{OC}$, such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. [2]

4

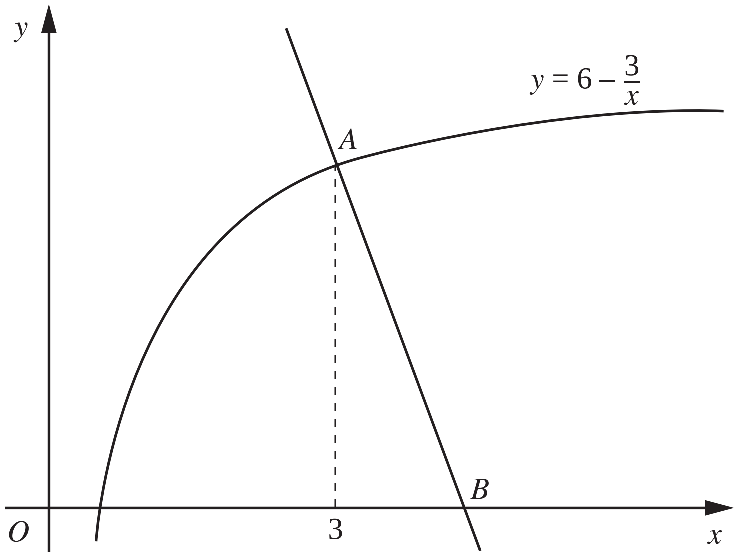


Variables x and y are such that, when y^2 is plotted against $\sec x$, a straight line graph passing through the points (2.4, 1.6) and (1.3, 3.8) is obtained.

(i) Express y^2 in terms of $\sec x$. [3]

(ii) Hence find the exact value of $\cos x$ when $y = 2$. [2]

5



The diagram shows part of the curve $y = 6 - \frac{3}{x}$ which passes through the point A where $x = 3$. The normal to the curve at the point A meets the x-axis at the point B. Find the coordinates of the point B. [5]

6 **(a)** **(i)** On the same diagram, sketch the curves $y = \cos x$ and $y = 1 + \cos 2x$ for $0 \leqslant x \leqslant 2\pi$. [3]

(ii) Hence state the **number** of solutions of the equation

$$\cos 2x - \cos x + 1 = 0 \quad \text{where} \quad 0 \leqslant x \leqslant 2\pi.$$ [1]

(b) The function f is given by $f(x) = 5\sin 3x$. Find

(i) the amplitude of f, [1]

(ii) the period of f. [1]

7 The table shows values of the variables p and v which are related by the equation $p = kv^n$, where k and n are constants.

v	10	50	110	230
p	1412	151	53	19

(i) Using graph paper, plot $\lg p$ against $\lg v$ and draw a straight line graph. [3]

Use your graph to estimate

(ii) the value of n, [2]

(iii) the value of p when $v = 170$. [2]

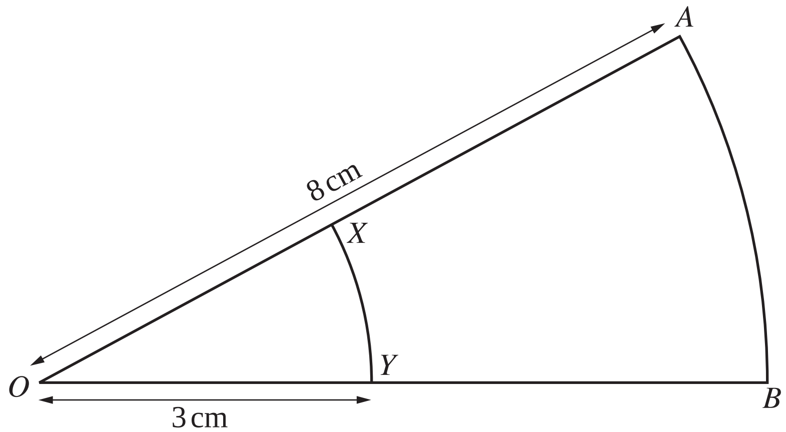
8 Given that $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 0 \\ 1 & 4 \end{pmatrix}$, find

(i) $3\mathbf{A} - 2\mathbf{B}$, [2]

(ii) $\mathbf{A}^{-1}$, [2]

(iii) the matrix $\mathbf{X}$ such that $\mathbf{XB}^{-1} = \mathbf{A}$. [3]

9



The diagram shows a sector OXY of a circle centre O, radius 3 cm and a sector OAB of a circle centre O, radius 8 cm. The point X lies on the line OA and the point Y lies on the line OB. The perimeter of the region $XABYX$ is 15.5 cm. Find

(i) the angle AOB in radians, [3]

(ii) the ratio of the area of the sector OXY to the area of the region $XABYX$ in the form $p : q$, where p and q are integers. [4]

10 A music student needs to select 7 pieces of music from 6 classical pieces and 4 modern pieces. Find the number of different selections that she can make if

(i) there are no restrictions, [1]

(ii) there are to be only 2 modern pieces included, [2]

(iii) there are to be more classical pieces than modern pieces. [4]

11 A particle moves in a straight line such that its displacement, x m, from a fixed point O on the line at time t seconds is given by $x = 12\{1n(2t + 3)\}$. Find

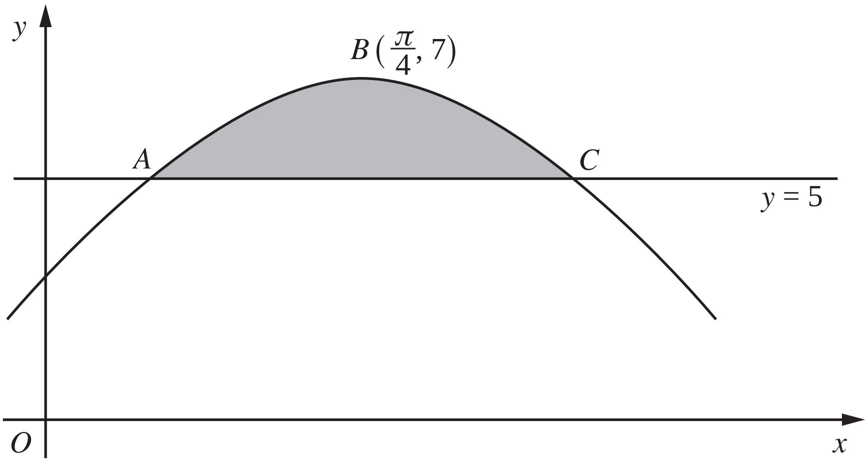
(i) the value of t when the displacement of the particle from O is 48 m, [3]

(ii) the velocity of the particle when $t = 1$, [3]

(iii) the acceleration of the particle when $t = 1$. [3]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows part of a curve for which $\frac{dy}{dx} = 8\cos 2x$. The curve passes through the point $B\left(\frac{\pi}{4}, 7\right)$. The line $y = 5$ meets the curve at the points A and C.

(i) Show that the curve has equation $y = 3 + 4\sin 2x$. [3]

(ii) Find the x-coordinate of the point A and of the point C. [4]

(iii) Find the area of the shaded region. [5]

OR

A curve is such that $\frac{dy}{dx} = 6e^{3x} - 12$. The curve passes through the point (0, 1).

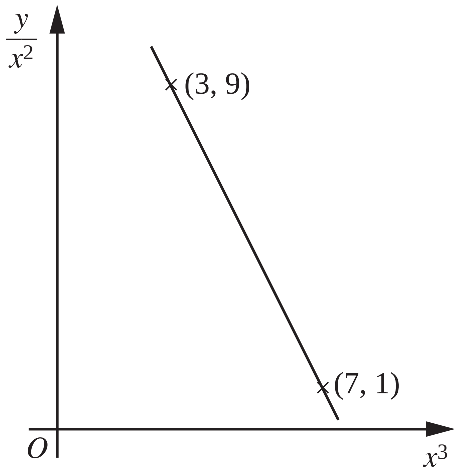
(i) Find the equation of the curve. [4]

(ii) Find the coordinates of the stationary point of the curve. [3]

(iii) Determine the nature of the stationary point. [2]

(iv) Find the coordinates of the point where the tangent to the curve at the point (0, 1) meets the x-axis. [3]

1



The variables x and y are related so that, when $\frac{y}{x^2}$ is plotted against x^3, a straight line graph passing through (3, 9) and (7, 1) is obtained. Express y in terms of x. [4]

2 In a singing competition there are 8 contestants. Each contestant sings in the first round of this competition.

(i) In how many different orders could the contestants sing? [1]

After the first round 5 contestants are chosen.

(ii) In how many different ways can these 5 contestants be chosen? [2]

These 5 contestants sing again and then First, Second and Third prizes are awarded to three of them.

(iii) In how many different ways can the prizes be awarded? [2]

3 It is given that $x - 1$ is a factor of $f(x)$, where $f(x) = x^3 - 6x^2 + ax + b$.

(i) Express b in terms of a. [2]

(ii) Show that the remainder when $f(x)$ is divided by $x - 3$ is twice the remainder when $f(x)$ is divided by $x - 2$. [4]

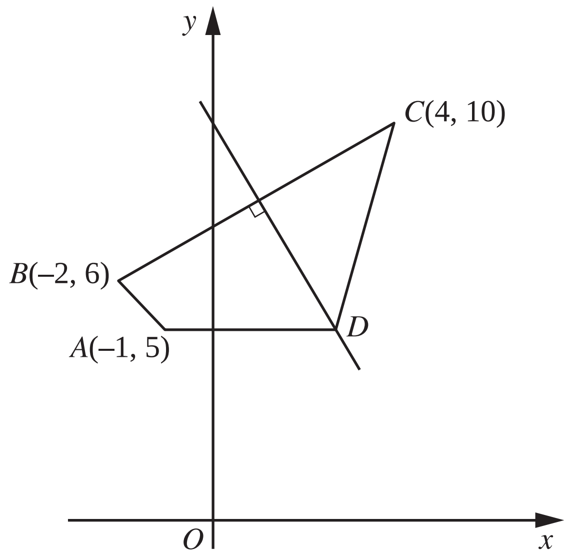
4 **(a)** Given that $\sin x = p$ and $\cos x = 2p$, where x is acute, find the exact value of p and the exact value of $\operatorname{cosec} x$. [3]

(b) Prove that $(\cot x + \tan x)(\cot x - \tan x) = \dfrac{1}{\sin^2 x} - \dfrac{1}{\cos^2 x}$. [3]

5 Given that a curve has equation $y = x^2 + 64\sqrt{x}$, find the coordinates of the point on the curve where $\dfrac{d^2y}{dx^2} = 0$. [7]

6 The line $y = x + 4$ intersects the curve $2x^2 + 3xy - y^2 + 1 = 0$ at the points A and B. Find the length of the line AB. [7]

7 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram the points $A(-1, 5)$, $B(-2, 6)$, $C(4, 10)$ and D are the vertices of a quadrilateral in which AD is parallel to the x-axis. The perpendicular bisector of BC passes through D. Find the area of the quadrilateral $ABCD$. [8]

8 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 & 7 \\ 1 & -5 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 8 & 6 \end{pmatrix}$, calculate

(i) $2\mathbf{A}$, [1]

(ii) $\mathbf{B}^2$, [2]

(iii) $\mathbf{BA}$. [2]

(b) **(i)** Given that $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 7 & 6 \end{pmatrix}$, find $\mathbf{C}^{-1}$. [2]

(ii) Given also that $\mathbf{D} = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$, find the matrix $\mathbf{X}$ such that $\mathbf{XC} = \mathbf{D}$. [2]

9 A particle starts from rest and moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, $v\,\text{m s}^{-1}$, is given by

$$v = 4\sin 2t.$$

(i) Find the distance travelled by the particle before it first comes to instantaneous rest. [5]

(ii) Find the acceleration of the particle when $t = 3$. [3]

10 In this question, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a unit vector due east and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is a unit vector due north.

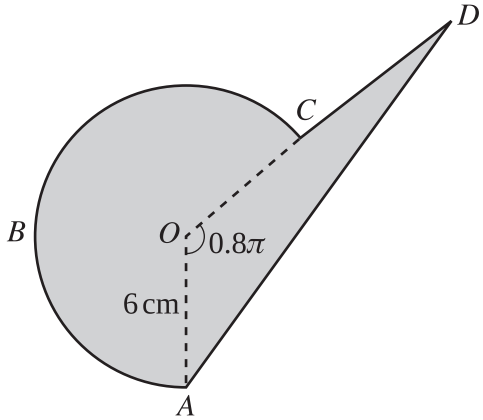
A lighthouse has position vector $\begin{pmatrix} 27 \\ 48 \end{pmatrix}$ km relative to an origin O. A boat moves in such a way that its position vector is given by $\begin{pmatrix} 4 + 8t \\ 12 + 6t \end{pmatrix}$ km, where t is the time, in hours, after 1200.

(i) Show that at 1400 the boat is 25 km from the lighthouse. [4]

(ii) Find the length of time for which the boat is less than 25 km from the lighthouse. [4]

11 Answer only **one** of the following two alternatives.

EITHER

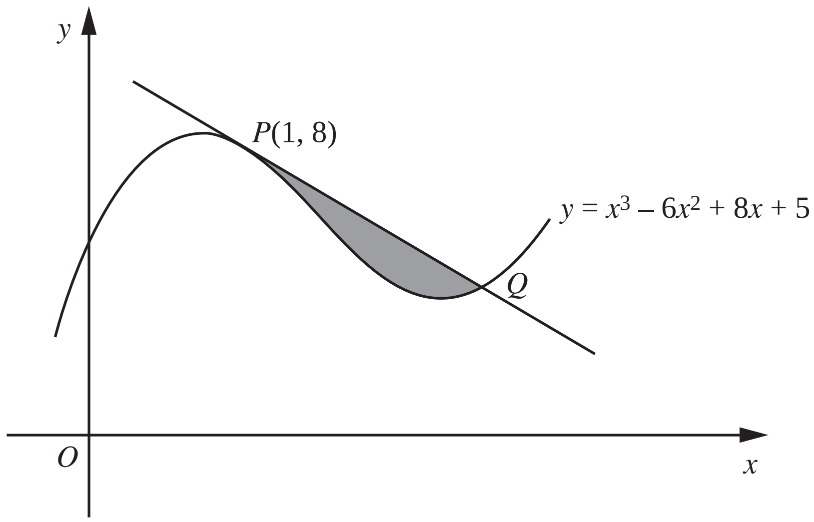


The diagram represents a company logo *ABCDA*, consisting of a sector *OABCO* of a circle, centre *O* and radius 6 cm, and a triangle *AOD*. Angle *AOC* = 0.8π radians and *C* is the mid-point of *OD*. Find

(i) the perimeter of the logo, [7]

(ii) the area of the logo. [5]

OR



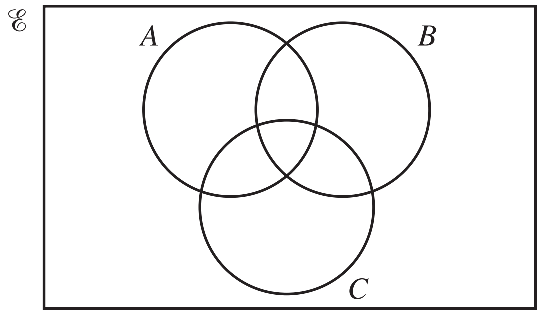
The diagram shows part of the curve $y = x^3 - 6x^2 + 8x + 5$. The tangent to the curve at the point $P(1, 8)$ cuts the curve at the point Q.

(i) Show that the x-coordinate of Q is 4. [6]

(ii) Find the area of the shaded region. [6]

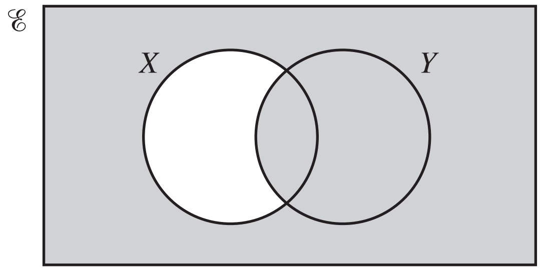
1 Find $\int \left(2 + 5x - \frac{1}{(x-2)^2}\right) dx$. [3]

2 **(a)**



Copy the diagram and shade the region which represents the set $A \cup (B \cap C')$. [1]

(b)



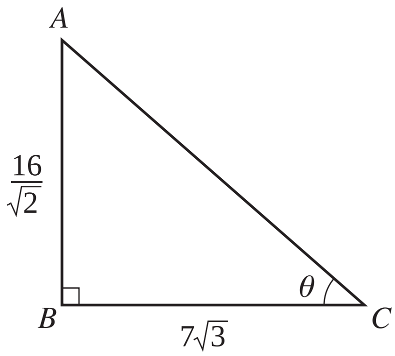
Express, in set notation, the set represented by the shaded region. [1]

(c) The universal set ℰ and the sets P and Q are such that n(ℰ) = 30, n(P) = 18 and n(Q) = 16. Given that n$(P \cup Q)' = 2$, find n$(P \cap Q)$. [2]

3 The volume V cm^3 of a spherical ball of radius r cm is given by $V = \frac{4}{3}\pi r^3$. Given that the radius is increasing at a constant rate of $\frac{1}{\pi}$ cm s^{-1}, find the rate at which the volume is increasing when $V = 288\pi$. [4]

[Turn over

4



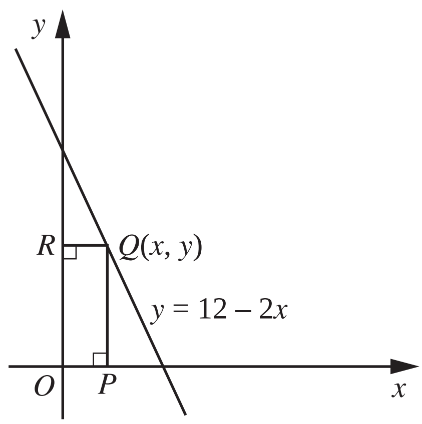
The diagram shows a right-angled triangle ABC in which the length of AB is $\frac{16}{\sqrt{2}}$, the length of BC is $7\sqrt{3}$ and angle BCA is θ.

(i) Find $\tan\theta$ in the form $\frac{a\sqrt{6}}{b}$, where a and b are integers. [2]

(ii) Calculate the length of AC, giving your answer in the form $c\sqrt{d}$, where c and d are integers and d is as small as possible. [3]

5 Solve the equation $2x^3 - 3x^2 - 11x + 6 = 0$. [6]

6



The diagram shows part of the line $y = 12 - 2x$. The point $Q(x, y)$ lies on this line and the points P and R lie on the coordinate axes such that $OPQR$ is a rectangle.

(i) Write down an expression, in terms of x, for the area A of the rectangle $OPQR$. [2]

(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iii) Find this stationary value of A and determine its nature. [2]

7 **(i)** Sketch the graph of $y = |3x + 9|$ for $-5 < x < 2$, showing the coordinates of the points where the graph meets the axes. [3]

(ii) On the same diagram, sketch the graph of $y = x + 6$. [1]

(iii) Solve the equation $|3x + 9| = x + 6$. [3]

8 **(a)** **(i)** Write down the first 4 terms, in ascending powers of x, of the expansion of $(1 - 3x)^7$. [3]

(ii) Find the coefficient of x^3 in the expansion of $(5 + 2x)(1 - 3x)^7$. [2]

(b) Find the term which is independent of x in the expansion of $\left(x^2 + \dfrac{2}{x}\right)^9$. [3]

9 **(i)** Given that $y = \dfrac{x + 2}{(4x + 12)^{\frac{1}{2}}}$, show that $\dfrac{dy}{dx} = \dfrac{k(x + 4)}{(4x + 12)^{\frac{3}{2}}}$, where k is a constant to be found. [5]

(ii) Hence evaluate $\displaystyle\int_1^{13} \frac{x + 4}{(4x + 12)^{\frac{3}{2}}}\,dx$. [3]

10 **(a)** Given that $\log_p X = 6$ and $\log_p Y = 4$, find the value of

(i) $\log_p \left(\dfrac{X^2}{Y}\right)$, [2]

(ii) $\log_Y X$. [2]

(b) Find the value of 2^z, where $z = 5 + \log_2 3$. [3]

(c) Express $\sqrt{512}$ as a power of 4. [2]

11 **(a)** Solve, for $0 < x < 3$ radians, the equation $4\sin x - 3 = 0$, giving your answers correct to 2 decimal places. [3]

(b) Solve, for $0° < y < 360°$, the equation $4\operatorname{cosec} y = 6\sin y + \cot y$. [6]

12 Answer only **one** of the following two alternatives.

EITHER

It is given that $f(x) = 4x^2 + kx + k$.

(i) Find the set of values of k for which the equation $f(x) = 3$ has no real roots. [5]

In the case where $k = 10$,

(ii) express $f(x)$ in the form $(ax + b)^2 + c$, [3]

(iii) find the least value of $f(x)$ and the value of x for which this least value occurs. [2]

OR

The functions f, g and h are defined, for $x \in \mathbb{R}$, by

$$f(x) = x^2 + 1,$$

$$g(x) = 2x - 5,$$

$$h(x) = 2^x.$$

(i) Write down the range of f. [1]

(ii) Find the value of gf(3). [2]

(iii) Solve the equation $fg(x) = g^{-1}(15)$. [5]

(iv) On the same axes, sketch the graph of $y = h(x)$ and the graph of the inverse function $y = h^{-1}(x)$, indicating clearly which graph represents h and which graph represents h^{-1}. [2]

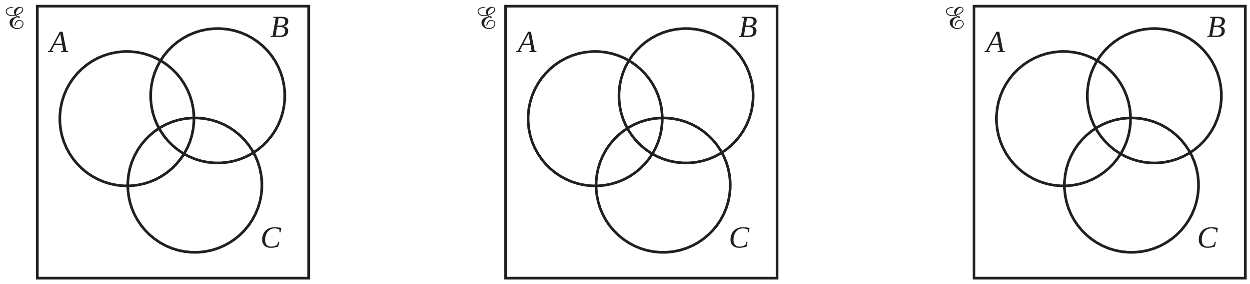
1 Show that $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}=2\operatorname{cosec}^2\theta$. [3]

For Examiner's Use

2 Express $\lg a + 3\lg b - 3$ as a single logarithm. [3]

3 **(a)** Shade the region corresponding to the set given below each Venn diagram.

For Examiner's Use



$(A \cup B) \cap C'$ $(A \cup B \cup C)'$ $(A \cap B) \cup (B \cap C) \cup (C \cap A)$

[3]

(b) Given that $P = \{p : \tan p = 1 \text{ for } 0^\circ \leqslant p \leqslant 540^\circ\}$, find $n(P)$. [1]

---

4 **(a)** Solve the equation $16^{3x-2} = 8^{2x}$. [3]

For Examiner's Use

(b) Given that $\dfrac{\sqrt{a^{\frac{4}{3}}b^{-\frac{2}{5}}}}{a^{-\frac{1}{3}}b^{\frac{3}{5}}} = a^p b^q$, find the value of p and of q. [2]

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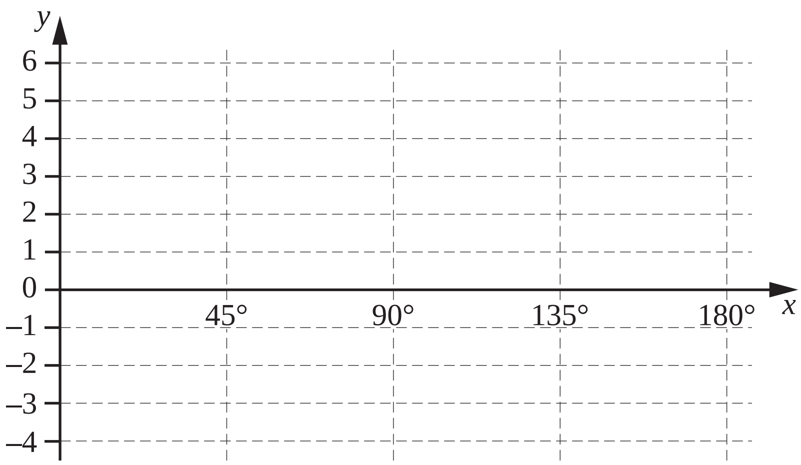
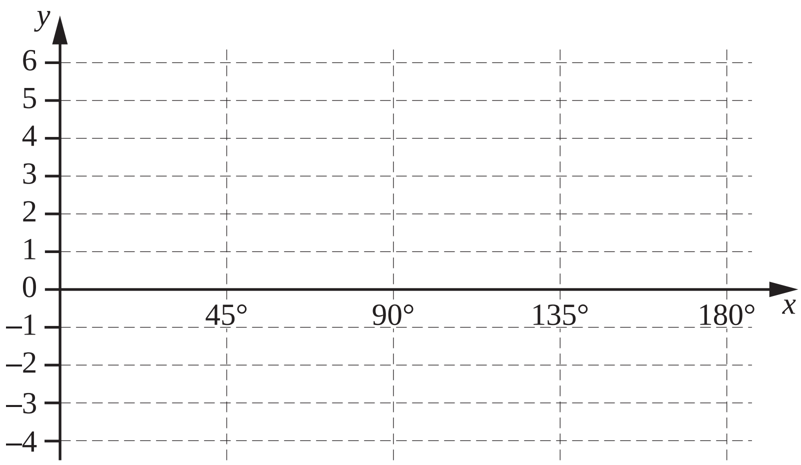
5 **(i)**



On the diagram above, sketch the curve $y = 1 + 3\sin 2x$ for $0° \leqslant x \leqslant 180°$. [3]

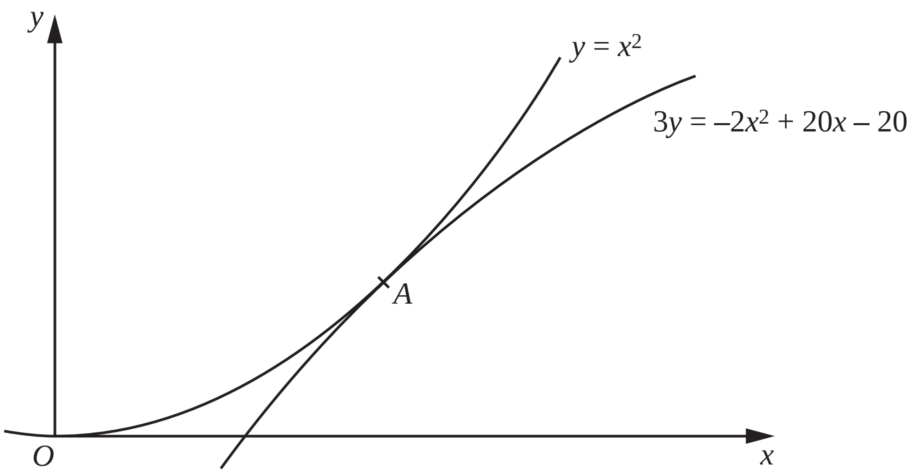
(ii)



On the diagram above, sketch the curve $y = |1 + 3\sin 2x|$ for $0° \leqslant x \leqslant 180°$. [1]

(iii) Write down the number of solutions of the equation $|1 + 3\sin 2x| = 1$ for $0° \leqslant x \leqslant 180°$. [1]

6 The curves $y = x^2$ and $3y = -2x^2 + 20x - 20$ meet at the point A.



(i) Show that the x-coordinate of A is 2. [1]

(ii) Show that the gradients of the two curves are equal at A. [3]

(iii) Find the equation of the tangent to the curves at A. [1]

7 The points A and B have coordinates (–2, 15) and (3, 5) respectively. The perpendicular to the line AB at the point A (–2, 15) crosses the y-axis at the point C. Find the area of the triangle ABC. [6]

For Examiner's Use

For Examiner's Use

8 **(a)** The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 2 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 1 & 3 & 4 \\ 1 & 5 & 6 & 7 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 9 \\ 10 \end{pmatrix}$. Write down, but do not evaluate, matrix products which may be calculated from the matrices **A**, **B** and **C**. [2]

(b) Given that $\mathbf{X} = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} 2x & 3y \\ x & 4y \end{pmatrix}$, find the value of x and of y such that

$\mathbf{X}^{-1}\mathbf{Y} = \begin{pmatrix} -12x + 3y & 6 \\ -7x + 3y & 6 \end{pmatrix}$. [6]

9 A body moves in a straight line such that, t s after passing through a fixed point O, its displacement from O is s m. The velocity v ms^{-1} of the body is such that $v = 5\cos 4t$.

For Examiner's Use

(i) Write down the velocity of the body as it passes through O. [1]

(ii) Find the value of t when the acceleration of the body is first equal to 10 ms^{-2}. [4]

(iii) Find the value of s when $t = 5$. [4]

10 **(a)** A curve is such that $\frac{dy}{dx} = ae^{1-x} - 3x^2$, where a is a constant. At the point (1, 4), the gradient of the curve is 2.

For Examiner's Use

(i) Find the value of a. [1]

(ii) Find the equation of the curve. [5]

For Examiner's Use

(b) **(i)** Find $\int (7x + 8)^{\frac{1}{3}} \mathrm{d}x$. [2]

(ii) Hence evaluate $\int_0^8 (7x + 8)^{\frac{1}{3}} \mathrm{d}x$. [2]

11 **(a)** The function f is such that $f(x) = 2x^2 - 8x + 5$.

(i) Show that $f(x) = 2(x + a)^2 + b$, where a and b are to be found. [2]

(ii) Hence, or otherwise, write down a suitable domain for f so that f^{-1} exists. [1]

(b) The functions g and h are defined respectively by

$$g(x) = x^2 + 4, \quad x \geqslant 0, \quad h(x) = 4x - 25, \; x \geqslant 0.$$

(i) Write down the range of g and of h^{-1}. [2]

(ii) On the axes below, sketch the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$, showing the coordinates of any points where the curves meet the coordinate axes. [3]

(iii) Find the value of x for which $\mathrm{gh}(x) = 85$. [4]

12 Answer only **one** of the following two alternatives.

EITHER

The equation of a curve is $y = (x - 1)(x^2 - 6x + 2)$.

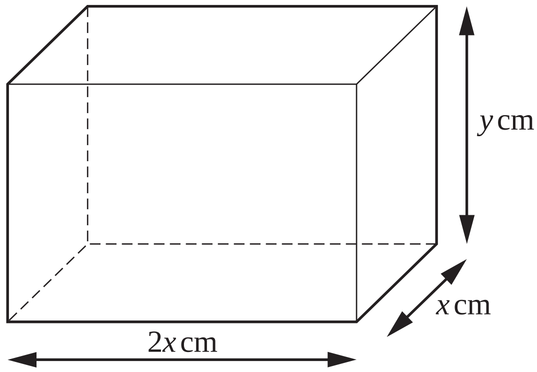
(i) Find the x-coordinates of the stationary points on the curve and determine the nature of each of these stationary points. [6]

(ii) Given that $z = y^2$ and that z is increasing at the constant rate of 10 units per second, find the rate of change of y when $x = 2$. [2]

(iii) Hence find the rate of change of x when $x = 2$. [2]

OR

The diagram shows a cuboid with a rectangular base of sides x cm and $2x$ cm. The height of the cuboid is y cm and its volume is $72\,\text{cm}^3$.



(i) Show that the surface area $A\,\text{cm}^2$ of the cuboid is given by

$$A = 4x^2 + \frac{216}{x}.$$

[3]

(ii) Given that x can vary, find the dimensions of the cuboid when A is a minimum. [4]

(iii) Given that x increases from 2 to $2 + p$, where p is small, find, in terms of p, the corresponding approximate change in A, stating whether this change is an increase or a decrease. [3]

1 Find the value of k for which the x-axis is a tangent to the curve

$$y = x^2 + (2k + 10)x + k^2 + 5.$$ [3]

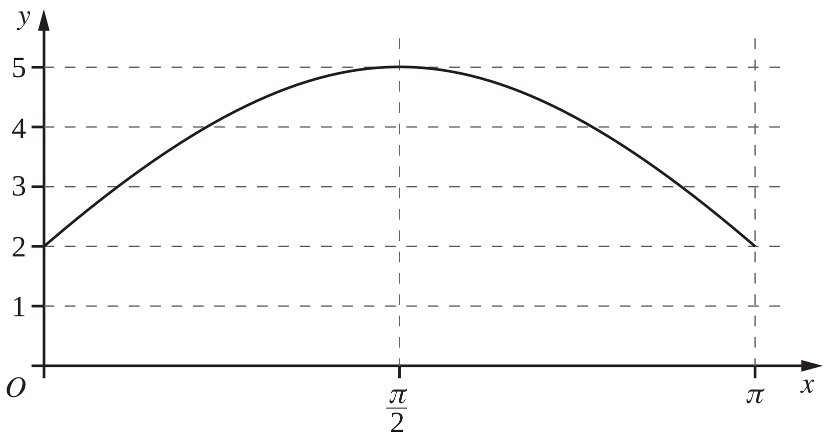
For Examiner's Use

2 The coefficient of x^3 in the expansion of $(2 + ax)^5$ is 10 times the coefficient of x^2 in the expansion of $\left(1 + \dfrac{ax}{3}\right)^4$. Find the value of a. [4]

For Examiner's Use

For Examiner's Use

3 **(a)**



The figure shows the graph of $y = k + m\sin px$ for $0 \leqslant x \leqslant \pi$, where k, m and p are positive constants. Complete the following statements.

k = ... m = ... p = ... [3]

(b) The function g is such that $\mathrm{g}(x) = 1 + 5\cos 3x$. Write down

(i) the amplitude of g, [1]

(ii) the period of g in terms of π. [1]

4 You must not use a calculator in Question 4.

For Examiner's Use

In the triangle ABC, angle $B = 90°$, $AB = 4 + 2\sqrt{2}$ and $BC = 1 + \sqrt{2}$.

(i) Find $\tan C$, giving your answer in the form $k\sqrt{2}$. [2]

(ii) Find the area of the triangle ABC, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers. [2]

(iii) Find the area of the square whose side is of length AC, giving your answer in the form $s + t\sqrt{2}$, where s and t are integers. [2]

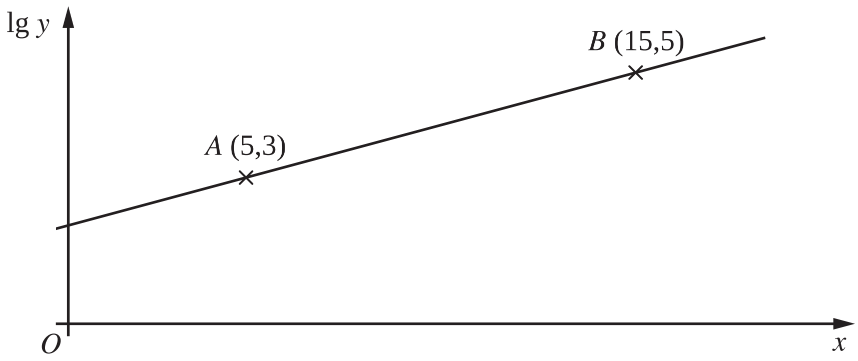
For Examiner's Use

5 **(i)** Show that $2x - 1$ is a factor of $2x^3 - 5x^2 + 10x - 4$. [2]

(ii) Hence show that $2x^3 - 5x^2 + 10x - 4 = 0$ has only one real root and state the value of this root. [4]

6 The figure shows the graph of a straight line with $\lg y$ plotted against x. The straight line passes through the points $A(5,3)$ and $B(15,5)$.

For Examiner's Use



(i) Express $\lg y$ in terms of x. [3]

(ii) Show that $y = a(10^{bx})$ where a and b are to be found. [3]

7 A team of 6 members is to be selected from 6 women and 8 men.

For Examiner's Use

(i) Find the number of different teams that can be selected. [1]

(ii) Find the number of different teams that consist of 2 women and 4 men. [3]

(iii) Find the number of different teams that contain no more than 1 woman. [3]

For Examiner's Use

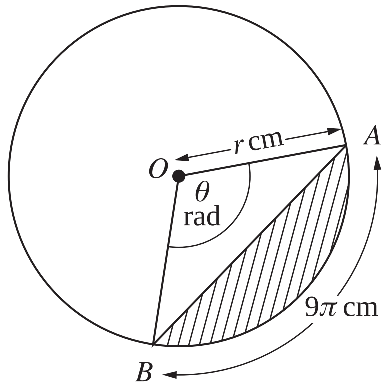
8 **(i)** Sketch the curve $y = (2x - 5)(2x + 1)$ for $-1 \leqslant x \leqslant 3$, stating the coordinates of the points where the curve meets the coordinate axes. [4]

(ii) State the coordinates of the stationary point on the curve. [1]

(iii) Using your answers to parts **(i)** and **(ii)**, sketch the curve $y = |(2x - 5)(2x + 1)|$ for $-1 \leqslant x \leqslant 3$. [2]

9 The figure shows a circle, centre O, radius r cm. The length of the arc AB of the circle is 9π cm. Angle AOB is θ radians and is 3 times angle OBA.

For Examiner's Use



(i) Show that $\theta = \dfrac{3\pi}{5}$. [2]

(ii) Find the value of r. [2]

(iii) Find the area of the shaded region. [3]

10 Relative to an origin O, points A and B have position vectors $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ and $\begin{pmatrix} 29 \\ -13 \end{pmatrix}$ respectively.

(i) Find a unit vector parallel to $\overrightarrow{AB}$. [3]

The points A, B and C lie on a straight line such that $2\overrightarrow{AC} = 3\overrightarrow{AB}$.

(ii) Find the position vector of the point C. [4]

11 Solve

For Examiner's Use

(i) $2\cot^2 x - 5\operatorname{cosec} x - 1 = 0$ for $0° < x < 180°$, [5]

(ii) $5\cos 2y - 4\sin 2y = 0$ for $0° < y < 180°$, [4]

(iii) $\cos\left(z + \frac{\pi}{6}\right) = -\frac{1}{2}$ for $0 < z < 2\pi$ radians. [3]

For Examiner's Use

12 Answer only **one** of the following two alternatives.

EITHER

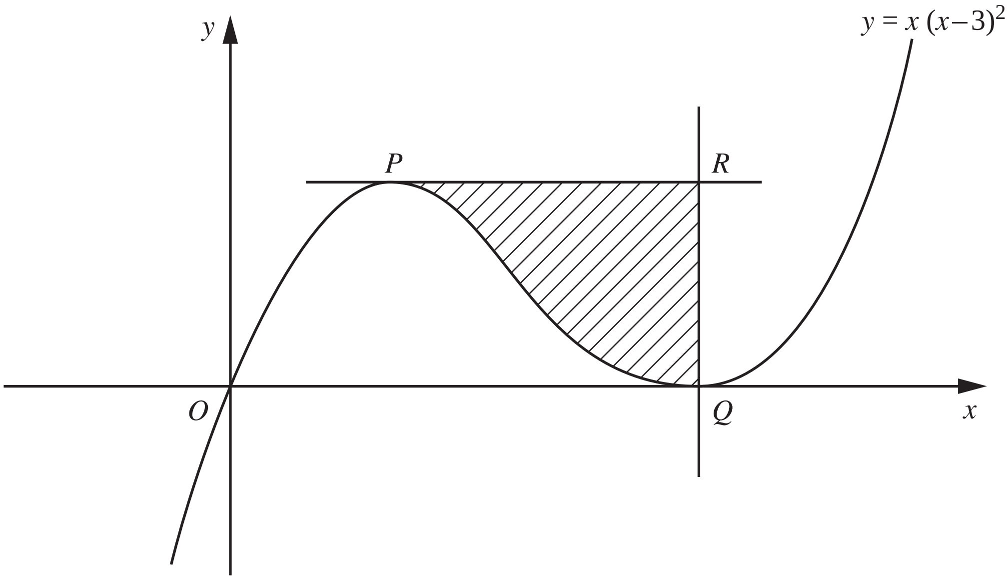
The tangent to the curve $y = 3x^3 + 2x^2 - 5x + 1$ at the point where $x = -1$ meets the y-axis at the point A.

(i) Find the coordinates of the point A. [3]

The curve meets the y-axis at the point B. The normal to the curve at B meets the x-axis at the point C. The tangent to the curve at the point where $x = -1$ and the normal to the curve at B meet at the point D.

(ii) Find the area of the triangle ACD. [7]

OR



The diagram shows the curve $y = x(x - 3)^2$. The curve has a maximum at the point P and touches the x-axis at the point Q. The tangent at P and the normal at Q meet at the point R. Find the area of the shaded region PQR. [10]

For Examiner's Use

1 Without using a calculator, express $\dfrac{(5 + 2\sqrt{3})^2}{2 + \sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers. [4]

---

2 **(i)** Find the coefficient of x^3 in the expansion of $\left(1 - \dfrac{x}{2}\right)^{12}$. [2]

(ii) Find the coefficient of x^3 in the expansion of $(1 + 4x)\left(1 - \dfrac{x}{2}\right)^{12}$. [3]

3 Relative to an origin O, the position vectors of the points A and B are $\mathbf{i} - 4\mathbf{j}$ and $7\mathbf{i} + 20\mathbf{j}$ respectively. The point C lies on AB and is such that $\overrightarrow{AC} = \frac{2}{3}\overrightarrow{AB}$. Find the position vector of C and the magnitude of this vector. [5]

For Examiner's Use

4 Find the set of values of k for which the line $y = 2x - 5$ cuts the curve $y = x^2 + kx + 11$ in two distinct points. [6]

For Examiner's Use

5 The expression $x^3 + 8x^2 + px - 25$ leaves a remainder of R when divided by $x - 1$ and a remainder of $-R$ when divided by $x + 2$.

For Examiner's Use

(i) Find the value of p. [4]

(ii) Hence find the remainder when the expression is divided by $x + 3$. [2]

For Examiner's Use

6 **(a)** A shelf contains 8 different travel books, of which 5 are about Europe and 3 are about Africa.

(i) Find the number of different ways the books can be arranged if there are no restrictions. [2]

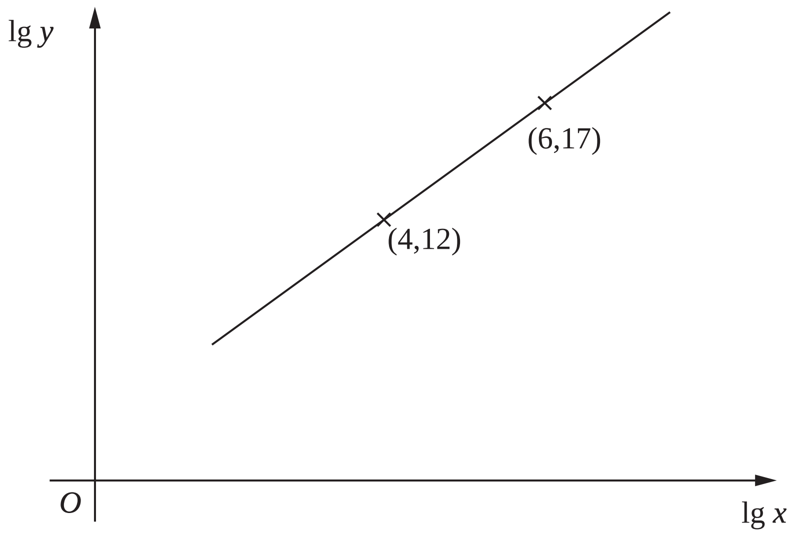
(ii) Find the number of different ways the books can be arranged if the 5 books about Europe are kept together. [2]

(b) 3 DVDs and 2 videotapes are to be selected from a collection of 7 DVDs and 5 videotapes.

Calculate the number of different selections that could be made. [3]

7 The variables x and y are related so that when $\lg y$ is plotted against $\lg x$ a straight line graph passing through the points (4, 12) and (6, 17) is obtained.

For Examiner's Use



(i) Express y in terms of x, giving your answer in the form $y = ax^b$. [6]

(ii) Find the value of x when $y = 300$. [2]

For Examiner's Use

8 The temperature, $T°$ Celsius, of an object, t minutes after it is removed from a heat source, is given by

$$T = 55e^{-0.1t} + 15.$$

(i) Find the temperature of the object at the instant it is removed from the heat source. [1]

(ii) Find the temperature of the object when $t = 8$. [1]

(iii) Find the value of t when $T = 25$. [3]

(iv) Find the rate of change of T when $t = 16$. [3]

For Examiner's Use

9 A coastguard station receives a distress call from a ship which is travelling at $15\,\text{km}\,\text{h}^{-1}$ on a bearing of 150°. A lifeboat leaves the coastguard station at 15 00 hours; at this time the ship is at a distance of 30 km on a bearing of 270°. The lifeboat travels in a straight line at constant speed and reaches the ship at 15 40 hours.

(i) Find the speed of the lifeboat. [5]

(ii) Find the bearing on which the lifeboat travelled. [3]

10 **(i)** Solve the equation $3\sin x + 4\cos x = 0$ for $0° < x < 360°$. [3]

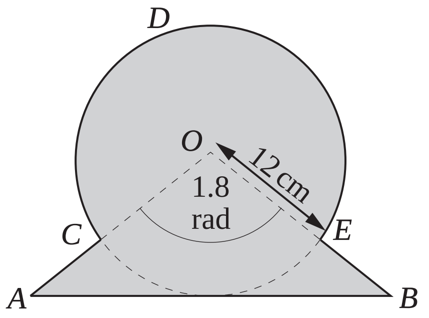
For Examiner's Use

(ii) Solve the equation $6\cos y + 6\sec y = 13$ for $0° < y < 360°$. [5]

(iii) Solve the equation $\sin(2z - 3) = 0.7$ for $0 < z < \pi$ radians. [3]

11 Answer only **one** of the following two alternatives.

EITHER



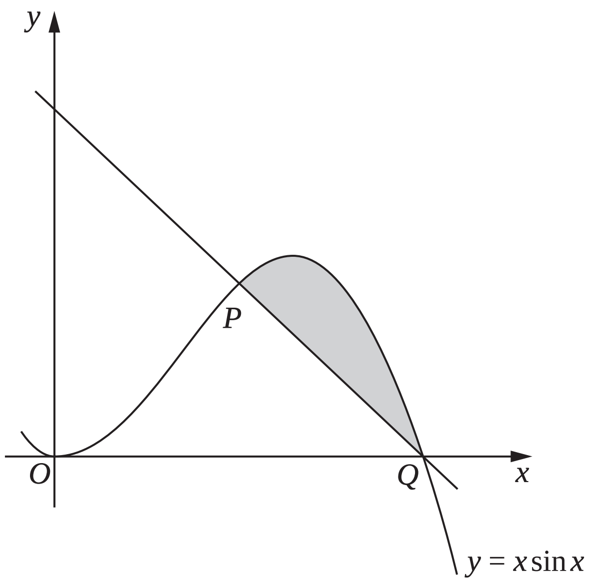
The diagram shows an isosceles triangle *AOB* and a sector *OCDEO* of a circle with centre *O*. The line *AB* is a tangent to the circle. Angle *AOB* = 1.8 radians and the radius of the circle is 12 cm.

(i) Show that the distance *AC* = 7.3 cm to 1 decimal place. [2]

(ii) Find the perimeter of the shaded region. [6]

(iii) Find the area of the shaded region. [4]

OR



The diagram shows part of the curve $y = x\sin x$ and the normal to the curve at the point $P\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$. The curve passes through the point $Q(\pi, 0)$.

(i) Show that the normal to the curve at *P* passes through the point *Q*. [4]

(ii) Given that $\frac{\mathrm{d}}{\mathrm{d}x}(x\cos x) = \cos x - x\sin x$, find $\int x\sin x\,\mathrm{d}x$. [3]

(iii) Find the area of the shaded region. [5]

1 **(i)** Given that $y = \sin 3x$, find $\frac{dy}{dx}$. [1]

(ii) Hence find the approximate increase in y as x increases from $\frac{\pi}{9}$ to $\frac{\pi}{9} + p$, where p is small. [2]

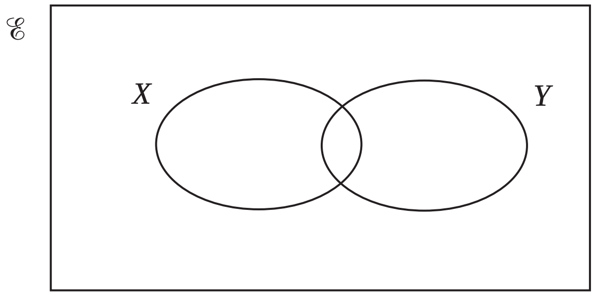
---

2 **(a)** An outdoor club has three sections, walking, biking and rock-climbing. Using $\mathscr{E}$ to denote the set of all members of the club and W, B and R to denote the members of the walking, biking and rock-climbing sections respectively, write each of the following statements using set notation.

(i) There are 72 members in the club. [1]

(ii) Every member of the rock-climbing section is also a member of the walking section. [1]

(b) **(i)**



On the diagram shade the region which represents the set $X \cup Y'$. [1]

(ii) Using set notation express the set $X \cup Y'$ in an alternative way. [1]

For Examiner's Use

3 **(i)** Given that $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -2 & 5 \end{pmatrix}$, find the inverse of the matrix $\mathbf{A} + \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. [3]

(ii) Hence, or otherwise, find the matrix $\mathbf{X}$ such that $\mathbf{AX} + \mathbf{X} = \mathbf{B}$, where $\mathbf{B} = \begin{pmatrix} 14 \\ 4 \end{pmatrix}$. [2]

4 **(a)** Prove that $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x \sec x$. [3]

For Examiner's Use

(b) An acute angle x is such that $\sin x = p$. Given that $\sin 2x = 2\sin x \cos x$, find an expression, in terms of p, for cosec $2x$. [3]

For Examiner's Use

5 **(i)** Given that $y = x\sqrt{2x + 15}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k(x+5)}{\sqrt{2x+15}}$, where k is a constant to be found. [3]

(ii) Hence find $\int \frac{x+5}{\sqrt{2x+15}}\,\mathrm{d}x$ and evaluate $\int_{-3}^{5} \frac{x+5}{\sqrt{2x+15}}\,\mathrm{d}x$. [3]

6 The line $y = 3x - 9$ intersects the curve $49x^2 - y^2 + 42x + 8y = 247$ at the points A and B. Find the length of the line AB. [7]

For Examiner's Use

For Examiner's Use

7 A particle moves in a straight line so that, t s after passing through a fixed point O, its velocity, v ms^{-1}, is given by $v = \dfrac{60}{(3t+4)^2}$.

(i) Find the velocity of the particle as it passes through O. [1]

(ii) Find the acceleration of the particle when $t = 2$. [3]

(iii) Find an expression for the displacement of the particle from O, t s after it has passed through O. [4]

8 **(a)** **(i)** Solve $3^x = 200$, giving your answer to 2 decimal places. [2]

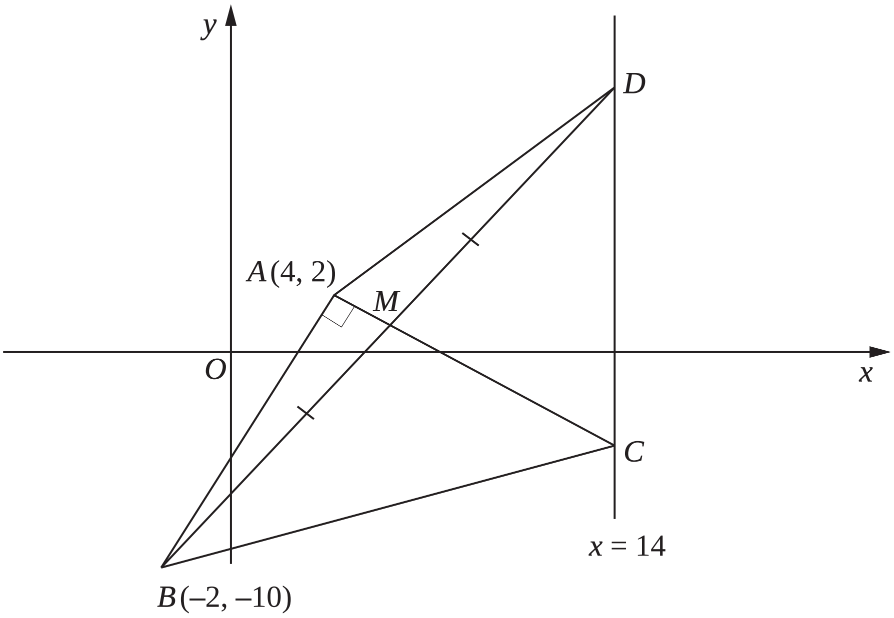
For Examiner's Use

(ii) Solve $\log_5 (5y + 40) - \log_5 (y + 2) = 2$. [4]

(b) Given that $\dfrac{(24z^3)^2}{27 \times 12z} = 2^a 3^b z^c$, evaluate a, b and c. [3]

9 Solutions to this question by accurate drawing will not be accepted.

For Examiner's Use



The diagram shows the quadrilateral *ABCD* in which *A* is the point (4, 2) and *B* is the point (–2, –10). The points *C* and *D* lie on the line $x = 14$. The diagonal *AC* is perpendicular to *AB* and passes through the mid-point, *M*, of the diagonal *BD*. Find the area of the quadrilateral *ABCD*. [9]

10 **(a)** **(i)** Express $18 + 16x - 2x^2$ in the form $a + b(x + c)^2$, where a, b and c are integers. [3]

For Examiner's Use

A function f is defined by $\mathrm{f} : x \to 18 + 16x - 2x^2$ for $x \in \mathbb{R}$.

(ii) Write down the coordinates of the stationary point on the graph of $y = \mathrm{f}(x)$. [1]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(b) A function g is defined by $g : x \rightarrow (x + 3)^2 - 7$ for $x > -3$.

For Examiner's Use

(i) Find an expression for $g^{-1}(x)$. [2]

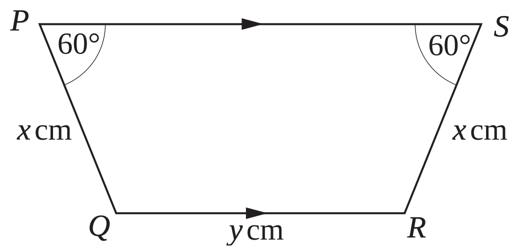
(ii) Solve the equation $g^{-1}(x) = g(0)$. [3]

11 Answer only **one** of the following two alternatives.

EITHER

(a) Using an equilateral triangle of side 2 units, find the exact value of sin 60° and of cos 60°. [3]

(b)



PQRS is a trapezium in which $PQ = RS = x$ cm and $QR = y$ cm.
Angle QPS = angle RSP = 60° and QR is parallel to PS.

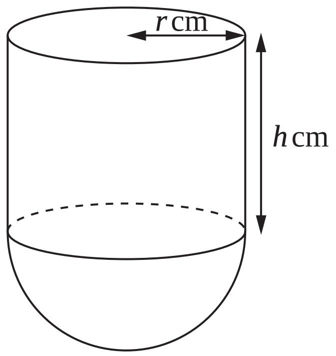
(i) Given that the perimeter of the trapezium is 60 cm, express y in terms of x. [2]

(ii) Given that the area of the trapezium is A cm^2, show that

$$A = \frac{\sqrt{3}\,(30x - x^2)}{2}.$$ [3]

(iii) Given that x can vary, find the value of x for which A has a stationary value and determine the nature of this stationary value. [4]

OR



For a sphere of radius r:

Volume = $\frac{4}{3}\pi r^3$

Surface area = $4\pi r^2$

The diagram shows a solid object in the form of a cylinder of height h cm and radius r cm on top of a hemisphere of radius r cm. Given that the volume of the object is 2880π cm^3,

(i) express h in terms of r, [2]

(ii) show that the external surface area, A cm^2, of the object is given by

$$A = \frac{5}{3}\pi r^2 + \frac{5760\pi}{r}.$$ [3]

Given that r can vary,

(iii) find the value of r for which A has a stationary value, [4]

(iv) find this stationary value of A, leaving your answer in terms of π, [2]

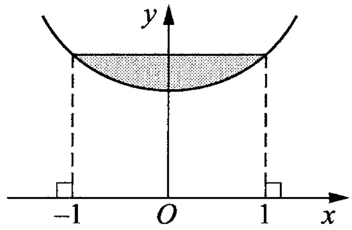
(v) determine the nature of this stationary value. [1]

1 Solve, for $0° \leqslant \theta \leqslant 360°$, the equation $4 \sin \theta + 3 \cos \theta = 0$. [3]

2 Find the values of m for which the line $y = mx - 9$ is a tangent to the curve $x^2 = 4y$. [4]

3 The speed v ms^{-1} of a particle travelling from A to B, at time t s after leaving A, is given by $v = 10t - t^2$. The particle starts from rest at A and comes to rest at B. Show that the particle has a speed of 5 ms^{-1} or greater for exactly $4\sqrt{5}$ s. [5]

4



The diagram shows part of the curve $y = e^x + e^{-x}$ for $-1 \leqslant x \leqslant 1$. Find, to 2 decimal places, the area of the shaded region. [6]

5 A company produces 4 types of central heating radiator, known as types A, B, C and D.
A builder buys radiators for all the houses on a new estate. There are 20 small houses, 30 medium-sized houses and 15 large houses.

A small house needs 3 radiators of type A, 2 of type B and 2 of type C.
A medium-sized house needs 2 radiators of type A, 3 of type C and 3 of type D.
A large house needs 1 radiator of type B, 6 of type C and 3 of type D.

The costs of the radiators are \$30 for type A, \$40 for B, \$50 for C and \$80 for D.

Using matrix multiplication twice, find the total cost to the builder of all the radiators for the estate. [6]

6 A curve is such that $\frac{dy}{dx} = \frac{6}{(2x-3)^2}$. Given that the curve passes through the point (3, 5), find the coordinates of the point where the curve crosses the x-axis. [6]

7 The function f is given by $f: x \mapsto x^3 + x - 1, x \in \mathbb{R}$.

(i) Determine whether or not the curve $y = f(x)$ has any turning points and hence explain why the function f has an inverse. [4]

(ii) Evaluate $f^{-1}(9)$. [3]

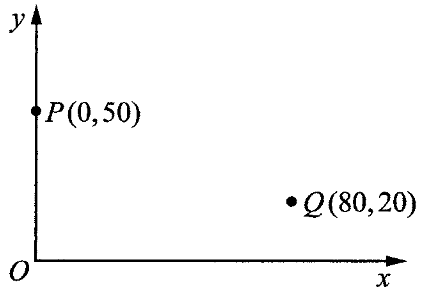
8 Solve the equation

(i) $e^x(2e^x - 1) = 10$, [3]

(ii) $\log_5(8y - 6) - \log_5(y - 5) = \log_4 16$. [4]

9 The line $2y = 3x - 6$ intersects the curve $xy = 12$ at the points P and Q. Find the equation of the perpendicular bisector of PQ. [8]

10



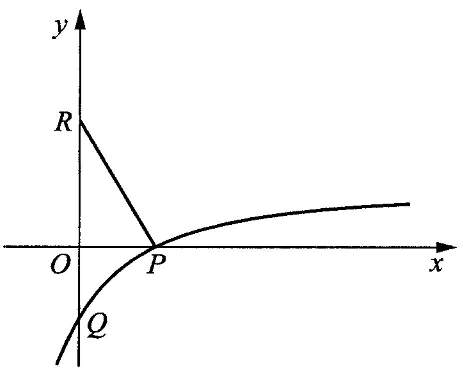
At 1200 hours, ship P is at the point with position vector $50\mathbf{j}$ km and ship Q is at the point with position vector $(80\mathbf{i} + 20\mathbf{j})$ km, as shown in the diagram. Ship P is travelling with velocity $(20\mathbf{i} + 10\mathbf{j})$ km h^{-1} and ship Q is travelling with velocity $(-10\mathbf{i} + 30\mathbf{j})$ km h^{-1}.

(i) Find an expression for the position vector of P and of Q at time t hours after 1200 hours. [3]

(ii) Use your answers to part **(i)** to determine the distance apart of P and Q at 1400 hours. [3]

(iii) Determine, with full working, whether or not P and Q will meet. [2]

11



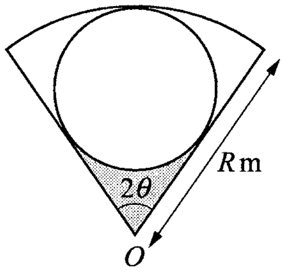
The diagram shows part of the curve $y = \dfrac{2x - 6}{x + 2}$ crossing the x-axis at P and the y-axis at Q. The normal to the curve at P meets the y-axis at R.

(i) Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k}{(x + 2)^2}$, evaluate k. [3]

(ii) Find the length of RQ. [6]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows a garden in the form of a sector of a circle, centre O, radius R m and angle 2θ. Within this garden a circular plot of the largest possible size is to be planted with roses. Given that the radius of this plot is r m,

(i) show that $R = r\left(1 + \frac{1}{\sin\theta}\right)$. [4]

Given also that $\theta = 30°$,

(ii) calculate the fraction of the garden that is to be planted with roses. [4]

When the circular plot has been constructed, the remainder of the garden consists of three regions. Given further that $R = 15$,

(iii) calculate, to 1 decimal place, the length of fencing required to fence along the perimeter of the shaded region. [3]

OR

A rectangle of area y m^2 has sides of length x m and $(Ax + B)$ m, where A and B are constants and x and y are variables. Values of x and y are given in the table below.

x	50	100	150	200	250
y	3700	11 000	21 600	36 000	53 500

(i) Use the data above in order to draw, on graph paper, the straight line graph of $\frac{y}{x}$ against x. [3]

(ii) Use your graph to estimate the value of A and of B. [4]

(iii) On the same diagram, draw the straight line representing the equation $y = x^2$ and explain the significance of the value of x given by the point of intersection of the two lines. [3]

(iv) State the value approached by the ratio of the two sides of the rectangle as x becomes increasingly large. [1]

1 Write down the inverse of the matrix $\begin{pmatrix} 4 & 3 \\ 7 & 6 \end{pmatrix}$ and use this to solve the simultaneous equations

$$4x + 3y + 7 = 0,$$
$$7x + 6y + 16 = 0.$$ [4]

2 Find the first three terms in the expansion, in ascending powers of x, of $(2 + x)^6$ and hence obtain the coefficient of x^2 in the expansion of $(2 + x - x^2)^6$. [4]

3 Given that $k = \frac{1}{\sqrt{3}}$ and that $p = \frac{1 + k}{1 - k}$, express in its simplest surd form

(i) p,

(ii) $p - \frac{1}{p}$.

[5]

4 Given that $\mathscr{E} = \{x: -5 < x < 5\}$,
$A = \{x: 8 > 2x + 1\}$,
$B = \{x: x^2 > x + 2\}$,

find the values of x which define the set $A \cap B$. [6]

5 **(a)** The producer of a play requires a total cast of 5, of which 3 are actors and 2 are actresses. He auditions 5 actors and 4 actresses for the cast. Find the total number of ways in which the cast can be obtained. [3]

(b) Find how many different odd 4-digit numbers less than 4000 can be made from the digits 1, 2, 3, 4, 5, 6, 7 if no digit may be repeated. [3]

6 The cubic polynomial f(x) is such that the coefficient of x^3 is -1 and the roots of the equation $f(x) = 0$ are 1, 2 and k. Given that f(x) has a remainder of 8 when divided by $x - 3$, find

(i) the value of k,

(ii) the remainder when f(x) is divided by $x + 3$.

[6]

7 **(i)** Differentiate $x \sin x$ with respect to x. [2]

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} x \cos x \, dx$. [4]

[Turn over

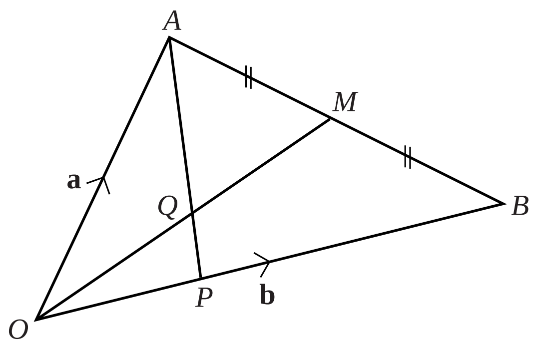
8 **(i)** Sketch the graph of $y = \ln x$. [2]

(ii) Determine the equation of the straight line which would need to be drawn on the graph of $y = \ln x$ in order to obtain a graphical solution of the equation $x^2 e^{x-2} = 1$. [4]

9 **(a)** Find, in its simplest form, the product of $a^{\frac{1}{3}} + b^{\frac{2}{3}}$ and $a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}$. [3]

(b) Given that $2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$, evaluate 10^x. [4]

10



In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AM} = \overrightarrow{MB}$ and $\overrightarrow{OP} = \frac{1}{3}\overrightarrow{OB}$.

(i) Express $\overrightarrow{AP}$ and $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

(ii) Given that $\overrightarrow{OQ} = \lambda\overrightarrow{OM}$, express $\overrightarrow{OQ}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [1]

(iii) Given that $\overrightarrow{AQ} = \mu\overrightarrow{AP}$, express $\overrightarrow{OQ}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(iv) Hence find the value of λ and of μ. [3]

11 A car moves on a straight road. As the driver passes a point A on the road with a speed of $20\,\text{ms}^{-1}$, he notices an accident ahead at a point B. He immediately applies the brakes and the car moves with an acceleration of $a\,\text{ms}^{-2}$, where $a = \dfrac{3t}{2} - 6$ and t s is the time after passing A. When $t = 4$, the car passes the accident at B. The car then moves with a constant acceleration of $2\,\text{ms}^{-2}$ until the original speed of $20\,\text{ms}^{-1}$ is regained at a point C. Find

(i) the speed of the car at B, [4]

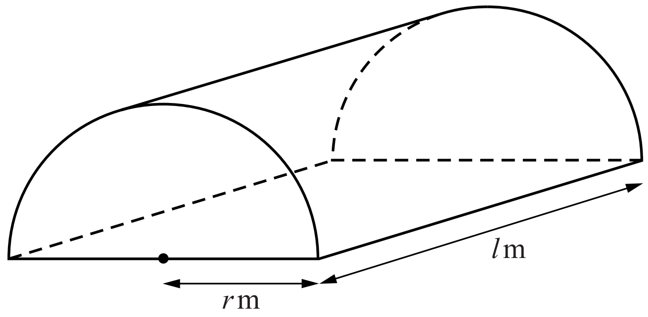
(ii) the distance AB, [3]

(iii) the time taken for the car to travel from B to C. [2]

Sketch the velocity-time graph for the journey from A to C. [2]

12 Answer only **one** of the following two alternatives.

EITHER



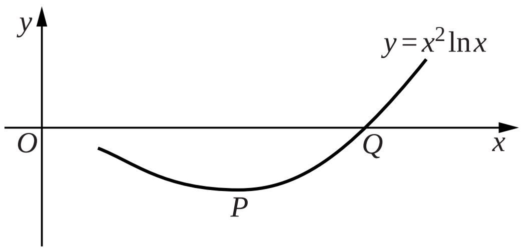
The diagram shows a greenhouse standing on a horizontal rectangular base. The vertical semicircular ends and the curved roof are made from polythene sheeting. The radius of each semicircle is r m and the length of the greenhouse is l m. Given that 120 m^2 of polythene sheeting is used for the greenhouse, express l in terms of r and show that the volume, V m^3, of the greenhouse is given by

$$V = 60r - \frac{\pi r^3}{2}.$$ [4]

Given that r can vary, find, to 2 decimal places, the value of r for which V has a stationary value. [3]

Find this value of V and determine whether it is a maximum or a minimum. [3]

OR



The diagram shows part of the curve $y = x^2 \ln x$, crossing the x-axis at Q and having a minimum point at P.

(i) Find the value of $\frac{dy}{dx}$ at Q. [4]

(ii) Show that the x-coordinate of P is $\frac{1}{\sqrt{e}}$. [3]

(iii) Find the value of $\frac{d^2y}{dx^2}$ at P. [3]

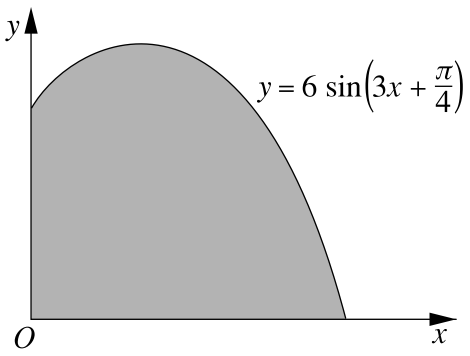
1 Find the values of k for which the line $x + 3y = k$ and the curve $y^2 = 2x + 3$ do not intersect. [4]

2 Without using a calculator, solve the equation $\dfrac{2^{x-3}}{8^{-x}} = \dfrac{32}{4^{\frac{1}{2}x}}$. [4]

3 The expression $x^3 + ax^2 + bx - 3$, where a and b are constants, has a factor of $x - 3$ and leaves a remainder of 15 when divided by $x + 2$. Find the value of a and of b. [5]

4 A rectangular block has a square base. The length of each side of the base is $(\sqrt{3} - \sqrt{2})$ m and the volume of the block is $(4\sqrt{2} - 3\sqrt{3})\ \text{m}^3$. Find, without using a calculator, the height of the block in the form $(a\sqrt{2} + b\sqrt{3})$ m, where a and b are integers. [5]

5



The diagram shows part of the curve $y = 6\sin\left(3x + \dfrac{\pi}{4}\right)$. Find the area of the shaded region bounded by the curve and the coordinate axes. [6]

6 In this question, **i** is a unit vector due east and **j** is a unit vector due north.

A plane flies from P to Q. The velocity, in still air, of the plane is $(280\mathbf{i} - 40\mathbf{j})\ \text{km h}^{-1}$ and there is a constant wind blowing with velocity $(50\mathbf{i} - 70\mathbf{j})\ \text{km h}^{-1}$. Find

(i) the bearing of Q from P, [4]

(ii) the time of flight, to the nearest minute, given that the distance PQ is 273 km. [2]

7 A small manufacturing firm produces four types of product, A, B, C and D. Each product requires three processes – assembly, finishing and packaging. The number of minutes required for each type of product for each process and the cost, in \$ per minute, of each process are given in the following table.

	Number of minutes				Cost per minute (\$)
Process \ Type	A	B	C	D	
Assembly	8	6	6	5	0.60
Finishing	5	4	3	2	0.20
Packaging	3	3	2	2	0.50

The firm receives an order for 40 of type A, 50 of type B, 50 of type C and 60 of type D. Write down three matrices such that matrix multiplication will give the total cost of meeting this order. Hence evaluate this total cost. [6]

8 Given that $y = \dfrac{\ln x}{2x+3}$, find

(i) $\dfrac{dy}{dx}$, [3]

(ii) the approximate change in y as x increases from 1 to $1 + p$, where p is small, [2]

(iii) the rate of change of x at the instant when $x = 1$, given that y is changing at the rate of 0.12 units per second at this instant. [2]

9 **(a)** Solve, for $0° < x < 360°$, the equation $4\tan^2 x + 8\sec x = 1$. [4]

(b) Given that $y < 4$, find the largest value of y such that $5\tan(2y + 1) = 16$. [4]

10 The function f is given by $\mathrm{f}: x \mapsto 5 - 3e^{\frac{1}{2}x}$, $x \in \mathbb{R}$.

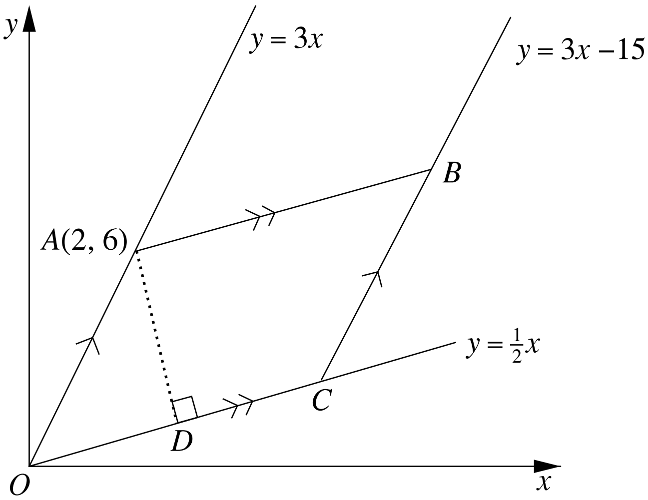
(i) State the range of f. [1]

(ii) Solve the equation $\mathrm{f}(x) = 0$, giving your answer correct to two decimal places. [2]

(iii) Sketch the graph of $y = \mathrm{f}(x)$, showing on your diagram the coordinates of the points of intersection with the axes. [2]

(iv) Find an expression for f^{-1} in terms of x. [3]

11 Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a parallelogram $OABC$ where O is the origin and A is the point $(2, 6)$. The equations of OA, OC and CB are $y = 3x$, $y = \frac{1}{2}x$ and $y = 3x - 15$ respectively. The perpendicular from A to OC meets OC at the point D. Find

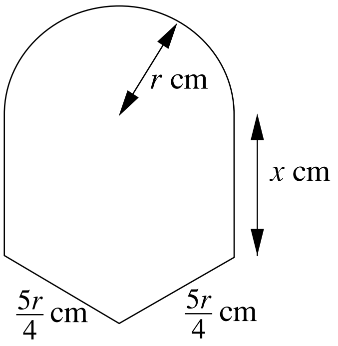
(i) the coordinates of C, B and D, [8]

(ii) the perimeter of the parallelogram $OABC$, correct to 1 decimal place. [3]

[Question 12 is printed on the next page.]

12 Answer only **one** of the following two alternatives.

EITHER



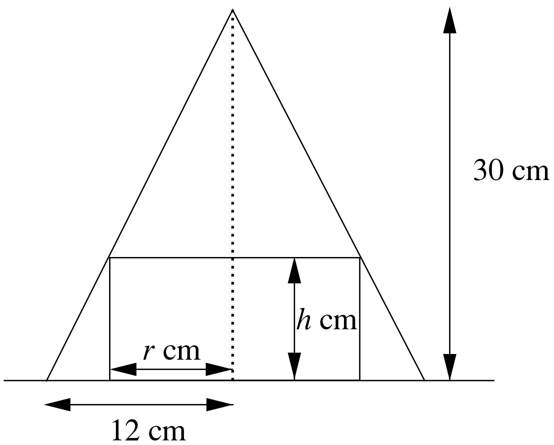
A piece of wire, 125 cm long, is bent to form the shape shown in the diagram. This shape encloses a plane region, of area A cm^2, consisting of a semi-circle of radius r cm, a rectangle of length x cm and an isosceles triangle having two equal sides of length $\frac{5r}{4}$ cm.

(i) Express x in terms of r and hence show that $A = 125r - \frac{\pi r^2}{2} - \frac{7r^2}{4}$. [6]

Given that r can vary,

(ii) calculate, to 1 decimal place, the value of r for which A has a maximum value. [4]

OR



The diagram shows the cross-section of a hollow cone of height 30 cm and base radius 12 cm and a solid cylinder of radius r cm and height h cm. Both stand on a horizontal surface with the cylinder inside the cone. The upper circular edge of the cylinder is in contact with the cone.

(i) Express h in terms of r and hence show that the volume, V cm^3, of the cylinder is given by $V = \pi(30r^2 - \frac{5}{2}r^3)$. [4]

Given that r can vary,

(ii) find the volume of the largest cylinder which can stand inside the cone and show that, in this case, the cylinder occupies $\frac{4}{9}$ of the volume of the cone. [6]

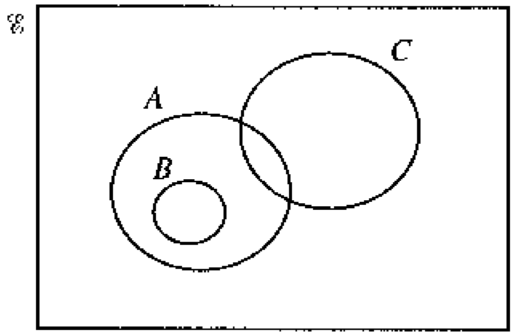
[The volume, V, of a cone of height H and radius R is given by $V = \frac{1}{3}\pi R^2 H$.]

1 The line $4y = x + 11$ intersects the curve $y^2 = 2x + 7$ at the points A and B. Find the coordinates of the mid-point of the line AB. [4]

2 Show that $\cos\theta\left(\dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta}\right)$ can be written in the form $k\tan\theta$ and find the value of k. [4]

3 Solve the equation $\log_2 x - \log_4(x - 4) = 2$. [4]

4

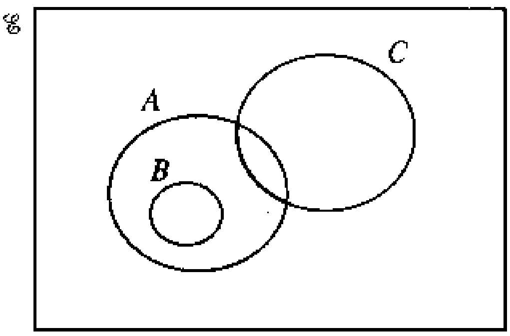


The diagram shows a universal set ℰ and the three sets A, B and C.

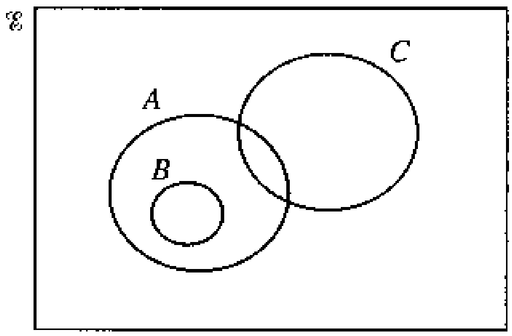
(i) Copy the above diagram and shade the region representing $(A \cup C) \cap B'$.

For each of the diagrams below, express, in set notation, the set represented by the shaded area in terms of A, B and C.

(ii)



(iii)



[4]

5 Obtain

(i) the first 3 terms in the expansion, in descending powers of x, of $(3x-1)^5$, [3]

(ii) the coefficient of x^4 in the expansion of $(3x-1)^5(2x+1)$. [2]

6 A particle travels in a straight line so that, t s after passing a fixed point A, its speed, v ms^{-1}, is given by

$$v = 40(e^{-t} - 0.1).$$

The particle comes to instantaneous rest at B. Calculate the distance AB. [6]

7 Given $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix}$, write down the inverse of **A** and of **B**. [3]

Hence find

(i) the matrix **C** such that $2\mathbf{A}^{-1} + \mathbf{C} = \mathbf{B}$, [2]

(ii) the matrix **D** such that $\mathbf{BD} = \mathbf{A}$. [2]

8 A garden centre sells 10 different varieties of rose bush. A gardener wishes to buy 6 rose bushes, all of different varieties.

(i) Calculate the number of ways she can make her selection. [2]

Of the 10 varieties, 3 are pink, 5 are red and 2 are yellow. Calculate the number of ways in which her selection of 6 rose bushes could contain

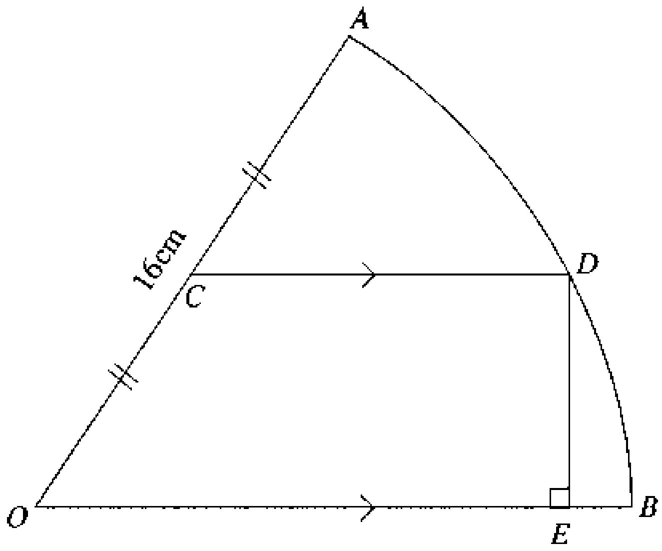
(ii) no pink rose bush, [1]

(iii) at least one rose bush of each colour. [4]

9 **(i)** Given that $y = (2x+3)\sqrt{4x-3}$, show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{kx}{\sqrt{4x-3}}$ and state the value of k. [5]

(ii) Hence evaluate $\displaystyle\int_1^7 \frac{x}{\sqrt{4x-3}}\,dx$. [3]

10



In the diagram, OAB is a sector of a circle, centre O and radius 16 cm, and the length of the arc AB is 19.2 cm. The mid-point of OA is C and the line through C parallel to OB meets the arc AB at D. The perpendicular from D to OB meets OB at E.

(i) Find angle AOB in radians. [2]

(ii) Find the length of DE. [2]

(iii) Show that angle DOE is approximately 0.485 radians. [2]

(iv) Find the area of the shaded region. [4]

11 A particle, moving in a certain medium with speed $v\,\text{ms}^{-1}$, experiences a resistance to motion of R N. It is believed that R and v are related by the equation $R = kv^{\beta}$, where k and β are constants.
The table shows experimental values of the variables v and R.

v	5	10	15	20	25
R	32	96	180	290	410

(i) Using graph paper, plot lg R against lg v and draw a straight line graph. [3]

Use your graph to estimate

(ii) the value of k and of β, [5]

(iii) the speed for which the resistance is 75 N. [2]

12 Answer only **one** of the following two alternatives.

EITHER

Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f}: x \mapsto 3x - 2, \quad x \neq \tfrac{4}{3},$$
$$\mathrm{g}: x \mapsto \frac{4}{2-x}, \quad x \neq 2.$$

(i) Solve the equation $\mathrm{gf}(x) = 2$. [3]

(ii) Determine the number of real roots of the equation $\mathrm{f}(x) = \mathrm{g}(x)$. [2]

(iii) Express f^{-1} and g^{-1} in terms of x. [3]

(iv) Sketch, on a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, stating the coordinates of the point of intersection of the two graphs. [3]

OR

(i) Find the value of a and of b for which $1 - x^2 + 6x$ can be expressed in the form $a - (x + b)^2$. [3]

A function f is defined by $\mathrm{f}: x \mapsto 1 - x^2 + 6x$ for the domain $x \geqslant 4$.

(ii) Explain why f has an inverse. [2]

(iii) Find an expression for f^{-1} in terms of x. [2]

A function g is defined by $\mathrm{g}: x \mapsto 1 - x^2 + 6x$ for the domain $2 \leqslant x \leqslant 7$.

(iv) Find the range of g. [2]

(v) Sketch the graph of $y = |\mathrm{g}(x)|$ for $2 \leqslant x \leqslant 7$. [2]

1 The position vectors of points A, B and C, relative to an origin O, are $\mathbf{i} + 9\mathbf{j}$, $5\mathbf{i} - 3\mathbf{j}$ and $k(\mathbf{i} + 3\mathbf{j})$ respectively, where k is a constant. Given that C lies on the line AB, find the value of k. [4]

2 A youth club has facilities for members to play pool, darts and table-tennis. Every member plays at least one of the three games. P, D and T represent the sets of members who play pool, darts and table-tennis respectively. Express each of the following in set language and illustrate each by means of a Venn diagram.

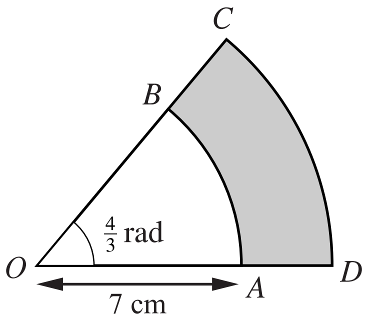
(i) The set of members who only play pool. [2]

(ii) The set of members who play exactly 2 games, neither of which is darts. [2]

3 Without using a calculator, solve, for x and y, the simultaneous equations

$$8^x \div 2^y = 64,$$
$$3^{4x} \times (\tfrac{1}{9})^{y-1} = 81.$$ [5]

4



The diagram shows a sector COD of a circle, centre O, in which angle $COD = \frac{4}{3}$ radians. The points A and B lie on OD and OC respectively, and AB is an arc of a circle, centre O, of radius 7 cm. Given that the area of the shaded region $ABCD$ is 48 cm^2, find the perimeter of this shaded region. [6]

5 Given that the expansion of $(a + x)(1 - 2x)^n$ in ascending powers of x is $3 - 41x + bx^2 + \ldots$, find the values of the constants a, n and b. [6]

6 The function f is defined, for $0 < x < \pi$, by $f(x) = 5 + 3\cos 4x$. Find

(i) the amplitude and the period of f, [2]

(ii) the coordinates of the maximum and minimum points of the curve $y = f(x)$. [4]

7 **(a)** Find the number of different arrangements of the 9 letters of the word SINGAPORE in which S does **not** occur as the first letter. [2]

(b) 3 students are selected to form a chess team from a group of 5 girls and 3 boys. Find the number of possible teams that can be selected in which there are more girls than boys. [4]

8 The function f is defined, for $x \in \mathbb{R}$, by

$$\mathrm{f} : x \mapsto \frac{3x + 11}{x - 3}, \quad x \neq 3.$$

(i) Find f^{-1} in terms of x and explain what this implies about the symmetry of the graph of $y = \mathrm{f}(x)$. [3]

The function g is defined, for $x \in \mathbb{R}$, by

$$\mathrm{g} : x \mapsto \frac{x - 3}{2}.$$

(ii) Find the values of x for which $\mathrm{f}(x) = \mathrm{g}^{-1}(x)$. [3]

(iii) State the value of x for which $\mathrm{gf}(x) = -2$. [1]

9 **(a)** Solve, for $0° \leqslant x \leqslant 360°$, the equation $\sin^2 x = 3\cos^2 x + 4\sin x$. [4]

(b) Solve, for $0 < y < 4$, the equation $\cot 2y = 0.25$, giving your answers in radians correct to 2 decimal places. [4]

10 A curve has the equation $y = x^3 \ln x$, where $x > 0$.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

Hence

(ii) calculate the value of $\ln x$ at the stationary point of the curve, [2]

(iii) find the approximate increase in y as x increases from e to $\mathrm{e} + p$, where p is small, [2]

(iv) find $\int x^2 \ln x \,\mathrm{d}x$. [3]

11 The line $4y = 3x + 1$ intersects the curve $xy = 28x - 27y$ at the point $P(1, 1)$ and at the point Q. The perpendicular bisector of PQ intersects the line $y = 4x$ at the point R. Calculate the area of triangle PQR. [9]

12 Answer only **one** of the following two alternatives.

EITHER

(a) At the beginning of 1960, the number of animals of a certain species was estimated at 20 000. This number decreased so that, after a period of n years, the population was

$$20\,000\mathrm{e}^{-0.05n}.$$

Estimate

(i) the population at the beginning of 1970, [1]

(ii) the year in which the population would be expected to have first decreased to 2000. [3]

(b) Solve the equation $3^{x+1} - 2 = 8 \times 3^{x-1}$. [6]

OR

A curve has the equation $y = \mathrm{e}^{\frac{1}{2}x} + 3\mathrm{e}^{-\frac{1}{2}x}$.

(i) Show that the exact value of the y-coordinate of the stationary point of the curve is $2\sqrt{3}$. [4]

(ii) Determine whether the stationary point is a maximum or a minimum. [2]

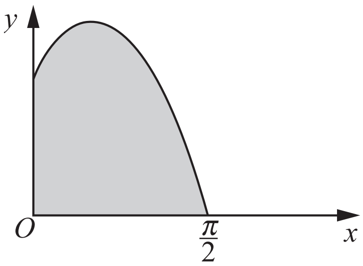
(iii) Calculate the area enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$. [4]

1 Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -5 & 4 \end{pmatrix}$, find $\mathbf{A}^{-1}$ and **hence** solve the simultaneous equations

$$2x + 3y + 4 = 0$$
$$-5x + 4y + 13 = 0.$$ [4]

2 Given that $\sqrt{a + b\sqrt{3}} = \dfrac{13}{4 + \sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and of b. [4]

3 The diagram shows part of the curve $y = 3\sin 2x + 4\cos x$.



Find the area of the shaded region, bounded by the curve and the coordinate axes. [5]

4 Find the values of k for which the line $y = x + 2$ meets the curve $y^2 + (x + k)^2 = 2$. [5]

5 Solve the equation $\log_{16}(3x - 1) = \log_4(3x) + \log_4(0.5)$. [6]

6 Given that $x = 3\sin\theta - 2\cos\theta$ and $y = 3\cos\theta + 2\sin\theta$,

(i) find the value of the acute angle θ for which $x = y$, [3]

(ii) show that $x^2 + y^2$ is constant for all values of θ. [3]

7 Given that $6x^3 + 5ax - 12a$ leaves a remainder of -4 when divided by $x - a$, find the possible values of a. [7]

8 A motor boat travels in a straight line across a river which flows at 3 ms^{-1} between straight parallel banks 200 m apart. The motor boat, which has a top speed of 6 ms^{-1} in still water, travels directly from a point A on one bank to a point B, 150 m downstream of A, on the opposite bank. Assuming that the motor boat is travelling at top speed, find, to the nearest second, the time it takes to travel from A to B. [7]

9 In order that each of the equations

(i) $y = ab^x$,

(ii) $y = Ax^k$,

(iii) $px + qy = xy$,

where a, b, A, k, p and q are unknown constants, may be represented by a straight line, they each need to be expressed in the form $Y = mX + c$, where X and Y are each functions of x and/or y, and m and c are constants. Copy the following table and insert in it an expression for Y, X, m and c for each case.

	Y	X	m	c
$y = ab^x$				
$y = Ax^k$				
$px + qy = xy$				

[7]

10 The function f is defined by $\text{f}: x \mapsto |x^2 - 8x + 7|$ for the domain $3 \leqslant x \leqslant 8$.

(i) By first considering the stationary value of the function $x \mapsto x^2 - 8x + 7$, show that the graph of $y = \text{f}(x)$ has a stationary point at $x = 4$ and determine the nature of this stationary point. [4]

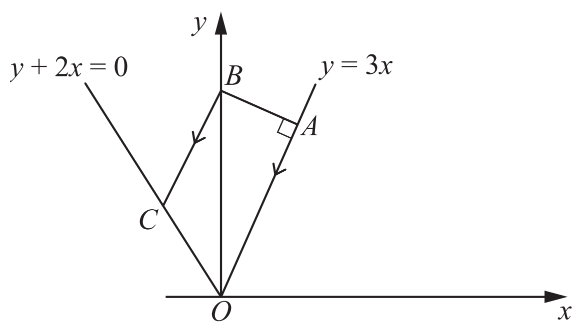
(ii) Sketch the graph of $y = \text{f}(x)$. [2]

(iii) Find the range of f. [2]

The function g is defined by $\text{g}: x \mapsto |x^2 - 8x + 7|$ for the domain $3 \leqslant x \leqslant k$.

(iv) Determine the largest value of k for which g^{-1} exists. [1]

11



The diagram shows a trapezium $OABC$, where O is the origin. The equation of OA is $y = 3x$ and the equation of OC is $y + 2x = 0$. The line through A perpendicular to OA meets the y-axis at B and BC is parallel to AO. Given that the length of OA is $\sqrt{250}$ units, calculate the coordinates of A, of B and of C. [10]

12 Answer only **one** of the following two alternatives.

EITHER

A particle, travelling in a straight line, passes a fixed point O on the line with a speed of 0.5 ms^{-1}. The acceleration, $a\,\text{ms}^{-2}$, of the particle, t s after passing O, is given by $a = 1.4 - 0.6t$.

(i) Show that the particle comes instantaneously to rest when $t = 5$. [4]

(ii) Find the total distance travelled by the particle between $t = 0$ and $t = 10$. [6]

OR

Each member of a set of curves has an equation of the form $y = ax + \frac{b}{x^2}$, where a and b are integers.

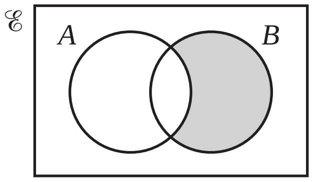
(i) For the curve where $a = 3$ and $b = 2$, find the area bounded by the curve, the x-axis and the lines $x = 2$ and $x = 4$. [4]

Another curve of this set has a stationary point at (2, 3).

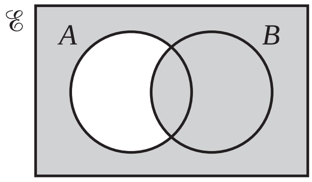
(ii) Find the value of a and of b in this case and determine the nature of the stationary point. [6]

1 Find the set of values of x for which $(x - 6)^2 > x$. [3]

2 **(a)** **(i)**

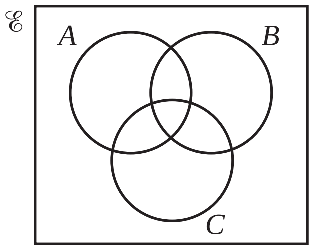


(ii)



For each of the Venn diagrams above, express the shaded region in set notation. [2]

(b)



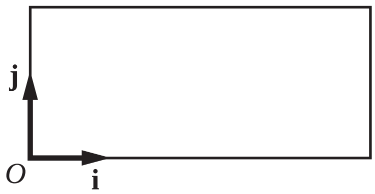
(i) Copy the Venn diagram above and shade the region that represents $A \cap B \cap C'$. [1]

(ii) Copy the Venn diagram above and shade the region that represents $A' \cap (B \cup C)$. [1]

3 Find the values of the constant c for which the line $2y = x + c$ is a tangent to the curve $y = 2x + \frac{6}{x}$. [4]

4 A cuboid has a square base of side $(2 - \sqrt{3})$ m and a volume of $(2\sqrt{3} - 3)$ m^3. Find the height of the cuboid in the form $(a + b\sqrt{3})$ m, where a and b are integers. [4]

5 The diagram, which is not drawn to scale, shows a horizontal rectangular surface. One corner of the surface is taken as the origin O and $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the edges of the surface.



A fly, F, starts at the point with position vector $(\mathbf{i} + 12\mathbf{j})$ cm and crawls across the surface with a velocity of $(3\mathbf{i} + 2\mathbf{j})$ cm s^{-1}. At the instant that the fly starts crawling, a spider, S, at the point with position vector $(85\mathbf{i} + 5\mathbf{j})$ cm, sets off across the surface with a velocity of $(-5\mathbf{i} + k\mathbf{j})$ cm s^{-1}, where k is a constant. Given that the spider catches the fly, calculate the value of k. [6]

6 A particle starts from rest at a fixed point O and moves in a straight line towards a point A. The velocity, $v\,\mathrm{m\,s}^{-1}$, of the particle, t seconds after leaving O, is given by $v = 6 - 6e^{-3t}$. Given that the particle reaches A when $t = \ln 2$, find

(i) the acceleration of the particle at A, [3]

(ii) the distance OA. [4]

7 **(a)** Solve $\log_7(17y + 15) = 2 + \log_7(2y - 3)$. [4]

(b) Evaluate $\log_p 8 \times \log_{16} p$. [3]

8 A curve has the equation $y = (x + 2)\sqrt{x-1}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{kx}{\sqrt{x-1}}$, where k is a constant, and state the value of k. [4]

(ii) Hence evaluate $\displaystyle\int_2^5 \frac{x}{\sqrt{x-1}}\,dx$. [4]

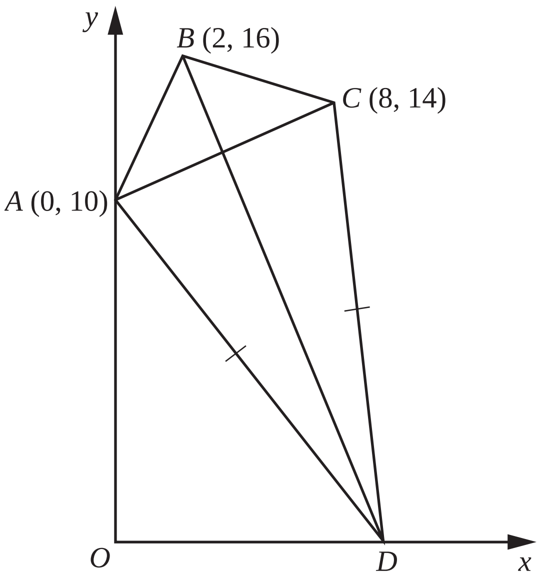
9 **(a)** Find all the angles between 0° and 360° which satisfy the equation

$$3\cos x = 8\tan x.$$ [5]

(b) Given that $4 \leqslant y \leqslant 6$, find the value of y for which

$$2\cos\left(\frac{2y}{3}\right) + \sqrt{3} = 0.$$ [3]

10 Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a quadrilateral $ABCD$ in which A is (0, 10), B is (2, 16) and C is (8, 14).

(i) Show that triangle ABC is isosceles. [2]

The point D lies on the x-axis and is such that $AD = CD$. Find

(ii) the coordinates of D, [4]

(iii) the ratio of the area of triangle ABC to the area of triangle ACD. [3]

11 A function f is defined by $\mathrm{f} : x \mapsto |2x - 3| - 4$, for $-2 \leqslant x \leqslant 3$.

(i) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(ii) State the range of f. [2]

(iii) Solve the equation $\mathrm{f}(x) = -2$. [3]

A function g is defined by $\mathrm{g}: x \mapsto |2x - 3| - 4$, for $-2 \leqslant x \leqslant k$.

(iv) State the largest value of k for which g has an inverse. [1]

(v) Given that g has an inverse, express g in the form $\mathrm{g}: x \mapsto ax + b$, where a and b are constants. [2]

12 Answer only **one** of the following two alternatives.

EITHER

Variables x and y are related by the equation $yx^n = a$, where a and n are constants. The table below shows measured values of x and y.

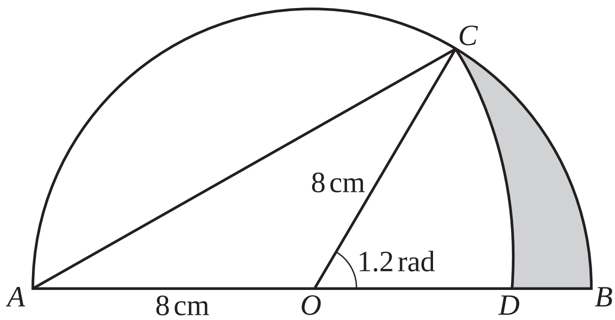
x	1.5	2	2.5	3	3.5
y	7.3	3.5	2.0	1.3	0.9

(i) On graph paper plot $\lg y$ against $\lg x$, using a scale of 2 cm to represent 0.1 on the $\lg x$ axis and 1 cm to represent 0.1 on the $\lg y$ axis. Draw a straight line graph to represent the equation $yx^n = a$. [3]

(ii) Use your graph to estimate the value of a and of n. [4]

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of x for which $x^{n+2} = a$. [3]

OR



The diagram shows a semicircle, centre O, of radius 8 cm. The radius OC makes an angle of 1.2 radians with the radius OB. The arc CD of a circle has centre A and the point D lies on OB. Find the area of

(i) sector COB, [2]

(ii) sector CAD, [5]

(iii) the shaded region. [3]

1 Variables V and t are related by the equation

$$V = 1000e^{-kt},$$

where k is a constant. Given that $V = 500$ when $t = 21$, find

(i) the value of k, [2]

(ii) the value of V when $t = 30$. [2]

2 The line $x + y = 10$ meets the curve $y^2 = 2x + 4$ at the points A and B. Find the coordinates of the mid-point of AB. [5]

3 **(i)** Given that $y = 1 + \ln(2x - 3)$, obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Hence find, in terms of p, the approximate value of y when $x = 2 + p$, where p is small. [3]

4 The function f is given by $f : x \mapsto 2 + 5 \sin 3x$ for $0° \leqslant x \leqslant 180°$.

(i) State the amplitude and period of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

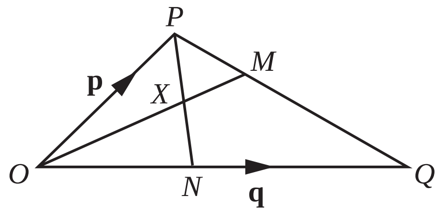
5 The binomial expansion of $(1 + px)^n$, where $n > 0$, in ascending powers of x is

$$1 - 12x + 28p^2x^2 + qx^3 + \dots .$$

Find the value of n, of p and of q. [6]

6 It is given that $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 5 & p \end{pmatrix}$ and that $\mathbf{A} + \mathbf{A}^{-1} = k\mathbf{I}$, where p and k are constants and $\mathbf{I}$ is the identity matrix. Evaluate p and k. [6]

7



In the diagram $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$, $\overrightarrow{PM} = \frac{1}{3}\overrightarrow{PQ}$ and $\overrightarrow{ON} = \frac{2}{5}\overrightarrow{OQ}$.

(i) Given that $\overrightarrow{OX} = m\overrightarrow{OM}$, express $\overrightarrow{OX}$ in terms of m, $\mathbf{p}$ and $\mathbf{q}$. [2]

(ii) Given that $\overrightarrow{PX} = n\overrightarrow{PN}$, express $\overrightarrow{OX}$ in terms of n, $\mathbf{p}$ and $\mathbf{q}$. [3]

(iii) Hence evaluate m and n. [2]

8 **(a)** Find the value of each of the integers p and q for which $\left(\frac{25}{16}\right)^{-\frac{3}{2}} = 2^p \times 5^q$. [2]

(b) **(i)** Express the equation $4^x - 2^{x+1} = 3$ as a quadratic equation in 2^x. [2]

(ii) Hence find the value of x, correct to 2 decimal places. [3]

9 The function $f(x) = x^3 - 6x^2 + ax + b$, where a and b are constants, is exactly divisible by $x - 3$ and leaves a remainder of –55 when divided by $x + 2$.

(i) Find the value of a and of b. [4]

(ii) Solve the equation $f(x) = 0$. [4]

10 A curve is such that $\frac{d^2y}{dx^2} = 6x - 2$. The gradient of the curve at the point (2, –9) is 3.

(i) Express y in terms of x. [5]

(ii) Show that the gradient of the curve is never less than $-\frac{16}{3}$. [3]

11 **(a)** Each day a newsagent sells copies of 10 different newspapers, one of which is *The Times*. A customer buys 3 different newspapers. Calculate the number of ways the customer can select his newspapers

(i) if there is no restriction, [1]

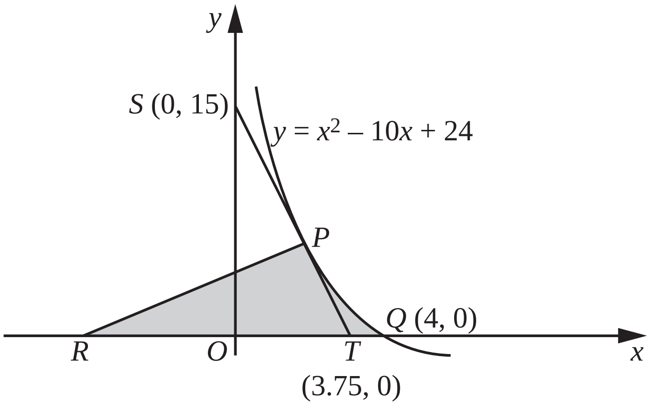
(ii) if 1 of the 3 newspapers is *The Times*. [1]

(b) Calculate the number of different 5-digit numbers which can be formed using the digits 0,1,2,3,4 without repetition and assuming that a number cannot begin with 0. [2]

How many of these 5-digit numbers are even? [4]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram, which is not drawn to scale, shows part of the curve $y = x^2 - 10x + 24$ cutting the x-axis at $Q(4, 0)$. The tangent to the curve at the point P on the curve meets the coordinate axes at $S(0, 15)$ and at $T\,(3.75, 0)$.

(i) Find the coordinates of P. [4]

The normal to the curve at P meets the x-axis at R.

(ii) Find the coordinates of R. [2]

(iii) Calculate the area of the shaded region bounded by the x-axis, the line PR and the curve PQ. [5]

OR

A curve has the equation $y = 2\cos x - \cos 2x$, where $0 < x \leqslant \frac{\pi}{2}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Given that $\sin 2x$ may be expressed as $2\sin x \cos x$, find the x-coordinate of the stationary point of the curve and determine the nature of this stationary point. [4]

(iii) Evaluate $\int_{\pi/3}^{\pi/2} y\,dx$. [3]

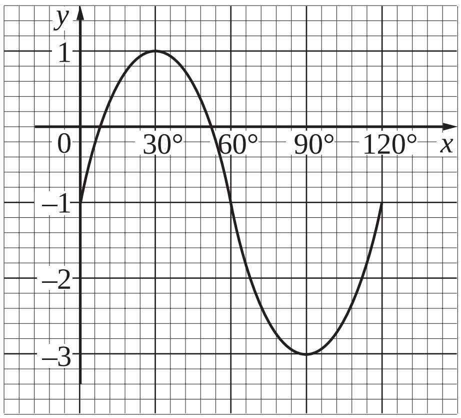
1 Express each of the following statements in appropriate set notation.

(i) x is not an element of set A.

(ii) The number of elements not in set B is 16.

(iii) Sets C and D have no common element. [3]

2



The diagram shows part of the graph of $y = a\sin(bx) + c$. State the value of

(i) a,

(ii) b,

(iii) c. [3]

3 The equation of a curve is $y = \dfrac{8}{(3x - 4)^2}$.

(i) Find the gradient of the curve where $x = 2$. [3]

(ii) Find the approximate change in y when x increases from 2 to $2 + p$, where p is small. [2]

4 The vector $\overrightarrow{OP}$ has a magnitude of 10 units and is parallel to the vector $3\mathbf{i} - 4\mathbf{j}$. The vector $\overrightarrow{OQ}$ has a magnitude of 15 units and is parallel to the vector $4\mathbf{i} + 3\mathbf{j}$.

(i) Express $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [3]

(ii) Given that the magnitude of $\overrightarrow{PQ}$ is $\lambda\sqrt{13}$, find the value of λ. [3]

5 A large airline has a fleet of aircraft consisting of 5 aircraft of type *A*, 8 of type *B*, 4 of type *C* and 10 of type *D*. The aircraft have 3 classes of seat known as Economy, Business and First. The table below shows the number of these seats in each of the 4 types of aircraft.

Class of seat / Type of aircraft	Economy	Business	First
A	300	60	40
B	150	50	20
C	120	40	0
D	100	0	0

(i) Write down two matrices whose product shows the total number of seats in each class.

(ii) Evaluate this product of matrices.

On a particular day, each aircraft made one flight. 5% of the Economy seats were empty, 10% of the Business seats were empty and 20% of the First seats were empty.

(iii) Write down a matrix whose product with the matrix found in part **(ii)** will give the total number of empty seats on that day.

(iv) Evaluate this total.

[6]

6 Given that the coefficient of x^2 in the expansion of $(k + x)\left(2 - \frac{x}{2}\right)^6$ is 84, find the value of the constant k.

[6]

7 The function f is defined for the domain $-3 \leqslant x \leqslant 3$ by

$$f(x) = 9\left(x - \tfrac{1}{3}\right)^2 - 11.$$

(i) Find the range of f. [3]

(ii) State the coordinates and nature of the turning point of

(a) the curve $y = f(x)$,

(b) the curve $y = |f(x)|$.

[4]

8 **(a)** Solve the equation $\lg(x + 12) = 1 + \lg(2 - x)$. [3]

(b) Given that $\log_2 p = a$, $\log_8 q = b$ and $\dfrac{p}{q} = 2^c$, express c in terms of a and b. [4]

9 A curve has the equation $y = \dfrac{2x - 4}{x + 3}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$ and hence explain why the curve has no turning points. [3]

The curve intersects the x-axis at the point P. The tangent to the curve at P meets the y-axis at the point Q.

(ii) Find the area of the triangle POQ, where O is the origin. [5]

10 The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 1 and the roots of $f(x) = 0$ are 1, k and k^2. It is given that $f(x)$ has a remainder of 7 when divided by $x - 2$.

(i) Show that $k^3 - 2k^2 - 2k - 3 = 0$. [3]

(ii) Hence find a value for k and show that there are no other real values of k which satisfy this equation. [5]

11 **(a)** Solve, for $0° \leqslant x \leqslant 360°$, the equation

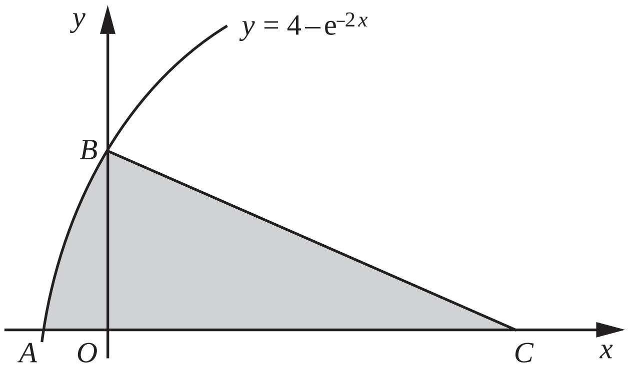
$$2\cot x = 1 + \tan x.$$ [5]

(b) Given that y is measured in radians, find the two smallest positive values of y such that

$$6\sin(2y + 1) + 5 = 0.$$ [5]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows part of the curve $y = 4 - e^{-2x}$ which crosses the axes at A and at B.

(i) Find the coordinates of A and of B. [2]

The normal to the curve at B meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Show that the area of the shaded region is approximately 10.3 square units. [5]

OR

The variables x and y are related by the equation $y = 10^{-A}b^x$, where A and b are constants. The table below shows values of x and y.

x	15	20	25	30	35	40
y	0.15	0.38	0.95	2.32	5.90	14.80

(i) Draw a straight line graph of $\lg y$ against x, using a scale of 2 cm to represent 5 units on the x-axis and 2 cm to represent 0.5 units on the $\lg y$-axis. [2]

(ii) Use your graph to estimate the value of A and of b. [4]

(iii) Estimate the value of x when $y = 10$. [2]

(iv) On the same diagram, draw the line representing $y^5 = 10^{-x}$ and hence find the value of x for which $10^{A - \frac{x}{5}} = b^x$. [3]

1 The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto x^3,$$

$$g : x \mapsto x + 2.$$

Express each of the following as a composite function, using only f, g, f^{-1} and/or g^{-1} :

(i) $x \mapsto x^3 + 2$, [1]

(ii) $x \mapsto x^3 - 2$, [1]

(iii) $x \mapsto (x + 2)^{\frac{1}{3}}$. [1]

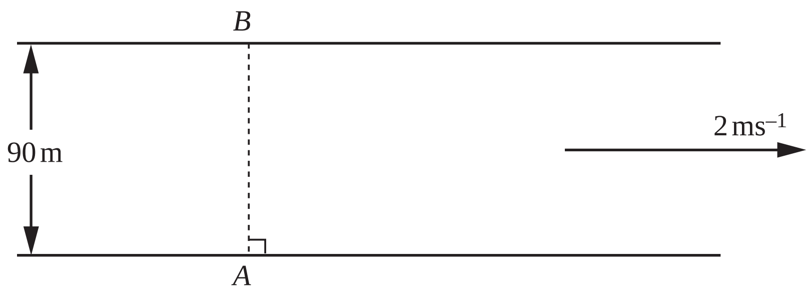
2 Prove the identity

$$\cos x \cot x + \sin x \equiv \operatorname{cosec} x\ .$$ [4]

3 Evaluate

$$\int_0^{\frac{\pi}{6}} \sin\left(2x + \frac{\pi}{6}\right) dx.$$ [4]

4



The diagram shows a river 90 m wide, flowing at $2\,\mathrm{ms}^{-1}$ between parallel banks. A ferry travels in a straight line from a point A to a point B directly opposite A. Given that the ferry takes exactly one minute to cross the river, find

(i) the speed of the ferry in still water, [3]

(ii) the angle to the bank at which the ferry must be steered. [2]

5 The straight line $2x + y = 14$ intersects the curve $2x^2 - y^2 = 2xy - 6$ at the points A and B. Show that the length of AB is $24\sqrt{5}$ units. [7]

6 A curve has equation $y = x^3 + ax + b$, where a and b are constants. The gradient of the curve at the point (2, 7) is 3. Find

(i) the value of a and of b, [5]

(ii) the coordinates of the other point on the curve where the gradient is 3. [2]

7 **(a)** Find the value of m for which the line $y = mx - 3$ is a tangent to the curve $y = x + \frac{1}{x}$ and find the x-coordinate of the point at which this tangent touches the curve. [5]

(b) Find the value of c and of d for which $\{x : -5 < x < 3\}$ is the solution set of $x^2 + cx < d$. [2]

8 Given that $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$, use the inverse matrix of **A** to

(i) solve the simultaneous equations

$$y - 4x + 8 = 0,$$

$$2y - 3x + 1 = 0,$$

(ii) find the matrix **B** such that $\mathbf{BA} = \begin{pmatrix} -2 & 3 \\ 9 & -1 \end{pmatrix}$.

[8]

9 **(a)** Express $\left(2 - \sqrt{5}\right)^2 - \frac{8}{3 - \sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers. [4]

(b) Given that $\frac{a^x}{b^{3-x}} \times \frac{b^y}{\left(a^{y+1}\right)^2} = ab^6$, find the value of x and of y. [4]

10 **(a)** How many different four-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if no digit may be repeated? [2]

(b) In a group of 13 entertainers, 8 are singers and 5 are comedians. A concert is to be given by 5 of these entertainers. In the concert there must be at least 1 comedian and there must be more singers than comedians. Find the number of different ways that the 5 entertainers can be selected. [6]

11 The equation of a curve is $y = xe^{-\frac{x}{2}}$.

(i) Show that $\frac{dy}{dx} = \frac{1}{2}(2 - x)e^{-\frac{x}{2}}$. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$. [2]

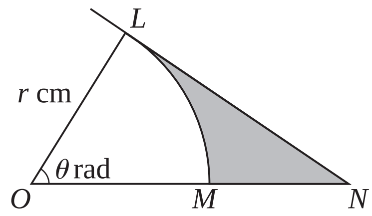
The curve has a stationary point at M.

(iii) Find the coordinates of M. [2]

(iv) Determine the nature of the stationary point at M. [2]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows a sector of a circle, centre O and radius r cm. Angle LOM is θ radians. The tangent to the circle at L meets the line through O and M at N. The shaded region shown has perimeter P cm and area A cm^2. Obtain an expression, in terms of r and θ, for

(i) P, [4]

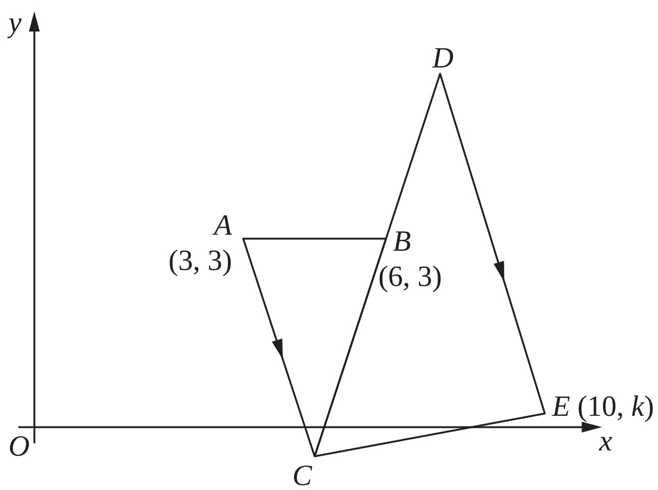
(ii) A. [3]

Given that $\theta = 1.2$ and that $P = 83$, find the value of

(iii) r, [2]

(iv) A. [1]

OR **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows an isosceles triangle ABC in which A is the point (3, 3), B is the point (6, 3) and C lies below the x-axis. Given that the area of triangle ABC is 6 square units,

(i) find the coordinates of C. [3]

The line CB is extended to the point D so that B is the mid-point of CD.

(ii) Find the coordinates of D. [2]

A line is drawn from D, parallel to AC, to the point E (10, k) and C is joined to E.

(iii) Find the value of k. [3]

(iv) Prove that angle CED is **not** a right angle. [2]

1 Given that $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, find the value of each of the constants m and n for which

$$\mathbf{A}^2 + m\mathbf{A} = n\mathbf{I},$$

where $\mathbf{I}$ is the identity matrix. [4]

2 Show that

$$\frac{1}{1-\cos\theta} - \frac{1}{1+\cos\theta} \equiv 2\operatorname{cosec}\theta\cot\theta.$$

[4]

3 Given that $p = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$, express in its simplest surd form,

(i) p, [3]

(ii) $p - \dfrac{1}{p}$. [2]

4 A badminton team of 4 men and 4 women is to be selected from 9 men and 6 women.

(i) Find the total number of ways in which the team can be selected if there are no restrictions on the selection. [3]

Two of the men are twins.

(ii) Find the number of ways in which the team can be selected if exactly one of the twins is in the team. [3]

5 In this question, $\mathbf{i}$ is a unit vector due east, and $\mathbf{j}$ is a unit vector due north.

A plane flies from P to Q where $\overrightarrow{PQ} = (960\mathbf{i} + 400\mathbf{j})$ km. A constant wind is blowing with velocity $(-60\mathbf{i} + 60\mathbf{j})$ km h^{-1}. Given that the plane takes 4 hours to travel from P to Q, find

(i) the velocity, in still air, of the plane, giving your answer in the form $(a\mathbf{i} + b\mathbf{j})$ km h^{-1}, [4]

(ii) the bearing, to the nearest degree, on which the plane must be directed. [2]

6 A curve is such that $\dfrac{dy}{dx} = \dfrac{6}{\sqrt{4x+1}}$, and (6, 20) is a point on the curve.

(i) Find the equation of the curve. [4]

A line with gradient $-\frac{1}{2}$ is a normal to the curve.

(ii) Find the coordinates of the points at which this normal meets the coordinate axes. [4]

7 **(i)** Use the substitution $u = 2^x$ to solve the equation $2^{2x} = 2^{x+2} + 5$. [5]

(ii) Solve the equation $2\log_9 3 + \log_5(7y - 3) = \log_2 8$. [4]

8 **(a)** The remainder when the expression $x^3 - 11x^2 + kx - 30$ is divided by $x - 1$ is 4 times the remainder when this expression is divided by $x - 2$. Find the value of the constant k. [4]

(b) Solve the equation $x^3 - 4x^2 - 8x + 8 = 0$, expressing non-integer solutions in the form $a \pm \sqrt{b}$, where a and b are integers. [5]

9

x	2	4	6	8	10
y	14.4	10.8	11.2	12.6	14.4

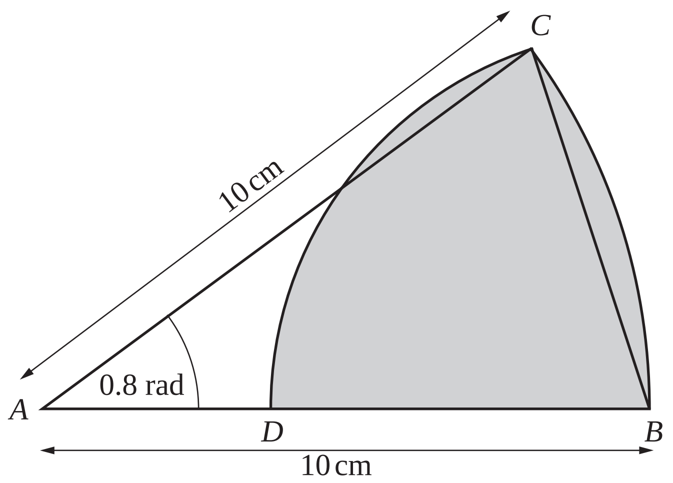
The table shows experimental values of two variables, x and y.

(i) Using graph paper, plot xy against x^2. [2]

(ii) Use the graph of xy against x^2 to express y in terms of x. [4]

(iii) Find the value of y for which $y = \dfrac{83}{x}$. [3]

10



The diagram shows a sector ABC of the circle, centre A and radius 10 cm, in which angle BAC = 0.8 radians. The arc CD of a circle has centre B and the point D lies on AB.

(i) Show that the length of the straight line BC is 7.79 cm, correct to 2 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [4]

11 Answer only **one** of the following two alternatives.

EITHER

A curve has the equation $y = xe^{2x}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [5]

(ii) Show that the y-coordinate of the stationary point of the curve is $-\frac{1}{2e}$. [3]

(iii) Determine the nature of this stationary point. [2]

OR

(i) Show that $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right) = \frac{1 - 2\ln x}{x^3}$. [3]

(ii) Show that the y-coordinate of the stationary point of the curve $y = \frac{\ln x}{x^2}$ is $\frac{1}{2e}$. [3]

(iii) Use the result from part **(i)** to find $\int\left(\frac{\ln x}{x^3}\right)dx$. [4]

1 The two variables x and y are related by the equation $yx^2 = 800$.

(i) Obtain an expression for $\frac{dy}{dx}$ in terms of x. [2]

(ii) Hence find the approximate change in y as x increases from 10 to $10 + p$, where p is small. [2]

2 Solve the equation $3\sin\left(\frac{x}{2} - 1\right) = 1$ for $0 < x < 6\pi$ radians. [5]

3 **(i)** Express 9^{x+1} as a power of 3. [1]

(ii) Express $\sqrt[3]{27^{2x}}$ as a power of 3. [1]

(iii) Express $\dfrac{54 \times \sqrt[3]{27^{2x}}}{9^{x+1} + 216(3^{2x-1})}$ as a fraction in its simplest form. [3]

4 A cycle shop sells three models of racing cycles, A, B and C. The table below shows the numbers of each model sold over a four-week period and the cost of each model in \$.

Week \ Model	A	B	C
1	8	12	4
2	7	10	2
3	10	12	0
4	6	8	4
Cost (\$)	300	500	800

In the first two weeks the shop banked 30% of all money received, but in the last two weeks the shop only banked 20% of all money received.

(i) Write down three matrices such that matrix multiplication will give the total amount of money banked over the four-week period. [2]

(ii) Hence evaluate this total amount. [4]

5 **(i)** Expand $(1 + x)^5$. [1]

(ii) Hence express $(1 + \sqrt{2})^5$ in the form $a + b\sqrt{2}$, where a and b are integers. [3]

(iii) Obtain the corresponding result for $(1 - \sqrt{2})^5$ and **hence** evaluate $(1 + \sqrt{2})^5 + (1 - \sqrt{2})^5$. [2]

6 Two circular flower beds have a combined area of $\frac{29\pi}{2}$ m^2. The sum of the circumferences of the two flower beds is 10π m. Determine the radius of each flower bed. [6]

7 The position vectors of points A and B, relative to an origin O, are $2\mathbf{i} + 4\mathbf{j}$ and $6\mathbf{i} + 10\mathbf{j}$ respectively. The position vector of C, relative to O, is $k\mathbf{i} + 25\mathbf{j}$, where k is a positive constant.

(i) Find the value of k for which the length of BC is 25 units. [3]

(ii) Find the value of k for which ABC is a straight line. [3]

8 Given that $x \in \mathbb{R}$ and that $\mathscr{E} = \{x : 2 < x < 10\}$,

$$A = \{x : 3x + 2 < 20\}$$

and $B = \{x : x^2 < 11x - 28\}$,

find the set of values of x which define

(i) $A \cap B$,

(ii) $(A \cup B)'$.

[7]

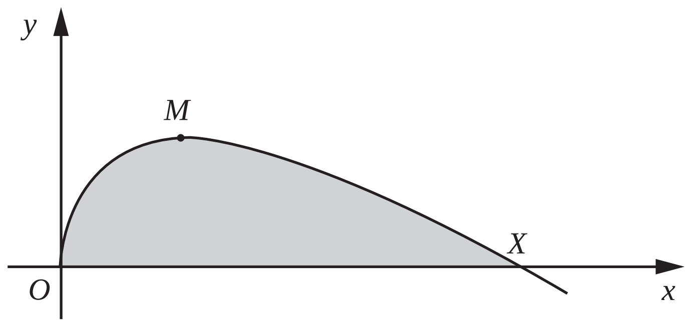
9 A particle travels in a straight line so that, t s after passing through a fixed point O, its speed, v ms^{-1}, is given by $v = 8\cos\left(\frac{t}{2}\right)$.

(i) Find the acceleration of the particle when $t = 1$. [3]

The particle first comes to instantaneous rest at the point P.

(ii) Find the distance OP. [4]

10

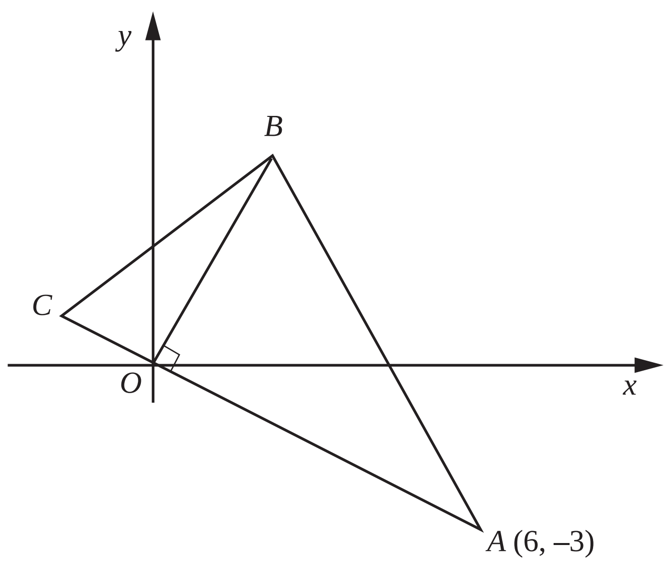


The diagram shows part of the curve $y = 4\sqrt{x} - x$. The origin O lies on the curve and the curve intersects the positive x-axis at X. The maximum point of the curve is at M. Find

(i) the coordinates of X and of M, [5]

(ii) the area of the shaded region. [4]

11 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a triangle ABC in which A is the point (6, –3). The line AC passes through the origin O. The line OB is perpendicular to AC.

(i) Find the equation of OB. [2]

The area of triangle AOB is 15 units^2.

(ii) Find the coordinates of B. [3]

The length of AO is 3 times the length of OC.

(iii) Find the coordinates of C. [2]

The point D is such that the quadrilateral $ABCD$ is a kite.

(iv) Find the area of $ABCD$. [2]

12 Answer only **one** of the following two alternatives.

EITHER

The function f is defined, for $x > 0$, by $\mathrm{f} : x \mapsto \ln x$.

(i) State the range of f. [1]

(ii) State the range of f^{-1}. [1]

(iii) On the same diagram, sketch and label the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [2]

The function g is defined, for $x > 0$, by $\mathrm{g} : x \mapsto 3x + 2$.

(iv) Solve the equation $\mathrm{fg}(x) = 3$. [2]

(v) Solve the equation $\mathrm{f}^{-1}\mathrm{g}^{-1}(x) = 7$. [4]

OR

(i) Find the values of k for which $y = kx + 2$ is a tangent to the curve $y = 4x^2 + 2x + 3$. [4]

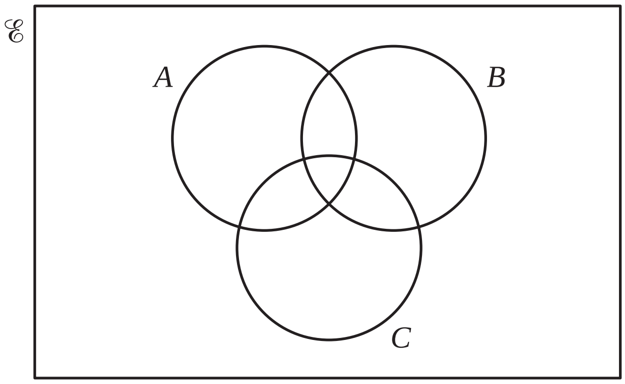
(ii) Express $4x^2 + 2x + 3$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(iii) Determine, with explanation, whether or not the curve $y = 4x^2 + 2x + 3$ meets the x-axis. [2]

The function f is defined by $\mathrm{f} : x \mapsto 4x^2 + 2x + 3$ where $x \geqslant p$.

(iv) Determine the smallest value of p for which f has an inverse. [1]

1



(i) Copy the Venn diagram above and shade the region that represents $A \cup (B \cap C)$. [1]

(ii) Copy the Venn diagram above and shade the region that represents $A \cap (B \cup C)$. [1]

(iii) Copy the Venn diagram above and shade the region that represents $(A \cup B \cup C)'$. [1]

2 Find the set of values of x for which $(2x + 1)^2 > 8x + 9$. [4]

3 Prove that $\dfrac{\sin A}{1 + \cos A} + \dfrac{1 + \cos A}{\sin A} \equiv 2\operatorname{cosec} A$. [4]

4 A function f is such that $f(x) = ax^3 + bx^2 + 3x + 4$. When $f(x)$ is divided by $x - 1$, the remainder is 3. When $f(x)$ is divided by $2x + 1$, the remainder is 6. Find the value of a and of b. [5]

5 Given that $\mathbf{a} = 5\mathbf{i} - 12\mathbf{j}$ and that $\mathbf{b} = p\mathbf{i} + \mathbf{j}$, find

(i) the unit vector in the direction of $\mathbf{a}$, [2]

(ii) the values of the constants p and q such that $q\mathbf{a} + \mathbf{b} = 19\mathbf{i} - 23\mathbf{j}$. [3]

6 **(i)** Solve the equation $2t = 9 + \dfrac{5}{t}$. [3]

(ii) Hence, or otherwise, solve the equation $2x^{\frac{1}{2}} = 9 + 5x^{-\frac{1}{2}}$. [3]

7 **(i)** Express $4x^2 - 12x + 3$ in the form $(ax + b)^2 + c$, where a, b and c are constants and $a > 0$. [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve $y = 4x^2 - 12x + 3$. [2]

(iii) Given that $f(x) = 4x^2 - 12x + 3$, write down the range of f. [1]

8 A curve is such that $\dfrac{d^2y}{dx^2} = 4e^{-2x}$. Given that $\dfrac{dy}{dx} = 3$ when $x = 0$ and that the curve passes through the point $(2, e^{-4})$, find the equation of the curve. [6]

9 **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $(2 - 3x)^5$. [3]

The first 3 terms in the expansion of $(a + bx)(2 - 3x)^5$ in ascending powers of x are $64 - 192x + cx^2$.

(ii) Find the value of a, of b and of c. [5]

10 **(a)** Functions f and g are defined, for $x \in \mathbb{R}$, by

$$f(x) = 3 - x,$$
$$g(x) = \frac{x}{x+2}, \quad \text{where } x \neq -2.$$

(i) Find $fg(x)$. [2]

(ii) Hence find the value of x for which $fg(x) = 10$. [2]

(b) A function h is defined, for $x \in \mathbb{R}$, by $h(x) = 4 + \ln x$, where $x > 1$.

(i) Find the range of h. [1]

(ii) Find the value of $h^{-1}(9)$. [2]

(iii) On the same axes, sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$. [3]

11 Solve the equation

(i) $\tan 2x - 3\cot 2x = 0$, for $0° < x < 180°$, [4]

(ii) $\operatorname{cosec} y = 1 - 2\cot^2 y$, for $0° \leqslant y \leqslant 360°$, [5]

(iii) $\sec(z + \frac{\pi}{2}) = -2$, for $0 < z < \pi$ radians. [3]

12 Answer only **one** of the following two alternatives.

EITHER

A curve has equation $y = \dfrac{x^2}{x+1}$.

(i) Find the coordinates of the stationary points of the curve. [5]

The normal to the curve at the point where $x = 1$ meets the x-axis at M. The tangent to the curve at the point where $x = -2$ meets the y-axis at N.

(ii) Find the area of the triangle MNO, where O is the origin. [6]

OR

A curve has equation $y = e^{x-2} - 2x + 6$.

(i) Find the coordinates of the stationary point of the curve and determine the nature of the stationary point. [6]

The area of the region enclosed by the curve, the positive x-axis, the positive y-axis and the line $x = 3$ is $k + e - e^{-2}$.

(ii) Find the value of k. [5]

1 Given that $\mathbf{A} = \begin{pmatrix} 13 & 6 \\ 7 & 4 \end{pmatrix}$, find the inverse matrix $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$13x + 6y = 41,$$
$$7x + 4y = 24.$$

[4]

2 Variables x and y are connected by the equation $y = (2x - 9)^3$. Given that x is increasing at the rate of 4 units per second, find the rate of increase of y when $x = 7$. [4]

3 Find the set of values of m for which the line $y = mx + 2$ does not meet the curve $y = x^2 - 5x + 18$. [5]

4 **(i)** Differentiate $x \ln x$ with respect to x. [2]

(ii) Hence find $\int \ln x \, dx$. [3]

5 Solve the equation

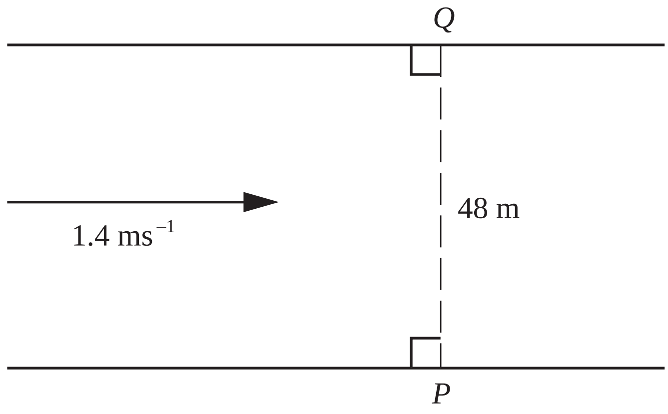
(i) $\dfrac{4^x}{2^{5-x}} = \dfrac{2^{4x}}{8^{x-3}}$, [3]

(ii) $\lg(2y + 10) + \lg y = 2$. [3]

6 **(a)** A sports team of 3 attackers, 2 centres and 4 defenders is to be chosen from a squad of 5 attackers, 3 centres and 6 defenders. Calculate the number of different ways in which this can be done. [3]

(b) How many different 4-digit numbers greater than 3000 can be formed using the six digits 1, 2, 3, 4, 5 and 6 if no digit can be used more than once? [3]

7



The diagram shows a river with parallel banks. The river is 48 m wide and is flowing with a speed of 1.4 ms^{-1}. A boat travels in a straight line from a point P on one bank to a point Q which is on the other bank directly opposite P. Given that the boat takes 10 seconds to cross the river, find

(i) the speed of the boat in still water, [4]

(ii) the angle to the bank at which the boat should be steered. [2]

8 The function f is defined, for $0 \leqslant x \leqslant 2\pi$, by

$$f(x) = 3 + 5 \sin 2x.$$

State

(i) the amplitude of f, [1]

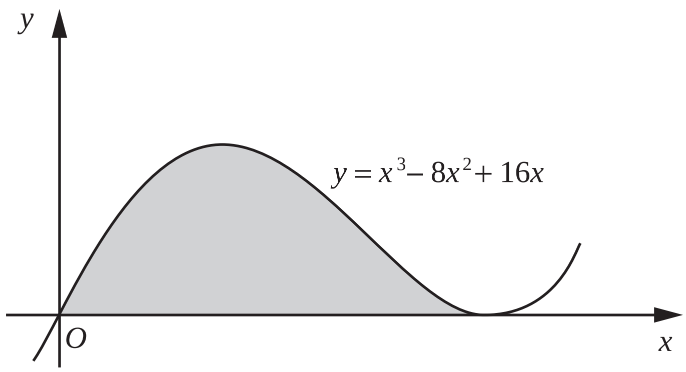
(ii) the period of f, [1]

(iii) the maximum and minimum values of f. [2]

Sketch the graph of $y = f(x)$. [3]

9 The line $y = 2x - 9$ intersects the curve $x^2 + y^2 + xy + 3x = 46$ at the points A and B. Find the equation of the perpendicular bisector of AB. [8]

10



The diagram shows part of the curve $y = x^3 - 8x^2 + 16x$.

(i) Show that the curve has a minimum point at (4, 0) and find the coordinates of the maximum point. [4]

(ii) Find the area of the shaded region enclosed by the x-axis and the curve. [4]

11 The table shows experimental values of two variables x and y.

x	2	4	6	8
y	2.25	0.81	0.47	0.33

(i) Using graph paper, plot xy against $\frac{1}{x}$ and draw a straight line graph. [3]

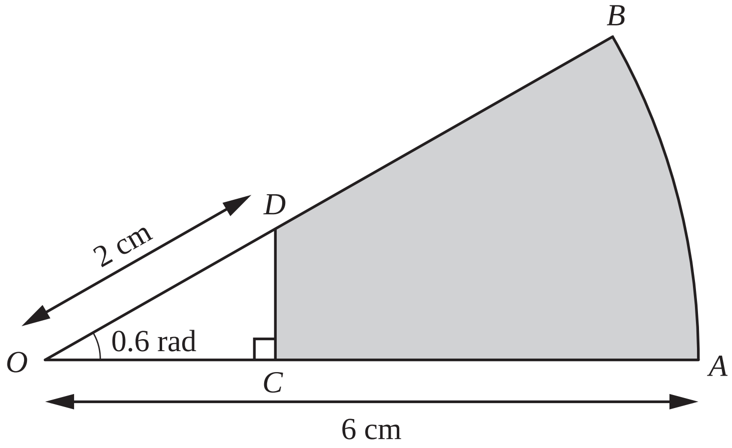
(ii) Use your graph to express y in terms of x. [5]

(iii) Estimate the value of x and of y for which $xy = 4$. [3]

[Question 12 is printed on the next page.]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows a sector AOB of a circle with centre O and radius 6 cm. Angle $AOB = 0.6$ radians. The point D lies on OB such that the length of OD is 2 cm. The point C lies on OA such that OCD is a right angle.

(i) Show that the length of OC is approximately 1.65 cm and find the length of CD. [4]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

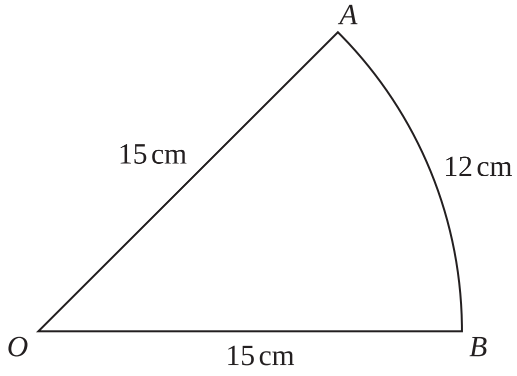
OR

A particle moves in a straight line so that t seconds after passing a fixed point O its acceleration, $a\,\text{ms}^{-2}$, is given by $a = 4t - 12$. Given that its speed at O is $16\,\text{ms}^{-1}$, find

(i) the values of t at which the particle is stationary, [5]

(ii) the distance the particle travels in the fifth second. [5]

1



The diagram shows a sector AOB of a circle, centre O, radius 15 cm. The length of the arc AB is 12 cm.

(i) Find, in radians, angle AOB. [2]

(ii) Find the area of the sector AOB. [2]

2 The equation of a curve is $y = x^3 - 8$. Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

3 Show that $\dfrac{1 - \cos^2\theta}{\sec^2\theta - 1} = 1 - \sin^2\theta$. [4]

4 The line $y = 5x - 3$ is a tangent to the curve $y = kx^2 - 3x + 5$ at the point A. Find

(i) the value of k, [3]

(ii) the coordinates of A. [2]

5 **(a)** Solve the equation $9^{2x-1} = 27^x$. [3]

(b) Given that $\dfrac{a^{-\frac{1}{2}}b^{\frac{2}{3}}}{\sqrt{a^3 b^{-\frac{2}{3}}}} = a^p b^q$, find the value of p and of q. [2]

6 Solve the equation $2x^3 + 3x^2 - 32x + 15 = 0$. [6]

7 **(i)** Find $\dfrac{d}{dx}\left(xe^{3x} - \dfrac{e^{3x}}{3}\right)$. [3]

(ii) Hence find $\int xe^{3x}dx$. [3]

8 A curve has equation $y = \dfrac{2x}{x^2 + 9}$.

(i) Find the x-coordinate of each of the stationary points of the curve. [4]

(ii) Given that x is increasing at the rate of 2 units per second, find the rate of increase of y when $x = 1$. [3]

9 At 10 00 hours, a ship P leaves a point A with position vector $(-4\mathbf{i} + 8\mathbf{j})$ km relative to an origin O, where $\mathbf{i}$ is a unit vector due East and $\mathbf{j}$ is a unit vector due North. The ship sails north-east with a speed of $10\sqrt{2}$ km h^{-1}. Find

(i) the velocity vector of P, [2]

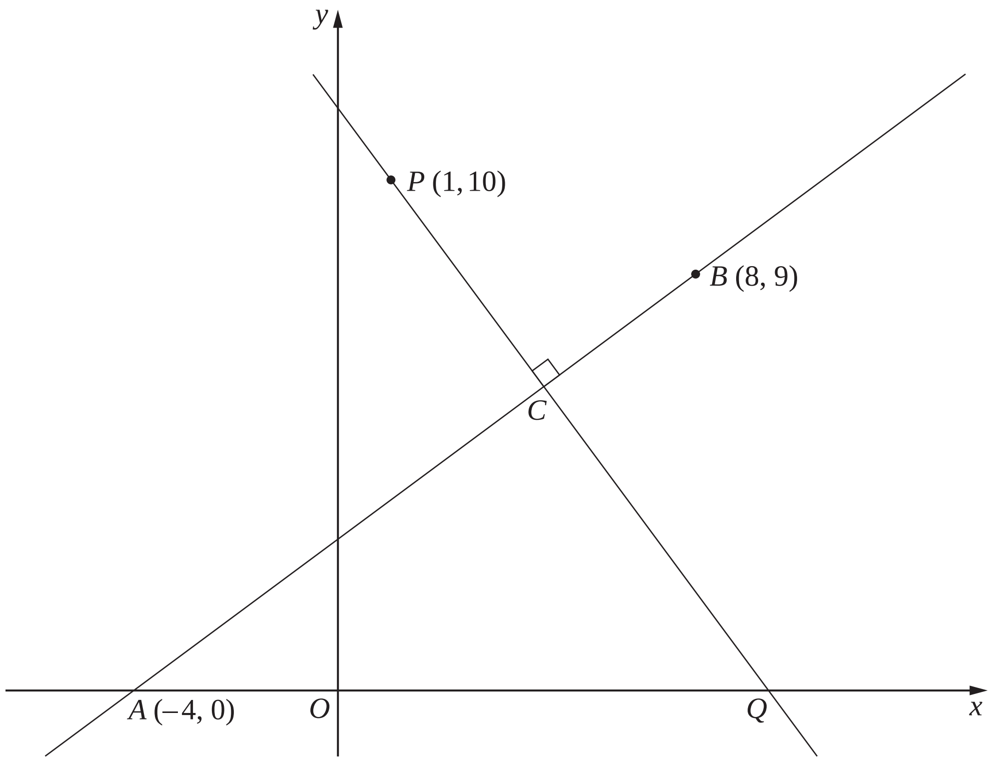
(ii) the position vector of P at 12 00 hours. [2]

At 12 00 hours, a second ship Q leaves a point B with position vector $(19\mathbf{i} + 34\mathbf{j})$ km travelling with velocity vector $(8\mathbf{i} + 6\mathbf{j})$ km h^{-1}.

(iii) Find the velocity of P relative to Q. [2]

(iv) Hence, or otherwise, find the time at which P and Q meet and the position vector of the point where this happens. [3]

10 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows the line AB passing through the points $A(-4, 0)$ and $B(8, 9)$. The line through the point $P(1, 10)$, perpendicular to AB, meets AB at C and the x-axis at Q. Find

(i) the coordinates of C and of Q, [7]

(ii) the area of triangle ACQ. [2]

11 The table shows experimental values of variables s and t.

t	5	15	30	70	100
s	1305	349	152	55	36

(i) By plotting a suitable straight line graph, show that s and t are related by the equation $s = kt^n$, where k and n are constants. [4]

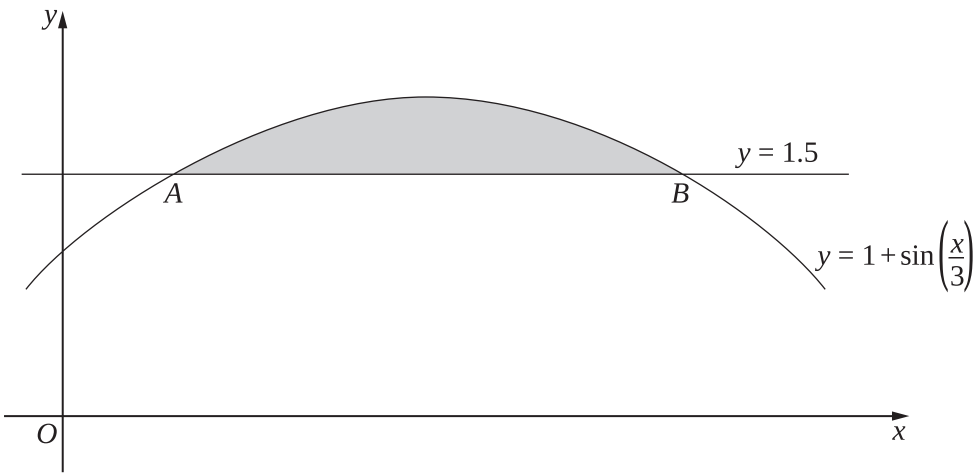
(ii) Use your graph to find the value of k and of n. [4]

12 Answer only **one** of the following two alternatives.

EITHER

(i) State the amplitude of $1 + \sin\left(\frac{x}{3}\right)$. [1]

(ii) State, in radians, the period of $1 + \sin\left(\frac{x}{3}\right)$. [1]



The diagram shows the curve $y = 1 + \sin\left(\frac{x}{3}\right)$ meeting the line $y = 1.5$ at points A and B. Find

(iii) the x-coordinate of A and of B, [3]

(iv) the area of the shaded region. [6]

OR

A particle moves in a straight line such that t s after passing through a fixed point O, its velocity, $v\ \mathrm{m\,s^{-1}}$, is given by $v = k\cos 4t$, where k is a positive constant. Find

(i) the value of t when the particle is first instantaneously at rest, [1]

(ii) an expression for the acceleration of the particle t s after passing through O. [2]

Given that the acceleration of the particle is $12\ \mathrm{m\,s^{-2}}$ when $t = \frac{3\pi}{8}$,

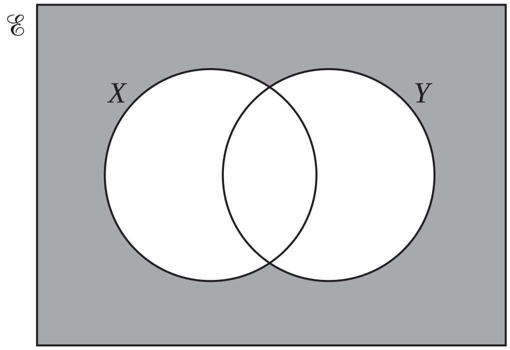
(iii) find the value of k. [2]

Using your value for k,

(iv) sketch the velocity-time curve for the particle for $0 \leqslant t \leqslant \pi$, [2]

(v) find the displacement of the particle from O when $t = \frac{\pi}{24}$. [4]

1 **(a)**



Express, in set notation, the set represented by the shaded region. [1]

(b) In a class of 30 students, 17 are studying politics, 14 are studying economics and 10 are studying both of these subjects.

(i) Illustrate this information using a Venn diagram. [1]

Find the number of students studying

(ii) neither of these subjects, [1]

(iii) exactly one of these subjects. [1]

2 Given that $\mathbf{A} = \begin{pmatrix} 7 & 6 \\ 3 & 4 \end{pmatrix}$, find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$7x + 6y = 17,$$
$$3x + 4y = 3.$$

[4]

3 Sketch the graph of $y = \left|x^2 - 8x + 12\right|$. [4]

4 Find the coefficient of x^4 in the expansion of

(i) $(1 + 2x)^6$, [2]

(ii) $\left(1 - \frac{x}{4}\right)(1 + 2x)^6$. [3]

5 Two variables, x and y, are related by the equation

$$y = 6x^2 + \frac{32}{x^3}.$$

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Use your expression to find the approximate change in the value of y when x increases from 2 to 2.04. [3]

6 The function f is defined by $f(x) = 2 + \sqrt{x-3}$ for $x \geqslant 3$. Find

(i) the range of f, [1]

(ii) an expression for $f^{-1}(x)$. [2]

The function g is defined by $g(x) = \frac{12}{x} + 2$ for $x > 0$. Find

(iii) gf(12). [2]

7 Given that $\log_p X = 9$ and $\log_p Y = 6$, find

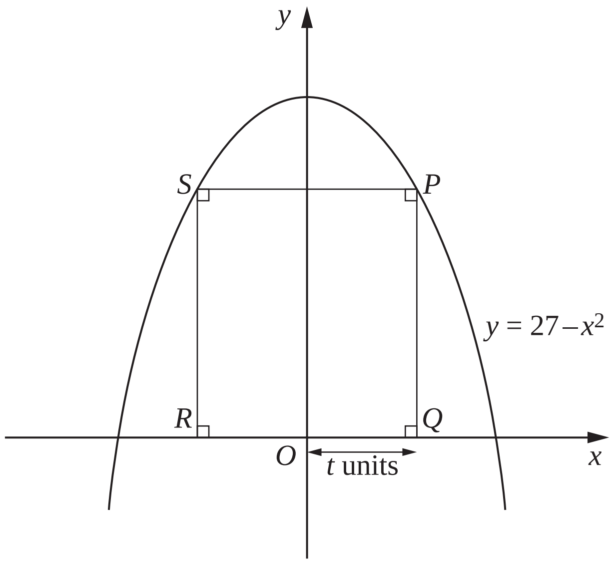
(i) $\log_p \sqrt{X}$, [1]

(ii) $\log_p \left(\frac{1}{X}\right)$, [1]

(iii) $\log_p (XY)$, [2]

(iv) $\log_Y X$. [2]

8



The diagram shows part of the curve $y = 27 - x^2$. The points P and S lie on this curve. The points Q and R lie on the x-axis and $PQRS$ is a rectangle. The length of OQ is t units.

(i) Find the length of PQ in terms of t and hence show that the area, A square units, of $PQRS$ is given by

$$A = 54t - 2t^3.$$ [2]

(ii) Given that t can vary, find the value of t for which A has a stationary value. [3]

(iii) Find this stationary value of A and determine its nature. [3]

9 A musician has to play 4 pieces from a list of 9. Of these 9 pieces 4 were written by Beethoven, 3 by Handel and 2 by Sibelius. Calculate the number of ways the 4 pieces can be chosen if

(i) there are no restrictions, [2]

(ii) there must be 2 pieces by Beethoven, 1 by Handel and 1 by Sibelius, [3]

(iii) there must be at least one piece by each composer. [4]

10 The line $2x + y = 12$ intersects the curve $x^2 + 3xy + y^2 = 176$ at the points A and B. Find the equation of the perpendicular bisector of AB. [9]

11 **(a)** Find all the angles between 0° and 360° which satisfy

12 Answer only **one** of the following two alternatives.

EITHER

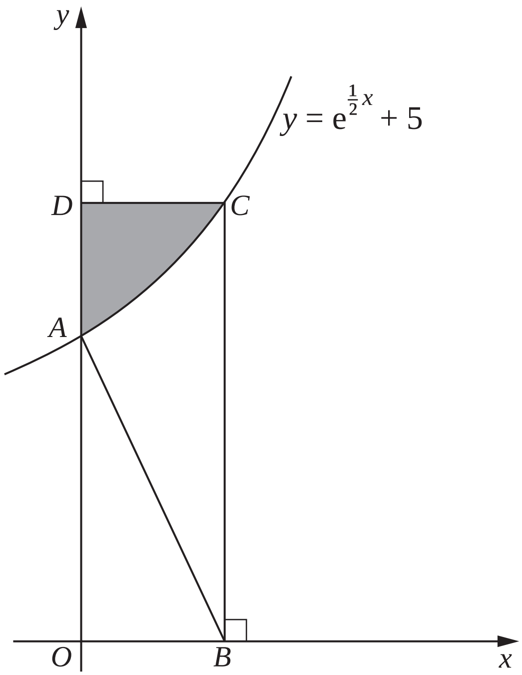
The point $P(0, 5)$ lies on the curve for which $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{\frac{1}{2}x}$. The point Q, with x-coordinate 2, also lies on the curve.

(i) Find, in terms of e, the y-coordinate of Q. [5]

The tangents to the curve at the points P and Q intersect at the point R.

(ii) Find, in terms of e, the x-coordinate of R. [5]

OR



The diagram shows part of the curve $y = \mathrm{e}^{\frac{1}{2}x} + 5$ crossing the y-axis at A. The normal to the curve at A meets the x-axis at B.

(i) Find the coordinates of B. [4]

The line through B, parallel to the y-axis, meets the curve at C. The line through C, parallel to the x-axis, meets the y-axis at D.

(ii) Find the area of the shaded region. [6]

1 The equation of a curve is given by $y = 2x^2 + ax + 14$, where a is a constant.

Given that this equation can also be written as $y = 2(x - 3)^2 + b$, where b is a constant, find

(i) the value of a and of b, [2]

(ii) the minimum value of y. [1]

2 **(i)** Sketch, on the same set of axes, the graphs of $y = \cos x$ and $y = \sin 2x$ for $0° \leqslant x \leqslant 180°$. [2]

(ii) Hence write down the number of solutions of the equation $\sin 2x - \cos x = 0$ for $0° \leqslant x \leqslant 180°$. [1]

3 Show that $\dfrac{\cos x}{1 - \sin x} + \dfrac{\cos x}{1 + \sin x} = 2 \sec x$. [4]

4 Factorise completely the expression $2x^3 - 11x^2 - 20x - 7$. [5]

5 A curve has the equation $y = 2x \sin x + \dfrac{\pi}{3}$. The curve passes through the point $P\left(\dfrac{\pi}{2}, a\right)$.

(i) Find, in terms of π, the value of a. [1]

(ii) Using your value of a, find the equation of the normal to the curve at P. [5]

6 **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $(2 - 5x)^6$, giving your answer in the form $a + bx + cx^2$, where a, b and c are integers. [3]

(ii) Find the coefficient of x in the expansion of $(2 - 5x)^6\left(1 + \dfrac{x}{2}\right)^{10}$. [3]

7 **(a)** Sets A and B are such that

$A = \{x : \sin x = 0.5 \text{ for } 0° \leqslant x \leqslant 360°\}$,

$B = \{x : \cos (x - 30°) = -0.5 \text{ for } 0° \leqslant x \leqslant 360°\}$.

Find the elements of

(i) A, [2]

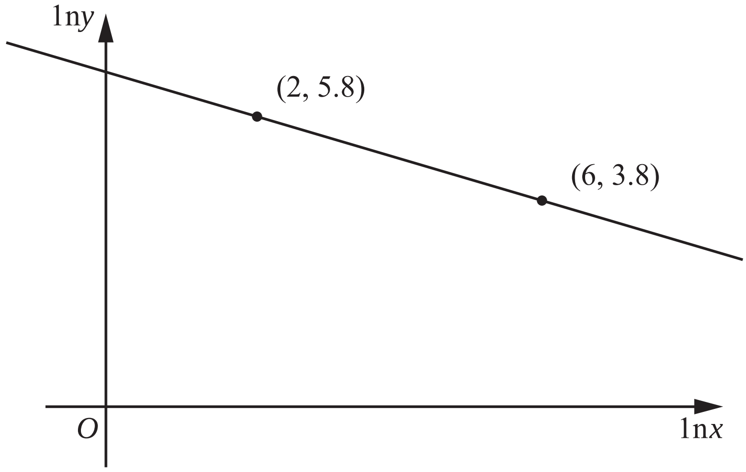
(ii) $A \cup B$. [2]

(b) Set C is such that

$C = \{x : \sec^2 3x = 1 \text{ for } 0° \leqslant x \leqslant 180°\}$.

Find $n(C)$. [3]

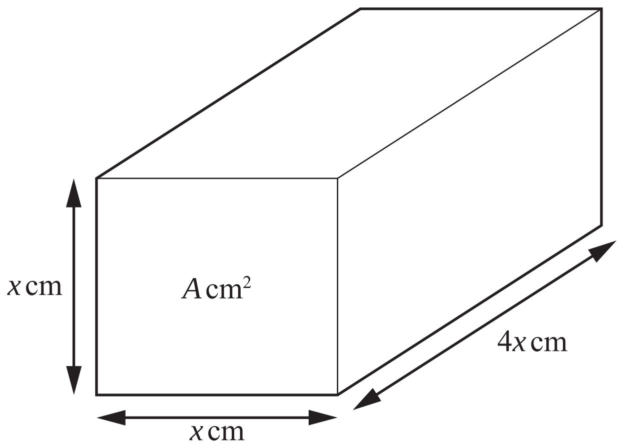
8 Variables x and y are such that, when $\ln y$ is plotted against $\ln x$, a straight line graph passing through the points (2, 5.8) and (6, 3.8) is obtained.



(i) Find the value of $\ln y$ when $\ln x = 0$. [2]

(ii) Given that $y = Ax^b$, find the value of A and of b. [5]

9



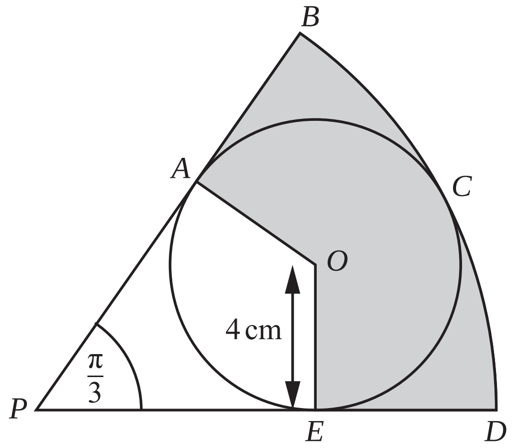
The figure shows a rectangular metal block of length $4x$ cm, with a cross-section which is a square of side x cm and area A cm^2. The block is heated and the area of the cross-section increases at a constant rate of 0.003 cm^{2}s^{-1}. Find

(i) $\frac{dA}{dx}$ in terms of x, [1]

(ii) the rate of increase of x when $x = 5$, [3]

(iii) the rate of increase of the volume of the block when $x = 5$. [4]

10



The diagram shows a circle, centre O, radius 4 cm, enclosed within a sector $PBCDP$ of a circle, centre P. The circle centre O touches the sector at points A, C and E. Angle BPD is $\frac{\pi}{3}$ radians.

(i) Show that $PA = 4\sqrt{3}$ cm and $PB = 12$ cm. [2]

Find, to 1 decimal place,

(ii) the area of the shaded region, [4]

(iii) the perimeter of the shaded region. [4]

11 **(i)** Find $\int \frac{1}{\sqrt{1+x}}\,dx$. [2]

(ii) Given that $y = \frac{2x}{\sqrt{1+x}}$, show that $\frac{dy}{dx} = \frac{A}{\sqrt{1+x}} + \frac{Bx}{\left(\sqrt{1+x}\right)^3}$, where A and B are to be found. [4]

(iii) Hence find $\int \frac{x}{\left(\sqrt{1+x}\right)^3}\,dx$ and evaluate $\int_0^3 \frac{x}{\left(\sqrt{1+x}\right)^3}\,dx$. [4]

12 Answer only **one** of the following two alternatives.

EITHER

A curve is such that $\frac{dy}{dx} = 4x^2 - 9$. The curve passes through the point (3, 1).

(i) Find the equation of the curve. [4]

The curve has stationary points at A and B.

(ii) Find the coordinates of A and of B. [3]

(iii) Find the equation of the perpendicular bisector of the line AB. [4]

OR

A curve has the equation $y = Ae^{2x} + Be^{-x}$ where $x \geqslant 0$. At the point where $x = 0$, $y = 50$ and $\frac{dy}{dx} = -20$.

(i) Show that $A = 10$ and find the value of B. [5]

(ii) Using the values of A and B found in part **(i)**, find the coordinates of the stationary point on the curve. [4]

(iii) Determine the nature of the stationary point, giving a reason for your answer. [2]

1 The equation of a curve is given by $y = 2x^2 + ax + 14$, where a is a constant.

Given that this equation can also be written as $y = 2(x - 3)^2 + b$, where b is a constant, find

(i) the value of a and of b, [2]

(ii) the minimum value of y. [1]

2 **(i)** Sketch, on the same set of axes, the graphs of $y = \cos x$ and $y = \sin 2x$ for $0° \leqslant x \leqslant 180°$. [2]

(ii) Hence write down the number of solutions of the equation $\sin 2x - \cos x = 0$ for $0° \leqslant x \leqslant 180°$. [1]

3 Show that $\dfrac{\cos x}{1 - \sin x} + \dfrac{\cos x}{1 + \sin x} = 2 \sec x$. [4]

4 Factorise completely the expression $2x^3 - 11x^2 - 20x - 7$. [5]

5 A curve has the equation $y = 2x \sin x + \dfrac{\pi}{3}$. The curve passes through the point $P\left(\dfrac{\pi}{2}, a\right)$.

(i) Find, in terms of π, the value of a. [1]

(ii) Using your value of a, find the equation of the normal to the curve at P. [5]

6 **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $(2 - 5x)^6$, giving your answer in the form $a + bx + cx^2$, where a, b and c are integers. [3]

(ii) Find the coefficient of x in the expansion of $(2 - 5x)^6\left(1 + \dfrac{x}{2}\right)^{10}$. [3]

7 **(a)** Sets A and B are such that

$A = \{x : \sin x = 0.5 \text{ for } 0° \leqslant x \leqslant 360°\}$,

$B = \{x : \cos(x - 30°) = -0.5 \text{ for } 0° \leqslant x \leqslant 360°\}$.

Find the elements of

(i) A, [2]

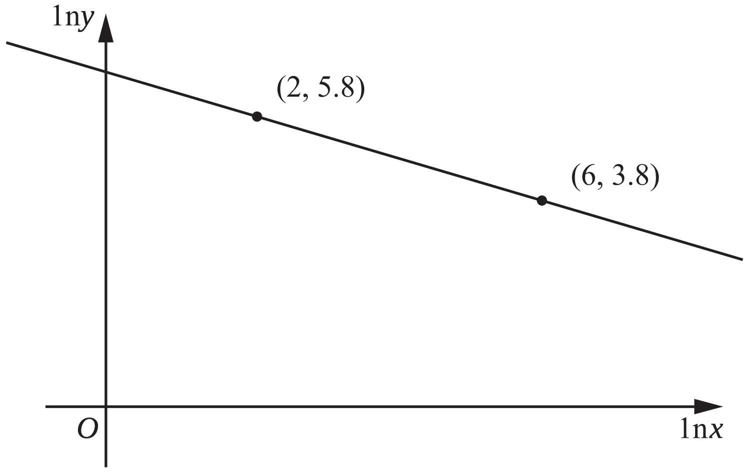
(ii) $A \cup B$. [2]

(b) Set C is such that

$C = \{x : \sec^2 3x = 1 \text{ for } 0° \leqslant x \leqslant 180°\}$.

Find $n(C)$. [3]

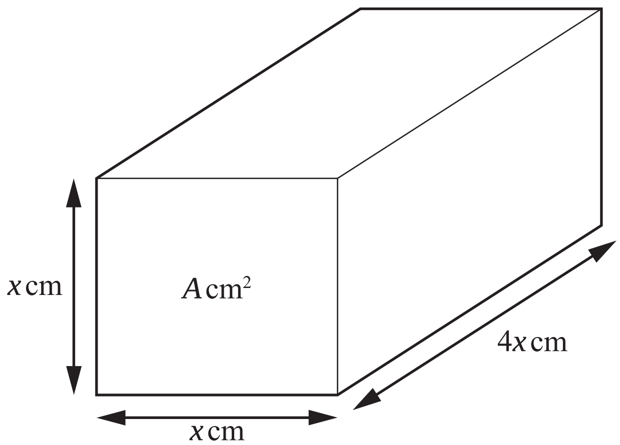
8 Variables x and y are such that, when $\ln y$ is plotted against $\ln x$, a straight line graph passing through the points (2, 5.8) and (6, 3.8) is obtained.



(i) Find the value of $\ln y$ when $\ln x = 0$. [2]

(ii) Given that $y = Ax^b$, find the value of A and of b. [5]

9



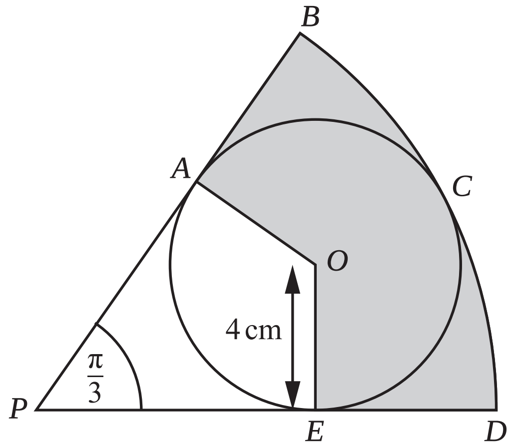
The figure shows a rectangular metal block of length $4x$ cm, with a cross-section which is a square of side x cm and area A cm^2. The block is heated and the area of the cross-section increases at a constant rate of 0.003 cm^{2}s^{-1}. Find

(i) $\frac{dA}{dx}$ in terms of x, [1]

(ii) the rate of increase of x when $x = 5$, [3]

(iii) the rate of increase of the volume of the block when $x = 5$. [4]

10



The diagram shows a circle, centre O, radius 4 cm, enclosed within a sector $PBCDP$ of a circle, centre P. The circle centre O touches the sector at points A, C and E. Angle BPD is $\frac{\pi}{3}$ radians.

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12 Answer only **one** of the following two alternatives.

EITHER

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The curve has stationary points at A and B.

(ii) Find the coordinates of A and of B. [3]

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OR

A curve has the equation $y = Ae^{2x} + Be^{-x}$ where $x \geqslant 0$. At the point where $x = 0$, $y = 50$ and $\frac{dy}{dx} = -20$.

(i) Show that $A = 10$ and find the value of B. [5]

(ii) Using the values of A and B found in part **(i)**, find the coordinates of the stationary point on the curve. [4]

(iii) Determine the nature of the stationary point, giving a reason for your answer. [2]

1 Show that $\sec x - \cos x = \sin x \tan x$. [3]

2 A 4-digit number is formed by using four of the seven digits 2, 3, 4, 5, 6, 7 and 8. No digit can be used more than once in any one number. Find how many different 4-digit numbers can be formed if

(i) there are no restrictions, [2]

(ii) the number is even. [2]

3 The line $y = mx + 2$ is a tangent to the curve $y = x^2 + 12x + 18$. Find the possible values of m. [4]

4 The remainder when the expression $x^3 + kx^2 - 5x - 3$ is divided by $x - 2$ is 5 times the remainder when the expression is divided by $x + 1$. Find the value of k. [4]

5 Solve the simultaneous equations

$$\log_3 a = 2 \log_3 b,$$

$$\log_3 (2a - b) = 1.$$ [5]

6 Solve the equation $3x^3 + 7x^2 - 22x - 8 = 0$. [6]

7 **(i)** Sketch the graph of $y = |3x - 5|$, for $-2 \leqslant x \leqslant 3$, showing the coordinates of the points where the graph meets the axes. [3]

(ii) On the same diagram, sketch the graph of $y = 8x$. [1]

(iii) Solve the equation $8x = |3x - 5|$. [3]

8 **(a)** A function f is defined, for $x \in \mathbb{R}$, by

$$f(x) = x^2 + 4x - 6.$$

(i) Find the least value of $f(x)$ and the value of x for which it occurs. [2]

(ii) Hence write down a suitable domain for $f(x)$ in order that $f^{-1}(x)$ exists. [1]

(b) Functions g and h are defined, for $x \in \mathbb{R}$, by

$$g(x) = \frac{x}{2} - 1,$$

$$h(x) = x^2 - x.$$

(i) Find $g^{-1}(x)$. [2]

(ii) Solve $gh(x) = g^{-1}(x)$. [3]

9 **(a)** Find $\int \left(x^{\frac{1}{3}} - 3\right)^2 \, \mathrm{d}x$. [3]

(b) **(i)** Given that $y = x\sqrt{x^2 + 6}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(ii) Hence find $\int \frac{x^2 + 3}{\sqrt{x^2 + 6}} \, \mathrm{d}x$. [2]

10 A particle travels in a straight line so that, t s after passing through a fixed point O, its displacement s m from O is given by $s = \ln(t^2 + 1)$.

(i) Find the value of t when $s = 5$. [2]

(ii) Find the distance travelled by the particle during the third second. [2]

(iii) Show that, when $t = 2$, the velocity of the particle is $0.8\,\mathrm{m\,s^{-1}}$. [2]

(iv) Find the acceleration of the particle when $t = 2$. [3]

11 Solve the equation

(i) $3 \sin x - 4 \cos x = 0$, for $0° \leqslant x \leqslant 360°$, [3]

(ii) $11 \sin y + 1 = 4 \cos^2 y$, for $0° \leqslant y \leqslant 360°$, [4]

(iii) $\sec\left(2z + \frac{\pi}{3}\right) = -2$, for $0 \leqslant z \leqslant \pi$ radians. [4]

12 Answer only **one** of the following two alternatives.

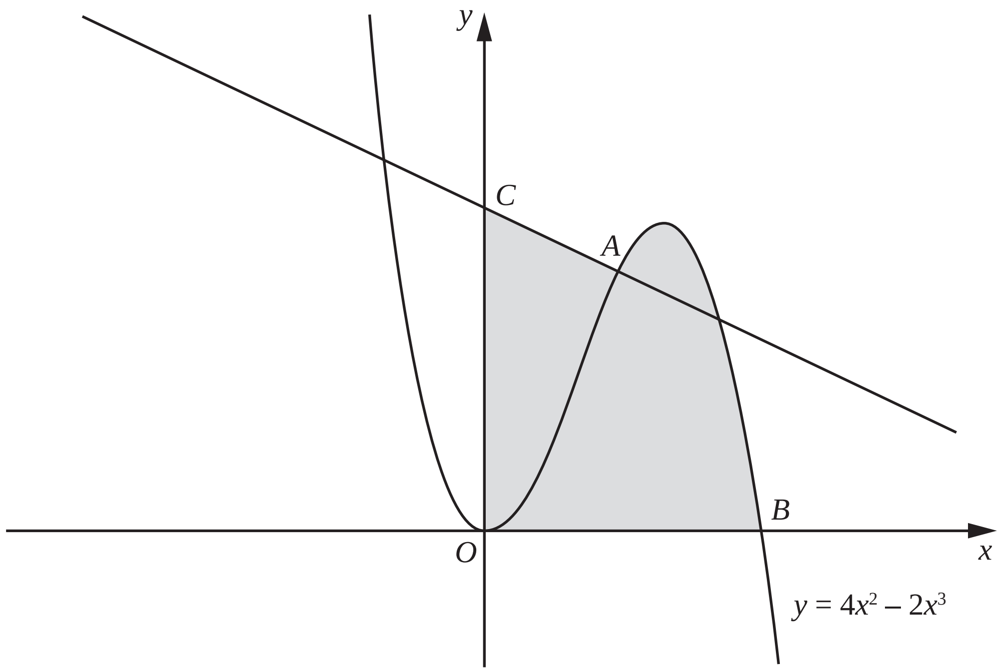
EITHER

A curve has the equation $y = A\sin 2x + B\cos 3x$. The curve passes through the point with coordinates $\left(\frac{\pi}{12}, 3\right)$ and has a gradient of -4 when $x = \frac{\pi}{3}$.

(i) Show that $A = 4$ and find the value of B. [6]

(ii) Given that, for $0 \leqslant x \leqslant \frac{\pi}{3}$, the curve lies above the x-axis, find the area of the region enclosed by the curve, the y-axis and the line $x = \frac{\pi}{3}$. [5]

OR



The diagram shows the curve $y = 4x^2 - 2x^3$. The point A lies on the curve and the x-coordinate of A is 1. The curve crosses the x-axis at the point B. The normal to the curve at the point A crosses the y-axis at the point C.

(i) Show that the coordinates of C are (0, 2.5). [5]

(ii) Find the area of the shaded region. [6]

1 Solve the equation $|2x + 10| = 7$. [3]

2 The expression $x^3 + ax^2 - 15x + b$ has a factor $x - 2$ and leaves a remainder of 75 when divided by $x + 3$. Find the value of a and of b. [5]

3 A number, N_0, of fish of a particular species are introduced to a lake. The number, N, of these fish in the lake, t weeks after their introduction, is given by $N = N_0 e^{-kt}$, where k is a constant. Calculate

(i) the value of k if, after 34 weeks, the number of these fish has fallen to $\frac{1}{2}$ of the number introduced, [2]

(ii) the number of weeks it takes for the number of these fish to have fallen to $\frac{1}{5}$ of the number introduced. [3]

4 Students take three multiple-choice tests, each with ten questions. A correct answer earns 5 marks. If no answer is given 1 mark is scored. An incorrect answer loses 2 marks. A student's final total mark is the sum of 20% of the mark in test 1, 30% of the mark in test 2 and 50% of the mark in test 3. One student's responses are summarized in the table below.

	Test 1	Test 2	Test 3
Correct answer	7	6	5
No answer	1	3	5
Incorrect answer	2	1	0

Write down three matrices such that matrix multiplication will give this student's final total mark and hence find this total mark. [5]

5 Find the set of values of m for which the line $y = mx - 2$ cuts the curve $y = x^2 + 8x + 7$ in two distinct points. [6]

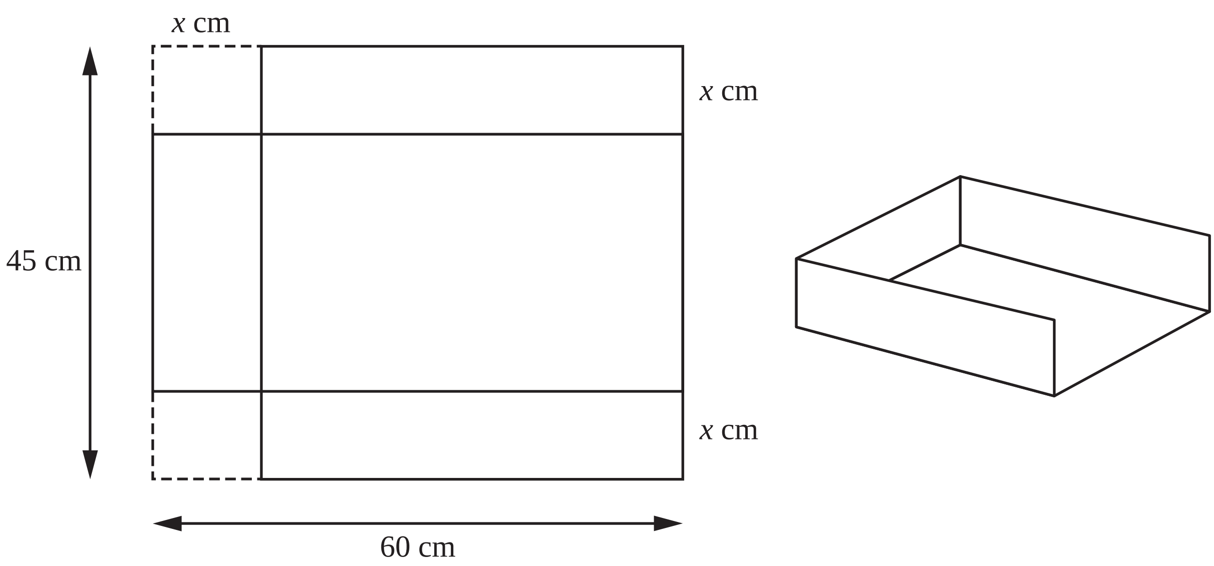
6 A 4-digit number is formed by using four of the seven digits 1, 3, 4, 5, 7, 8 and 9. No digit can be used more than once in any one number. Find how many different 4-digit numbers can be formed if

(i) there are no restrictions, [2]

(ii) the number is less than 4000, [2]

(iii) the number is even and less than 4000. [2]

7



A rectangular sheet of metal measures 60 cm by 45 cm. A scoop is made by cutting out squares, of side x cm, from two corners of the sheet and folding the remainder as shown.

(i) Show that the volume, V cm^3, of the scoop is given by

$$V = 2700x - 165x^2 + 2x^3.$$ [2]

(ii) Given that x can vary, find the value of x for which V has a stationary value. [4]

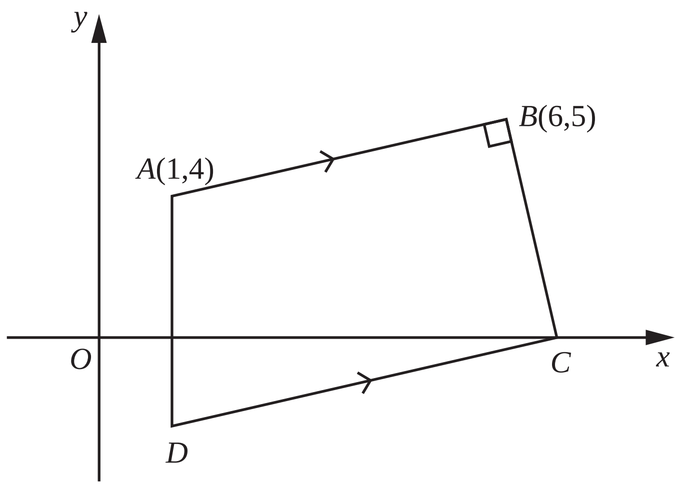
8 Solve the equation

(i) $\lg(5x + 10) + 2\lg 3 = 1 + \lg(4x + 12)$, [4]

(ii) $\dfrac{9^{2y}}{3^{7-y}} = \dfrac{3^{4y+3}}{27^{y-2}}$. [3]

9 A plane, whose speed in still air is 250 km h^{-1}, flies directly from A to B, where B is 500 km from A on a bearing of 060°. There is a constant wind of 80 km h^{-1} blowing from the south. Find, to the nearest minute, the time taken for the flight. [7]

10 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a quadrilateral $ABCD$ in which A is the point (1, 4) and B is the point (6, 5). Angle ABC is a right angle and the point C lies on the x-axis. The line AD is parallel to the y-axis and the line CD is parallel to BA. Find

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12 Answer only **one** of the following two alternatives.

EITHER

The functions f and g are defined, for $x > 1$, by

$$f(x) = (x + 1)^2 - 4,$$

$$g(x) = \frac{3x + 5}{x - 1}.$$

Find

(i) fg(9), [2]

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OR

A particle moves in a straight line so that, at time t s after passing a fixed point O, its velocity is $v\,\text{ms}^{-1}$, where

$$v = 6t + 4\cos 2t.$$

Find

(i) the velocity of the particle at the instant it passes O, [1]

(ii) the acceleration of the particle when $t = 5$, [4]

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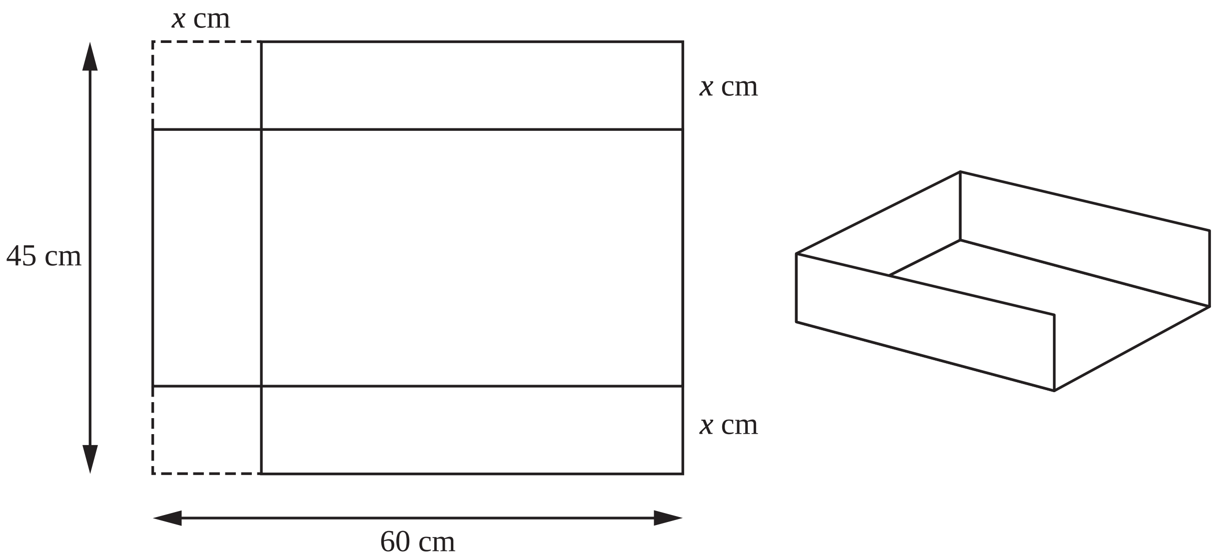

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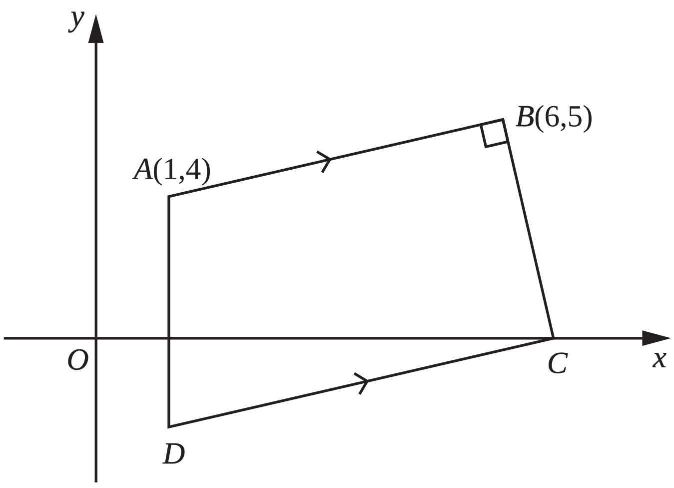

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The functions f and g are defined, for $x > 1$, by

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$$g(x) = \frac{3x + 5}{x - 1}.$$

Find

(i) fg(9), [2]

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OR

A particle moves in a straight line so that, at time t s after passing a fixed point O, its velocity is $v\,\text{m}\,\text{s}^{-1}$, where

$$v = 6t + 4\cos 2t.$$

Find

(i) the velocity of the particle at the instant it passes O, [1]

(ii) the acceleration of the particle when $t = 5$, [4]

(iii) the greatest value of the acceleration, [1]

(iv) the distance travelled in the fifth second. [4]

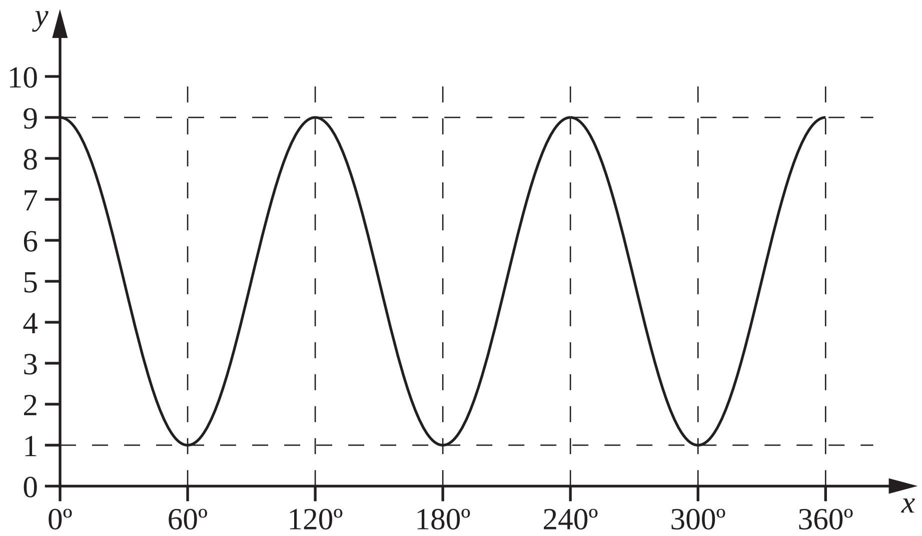
1 The two variables x and y are such that $y = \dfrac{10}{(x+4)^3}$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

(ii) Hence find the approximate change in y as x increases from 6 to $6+p$, where p is small. [2]

2 Find the equation of the curve which passes through the point (4, 22) and for which $\dfrac{dy}{dx} = 3x(x-2)$. [4]

3 **(a)**



The diagram shows the curve $y = A\cos Bx + C$ for $0° \leqslant x \leqslant 360°$. Find the value of

(i) A, **(ii)** B, **(iii)** C. [3]

(b) Given that $\mathrm{f}(x) = 6\sin 2x + 7$, state

(i) the period of f, [1]

(ii) the amplitude of f. [1]

4 **(i)** Find, in ascending powers of x, the first 4 terms of the expansion of $(1 + x)^6$. [2]

(ii) Hence find the coefficient of p^3 in the expansion of $(1 + p - p^2)^6$. [3]

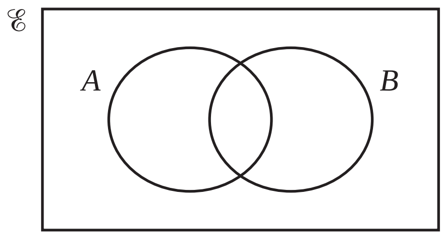
5 **(a)** Given that $\mathbf{A} = (2 \quad -4 \quad 1)$ and $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 0 & 5 \\ -2 & 7 \end{pmatrix}$, find the matrix product $\mathbf{AB}$. [2]

(b) Given that $\mathbf{C} = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 6 & -4 \\ 2 & 8 \end{pmatrix}$, find

(i) the inverse matrix $\mathbf{C}^{-1}$, [2]

(ii) the matrix $\mathbf{X}$ such that $\mathbf{CX} = \mathbf{D}$. [2]

6 **(a)**



Copy the diagram above and shade the region which represents the set $A' \cup B$. [1]

(b) The sets P, Q and R are such that

$$P \cap Q = \varnothing \text{ and } P \cup Q \subset R.$$

Draw a Venn diagram showing the sets P, Q and R. [2]

(c) In a group of 50 students F denotes the set of students who speak French and S denotes the set of students who speak Spanish. It is given that $n(F) = 24$, $n(S) = 18$, $n(F \cap S) = x$ and $n(F' \cap S') = 3x$. Write down an equation in x and hence find the number of students in the group who speak neither French nor Spanish. [3]

7 The line $y = 2x - 6$ meets the curve $4x^2 + 2xy - y^2 = 124$ at the points A and B. Find the length of the line AB. [7]

8 **(i)** Show that $(5 + 3\sqrt{2})^2 = 43 + 30\sqrt{2}$. [1]

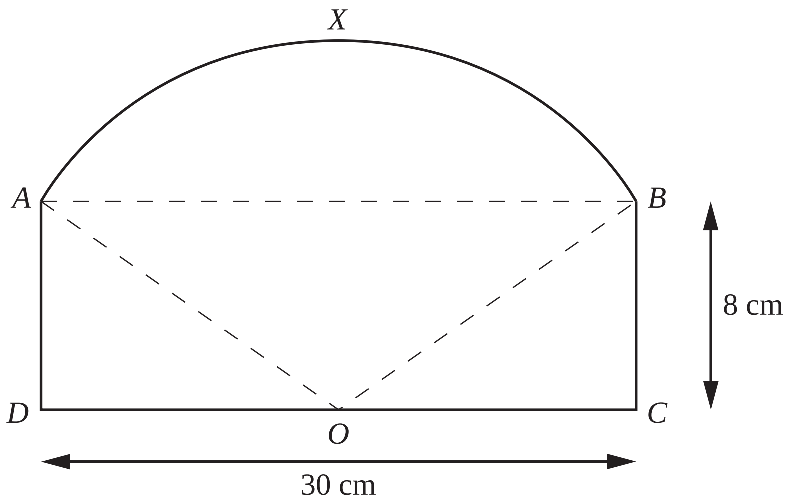
Hence find, **without using a calculator**, the positive square root of

(ii) $86 + 60\sqrt{2}$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers, [2]

(iii) $43 - 30\sqrt{2}$, giving your answer in the form $c + d\sqrt{2}$, where c and d are integers, [1]

(iv) $\dfrac{1}{43 + 30\sqrt{2}}$, giving your answer in the form $\dfrac{f + g\sqrt{2}}{h}$, where f, g and h are integers. [3]

9



The diagram shows a rectangle *ABCD* and an arc *AXB* of a circle with centre at *O*, the mid-point of *DC*. The lengths of *DC* and *BC* are 30 cm and 8 cm respectively. Find

(i) the length of *OA*, [2]

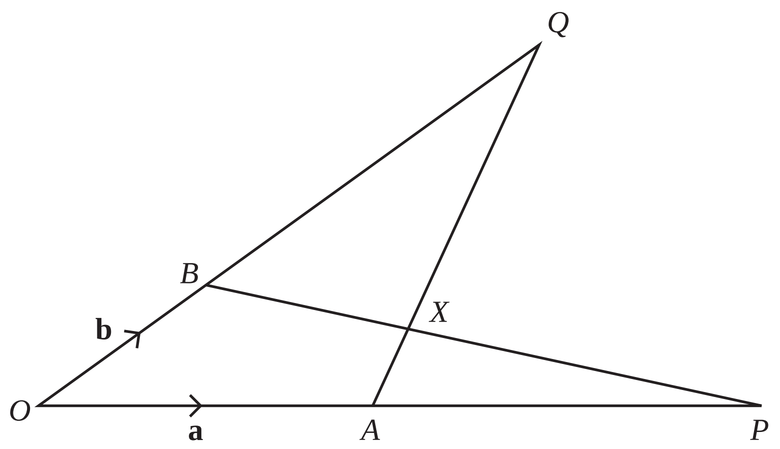
(ii) the angle *AOB*, in radians, [2]

(iii) the perimeter of figure *ADOCBXA*, [2]

(iv) the area of figure *ADOCBXA*. [2]

10 The equation of a curve is $y = x^2e^x$. The tangent to the curve at the point $P(1, e)$ meets the y-axis at the point A. The normal to the curve at P meets the x-axis at the point B. Find the area of the triangle OAB, where O is the origin. [9]

11



In the diagram $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OQ} = 3\mathbf{b}$.

(i) Given that $\overrightarrow{AX} = \mu\overrightarrow{AQ}$, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [3]

(ii) Given that $\overrightarrow{BX} = \lambda\overrightarrow{BP}$, express $\overrightarrow{OX}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [3]

(iii) Hence find the value of μ and of λ. [3]

12 Answer only **one** of the following two alternatives.

EITHER

The table shows values of the variables v and p which are related by the equation $p = \frac{a}{v^2} + \frac{b}{v}$, where a and b are constants.

v	2	4	6	8
p	6.22	2.84	1.83	1.35

(i) Using graph paper, plot $v^2 p$ on the y-axis against v on the x-axis and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of b. [4]

In another method of finding a and b from a straight line graph, $\frac{1}{v}$ is plotted along the x-axis. In this case, and without drawing a second graph,

(iii) state the variable that should be plotted on the y-axis, [2]

(iv) explain how the values of a and b could be obtained. [2]

OR

The table shows experimental values of two variables r and t.

t	2	8	24	54
r	22	134	560	1608

(i) Using the y-axis for $\ln r$ and the x-axis for $\ln t$, plot $\ln r$ against $\ln t$ to obtain a straight line graph. [2]

(ii) Find the gradient and the intercept on the y-axis of this graph and express r in terms of t. [6]

Another method of finding the relationship between r and t from a straight line graph is to plot $\lg r$ on the y-axis and $\lg t$ on the x-axis. Without drawing this second graph, find the value of the gradient and of the intercept on the y-axis for this graph. [2]

QUESTION BANK

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ADDITONAL MATHEMATICS

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