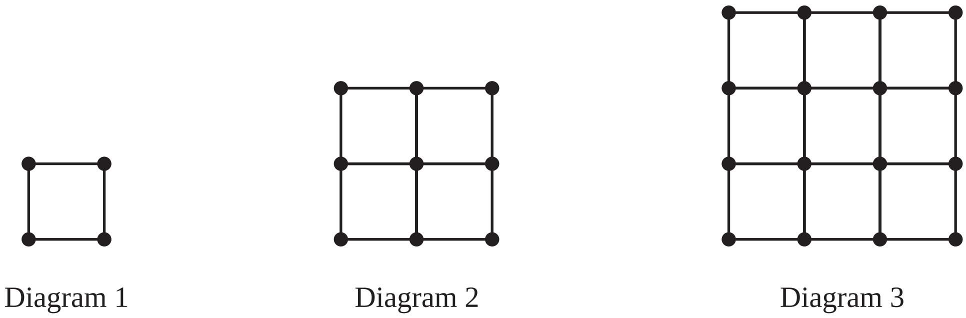
9



The first three diagrams in a sequence are shown above.

The diagrams are made up of dots and lines. Each line is one centimetre long.

(a) Make a sketch of the next diagram in the sequence. [1]

(b) The table below shows some information about the diagrams.

Diagram	1	2	3	4	---------	n
Area	1	4	9	16	---------	x
Number of dots	4	9	16	p	---------	y
Number of one centimetre lines	4	12	24	q	---------	z

(i) Write down the values of p and q. [2]

(ii) Write down each of x, y and z in terms of n. [4]

(c) The **total** number of one centimetre lines in the first n diagrams is given by the expression

$$\frac{2}{3}n^3 + fn^2 + gn.$$

(i) Use $n = 1$ in this expression to show that $f + g = \frac{10}{3}$. [1]

(ii) Use $n = 2$ in this expression to show that $4f + 2g = \frac{32}{3}$. [2]

(iii) Find the values of f and g. [3]

(iv) Find the total number of one centimetre lines in the first 10 diagrams. [1]

10

1	**2**	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**

A 3 by 3 square

x	b	c
d	e	f
g	h	i

can be chosen from the 6 by 6 grid above.

(a) One of these squares is

8	9	10
14	15	16
20	21	22

.

In this square, $x = 8$, $c = 10$, $g = 20$ and $i = 22$.

For this square, calculate the value of

(i) $(i - x) - (g - c)$, [1]

(ii) $cg - xi$. [1]

(b)

x	b	c
d	e	f
g	h	i

(i) $c = x + 2$. Write down g and i in terms of x. [2]

(ii) Use your answers to **part(b)(i)** to show that $(i - x) - (g - c)$ is constant. [1]

(iii) Use your answers to **part(b)(i)** to show that $cg - xi$ is constant. [2]

(c) The 6 by 6 grid is replaced by a 5 by 5 grid as shown.

1	**2**	**3**	**4**	**5**
6	**7**	**8**	**9**	**10**
11	**12**	**13**	**14**	**15**
16	**17**	**18**	**19**	**20**
21	**22**	**23**	**24**	**25**

A 3 by 3 square

x	***b***	***c***
d	***e***	***f***
g	***h***	***i***

can be chosen from the 5 by 5 grid.

For any 3 by 3 square chosen from this 5 by 5 grid, calculate the **value** of

(i) $(i - x) - (g - c)$, [1]

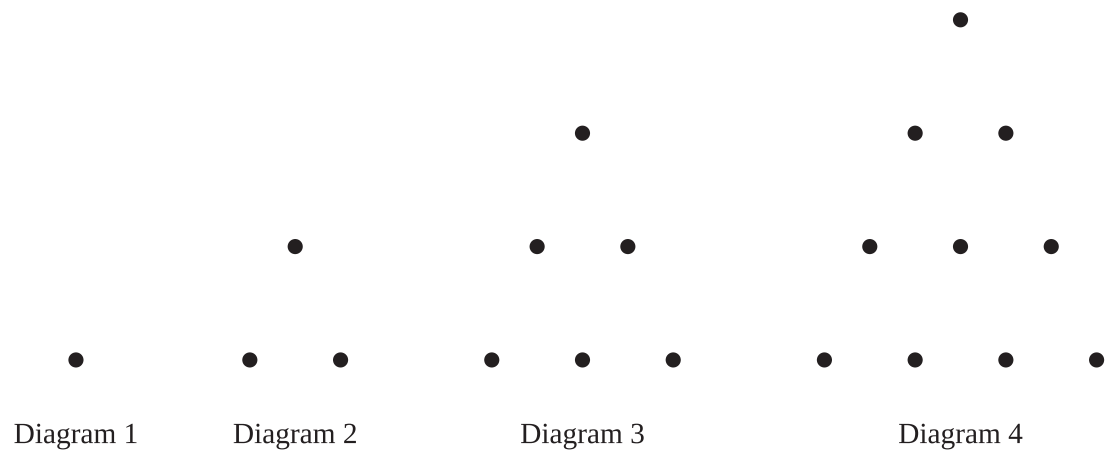
(ii) $cg - xi$. [1]

(d) A 3 by 3 square is chosen from an n by n grid.

(i) Write down the value of $(i - x) - (g - c)$. [1]

(ii) Find g and i in terms of x and n. [2]

(iii) Find $cg - xi$ in its simplest form. [1]



The first four terms in a sequence are 1, 3, 6 and 10.
They are shown by the number of dots in the four diagrams above.

(a) Write down the next four terms in the sequence.

Answer(a) .... , ... , ... , ... [2]

(b) **(i)** The sum of the two consecutive terms 3 and 6 is 9.
The sum of the two consecutive terms 6 and 10 is 16.

Complete the following statements using different pairs of terms.

The sum of the two consecutive terms .... and .... is ....

The sum of the two consecutive terms .... and .... is .... [1]

(ii) What special name is given to these sums?

Answer(b)(ii) .... [1]

(c) **(i)** The formula for the nth term in the sequence 1, 3, 6, 10… is $\frac{n(n+1)}{k}$,
where k is an integer.

Find the value of k.

Answer(c)(i) k = ... [1]

(ii) Test your formula when $n = 4$, **showing your working**.

Answer (c)(ii)

[1]

(iii) Find the value of the 180th term in the sequence.

Answer(c)(iii) .... [1]

(d) **(i)** Show clearly that the sum of the nth and the $(n + 1)$th terms is $(n + 1)^2$.

Answer (d)(i)

[3]

(ii) Find the values of the two consecutive terms which have a sum of 3481.

Answer(d)(ii) .... and .... [2]

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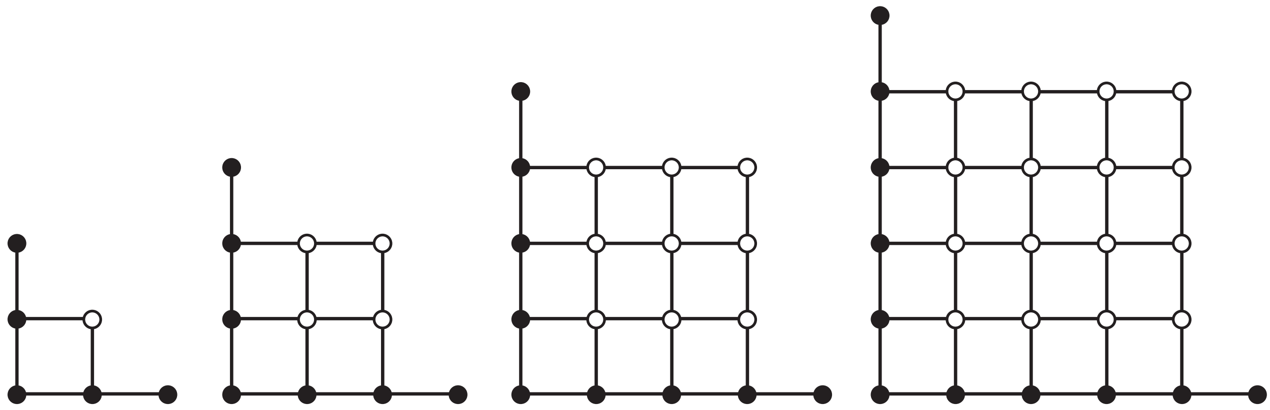


Diagram 1	Diagram 2	Diagram 3	Diagram 4
1 white dot	4 white dots	9 white dots	16 white dots
5 black dots	7 black dots	9 black dots	11 black dots
6 lines	14 lines	26 lines	42 lines

The four diagrams above are the first four of a pattern.

(a) Diagram 5 has been started below.
Complete this diagram and write down the information about the numbers of dots and lines.

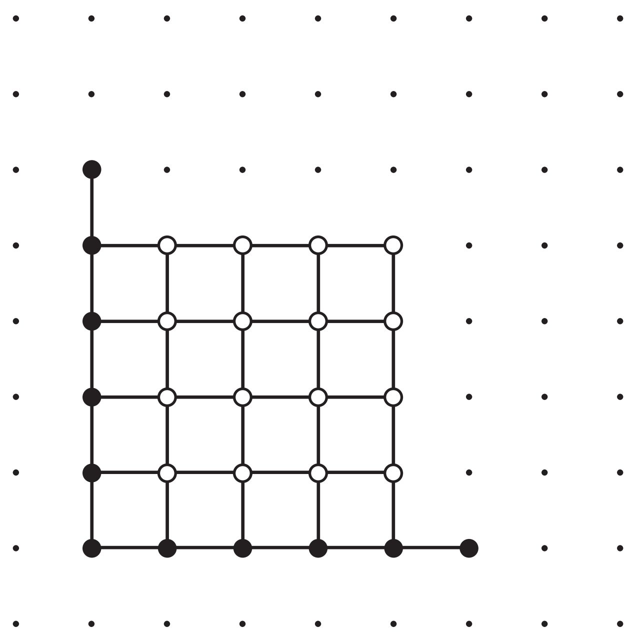


Diagram 5

.... white dots

.... black dots

.... lines

[4]

For Examiner's Use

(b) Complete the information about the number of dots and lines in Diagram 8.

Answer(b) .... white dots

.... black dots

.... lines [3]

(c) Complete the information about the number of dots in Diagram n.
Give your answers in terms of n.

Answer(c) .... white dots

.... black dots [2]

(d) The number of lines in diagram n is $k(n^2 + n + 1)$.

Find

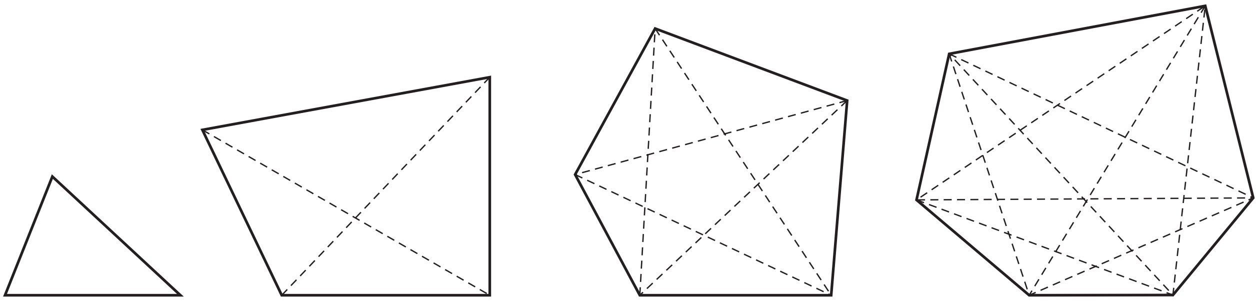
(i) the value of k,

Answer(d)(i) $k =$ .... [1]

(ii) the number of lines in Diagram 100.

Answer(d)(ii) .... [1]

10



The diagrams show some polygons and their diagonals.

(a) Complete the table.

Number of sides	Name of polygon	Total number of diagonals
3	triangle	0
4	quadrilateral	2
5		5
6	hexagon	9
7	heptagon	14
8		

[3]

(b) Write down the total number of diagonals in

(i) a decagon (a 10-sided polygon),

Answer(b)(i) .... [1]

(ii) a 12-sided polygon.

Answer(b)(ii) .... [1]

(c) A polygon with n sides has a total of $\frac{1}{p}n(n-q)$ diagonals, where p and q are integers.

(i) Find the values of p and q.

Answer(c)(i) p = ...

q = ... [3]

(ii) Find the total number of diagonals in a polygon with 100 sides.

Answer(c)(ii) .... [1]

(iii) Find the number of sides of a polygon which has a total of 170 diagonals.

Answer(c)(iii) .... [2]

(d) A polygon with $n + 1$ sides has 30 more diagonals than a polygon with n sides.

Find n.

Answer(d) n = ... [1]

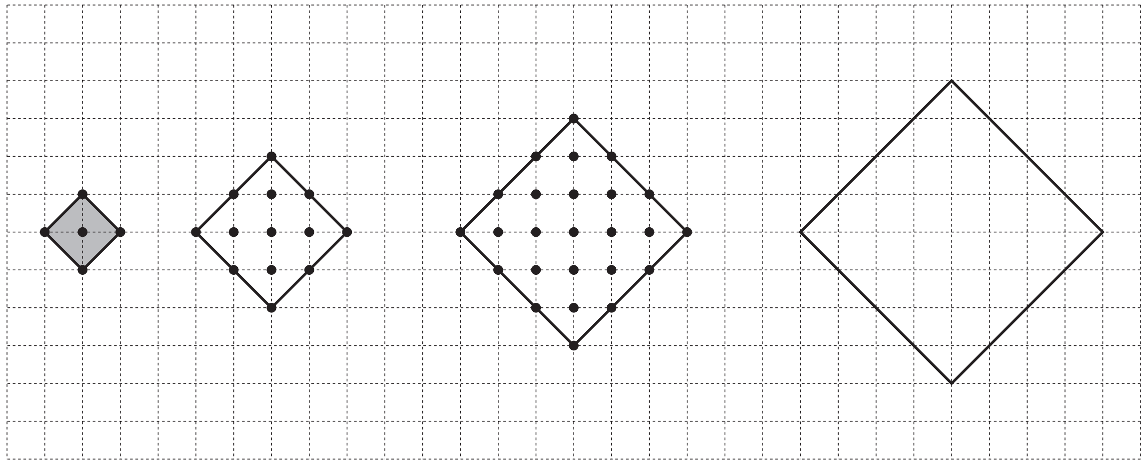


Diagram 1 Diagram 2 Diagram 3 Diagram 4

The diagrams show squares and dots on a grid.

Some dots are on the sides of each square and other dots are inside each square.

The area of the square (shaded) in Diagram 1 is 1 unit^2.

(a) Complete Diagram 4 by marking all the dots. [1]

(b) Complete the columns in the table below for Diagrams 4, 5 and n.

Diagram	1	2	3	4	5	-------	n
Number of units of area	1	4	9			-------	
Number of dots inside the square	1	5	13			-------	$(n-1)^2+n^2$
Number of dots on the sides of the square	4	8	12			-------	
Total number of dots	5	13	25			-------	

[7]

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(c) For Diagram 200, find the number of dots

(i) inside the square,

Answer(c)(i) .... [1]

(ii) on the sides of the square.

Answer(c)(ii) .... [1]

(d) Which diagram has 265 dots inside the square?

Answer(d) .... [1]

9 The table shows some terms of several sequences.

Term	1	2	3	4		8	
Sequence **P**	7	5	3	1		p	
Sequence **Q**	1	8	27	64		q	
Sequence **R**	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$		r	
Sequence **S**	4	9	16	25		s	
Sequence **T**	1	3	9	27		t	
Sequence **U**	3	6	7	−2		u	

(a) Find the values of p, q, r, s, t and u. [6]

(b) Find the nth term of sequence

(i) **P**, [1]

(ii) **Q**, [1]

(iii) **R**, [1]

(iv) **S**, [1]

(v) **T**, [1]

(vi) **U**. [1]

(c) Which term in sequence **P** is equal to −777? [2]

(d) Which term in sequence **T** is equal to 177 147? [2]

10 $$1 + 2 + 3 + 4 + 5 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}$$

(a) **(i)** Show that this formula is true for the sum of the first 8 natural numbers. [2]

(ii) Find the sum of the first 400 natural numbers. [1]

(b) **(i)** Show that $2 + 4 + 6 + 8 + \ldots\ldots\ldots\ldots\ldots\ldots + 2n = n(n + 1)$. [1]

(ii) Find the sum of the first 200 even numbers. [1]

(iii) Find the sum of the first 200 odd numbers. [1]

(c) **(i)** Use the formula at the beginning of the question to find the sum of the first $2n$ natural numbers. [1]

(ii) Find a formula, in its simplest form, for

$$1 + 3 + 5 + 7 + 9 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + (2n - 1).$$

Show your working. [2]

									Total
Row 1						1	=	1
Row 2				3	+	5	=	8
Row 3		7	+	9	+	11	=	27
Row 4	13	+ 15	+	17	+	19	=	64
Row 5								
Row 6								

The rows above show sets of consecutive odd numbers and their totals.

(a) Complete Row 5 and Row 6. [2]

(b) What is the special name given to the numbers 1, 8, 27, 64…?

Answer(b) .... [1]

(c) Write down in terms of n,

(i) how many consecutive odd numbers there are in Row n,

Answer(c)(i) .... [1]

(ii) the total of these numbers.

Answer(c)(ii) .... [1]

(d) The first number in Row n is given by $n^2 - n + 1$.

Show that this formula is true for Row 4.

Answer(d)

(e) The total of Row 3 is 27. This can be calculated by $(3 \times 7) + 2 + 4$.

The total of Row 4 is 64. This can be calculated by $(4 \times 13) + 2 + 4 + 6$.

The total of Row 7 is 343. Show how this can be calculated in the same way.

Answer(e)

[1]

(f) The total of the first n **even** numbers is $n(n + 1)$.

Write down a formula for the total of the first $(n - 1)$ **even** numbers.

Answer(f) .... [1]

(g) Use the results of **parts (d), (e)** and **(f)** to show clearly that the total of the numbers in Row n gives your answer to **part (c)(ii)**.

Answer(g)

[2]

9 **(a)** The first five terms P_1, P_2, P_3, P_4 and P_5 of a sequence are given below.

1	$= 1 = P_1$
1 + 2	$= 3 = P_2$
1 + 2 + 3	$= 6 = P_3$
1 + 2 + 3 + 4	$= 10 = P_4$
1 + 2 + 3 + 4 + 5	$= 15 = P_5$

(i) Write down the next term, P_6, in the sequence 1, 3, 6, 10, 15…

Answer(a)(i) .... [1]

(ii) The formula for the *n*th term of this sequence is

$$P_n = \frac{1}{2}n(n + 1).$$

Show this formula is true when $n = 6$.

Answer (a)(ii)

[1]

(iii) Use the formula to find P_{50}, the 50th term of this sequence.

Answer(a)(iii) .... [1]

(iv) Use your answer to **part (iii)** to find 3 + 6 + 9 + 12 +15 + ……….. + 150.

Answer(a)(iv) .... [1]

(v) Find 1 + 2 + 3 + 4 + 5 +……….. + 150.

Answer(a)(v) .... [1]

(vi) Use your answers to **parts (iv)** and **(v)** to find the sum of the numbers less than 150 which are **not** multiples of 3.

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(b) The first five terms, S_1, S_2, S_3, S_4 and S_5 of a different sequence are given below.

(1×1)	$= 1 = S_1$
$(1 \times 2) + (2 \times 1)$	$= 4 = S_2$
$(1 \times 3) + (2 \times 2) + (3 \times 1)$	$= 10 = S_3$
$(1 \times 4) + (2 \times 3) + (3 \times 2) + (4 \times 1)$	$= 20 = S_4$
$(1 \times 5) + (2 \times 4) + (3 \times 3) + (4 \times 2) + (5 \times 1)$	$= 35 = S_5$

(i) Work out the next term, S_6, in the sequence 1, 4, 10, 20, 35…

Answer(b)(i) .... [2]

(ii) The formula for the nth term of this sequence is

$$S_n = \frac{1}{6}n(n+1)(n+2).$$

Show this formula is true for $n = 6$.

Answer(b)(ii)

[1]

(iii) Find $(1 \times 20) + (2 \times 19) + (3 \times 18) \ldots\ldots\ldots\ldots + (20 \times 1)$.

Answer(b)(iii) .... [1]

(c) Show that $S_6 - S_5 = P_6$, where P_6 is your answer to **part (a)(i)**.

Answer(c)

[1]

(d) Show by algebra that $S_n - S_{n-1} = P_n$. $[P_n = \frac{1}{2}n(n+1)]$

Answer(d)

10 In all the following sequences, after the first two terms, the rule is to add the previous two terms to find the next term.

(a) Write down the next two terms in this sequence.

1 1 2 3 5 8 13 .... [1]

(b) Write down the first two terms of this sequence.

.... 3 11 14 [2]

(c) **(i)** Find the value of d and the value of e.

2 d e 10

Answer(c)(i) d = ...

e = ... [3]

(ii) Find the value of x, the value of y and the value of z.

-33 x y z 18

Answer(c)(ii) x = ...

y = ...

z = ... [5]

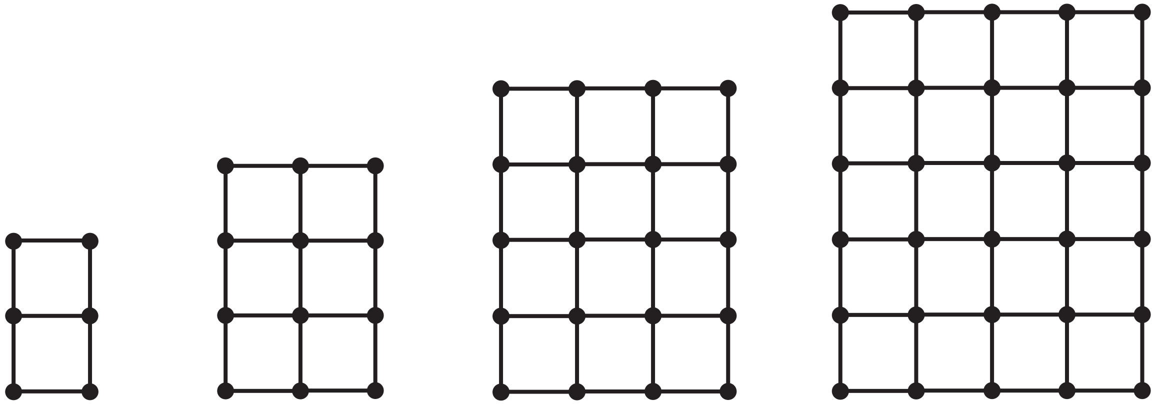

Diagram 1 Diagram 2 Diagram 3 Diagram 4

The first four Diagrams in a sequence are shown above.
Each Diagram is made from dots and one centimetre lines.
The area of each small square is $1\,\text{cm}^2$.

(a) Complete the table for Diagrams 5 and 6.

Diagram	1	2	3	4	5	6
Area (cm^2)	2	6	12	20		
Number of dots	6	12	20	30		
Number of one centimetre lines	7	17	31	49		

[4]

(b) The **area** of Diagram n is $n(n+1)$ cm^2.

(i) Find the **area** of Diagram 50.

Answer(b)(i) .... cm^2 [1]

(ii) Which Diagram has an **area** of $930\,\text{cm}^2$?

Answer(b)(ii) .... [1]

(c) Find, in terms of n, the number of **dots** in Diagram n.

(d) The number of one centimetre lines in Diagram n is $2n^2 + pn + 1$.

(i) Show that $p = 4$.

Answer(d)(i)

[2]

(ii) Find the number of one centimetre lines in Diagram 10.

Answer(d)(ii) .... [1]

(iii) Which Diagram has 337 one centimetre lines?

Answer(d)(iii) .... [3]

(e) For **each** Diagram, the number of squares of area 1 cm^2 is A, the number of dots is D and the number of one centimetre lines is L.

Find a connection between A, D and L that is true for each Diagram.