UNIT 6 (Extended) FURTHER NUMBER AND PROBABILITY

Recommended Prior Knowledge
It is strongly recommended that candidates have a thorough knowledge and understanding of the topics in Unit 1.

Context
The unit draws on the skills that candidates have developed whilst studying the topics in Unit 1. Further mathematical concepts in Number and Probability are explored together with the idea of modelling practical situations with straight line graphs and interpreting results. Candidates should use calculators where appropriate, however, it is recommended that regular non-calculator work is completed to strengthen candidates' mental arithmetic.

Outline
The topics in this unit may be studied sequentially. There is some element of choice, however, and Centres may wish to teach topics in a different order, for example Sets and Probability need not be studied last. Work on indices, standard form, accuracy, direct and inverse proportion, measures of rate, distance-time and speed-time graphs, personal and household finance is completed together with further work on straight line graphs. With all sections it is expected that candidates will be set questions of varying difficulty to complete for themselves. The unit gives candidates the opportunity to work investigatively and thus establish the skills needed for the submission of coursework.

Learning Outcomes		Suggested Teaching Activities	Resources
23	Use and interpret positive, negative, fractional and zero indices.	**Class activity:** Revise writing an integer as a product of primes, writing answers using index notation. Use simple examples to illustrate the rules of indices. Introduce negative indices, e.g. $2^{-1} = 2^{(2-3)} = \frac{2^2}{2^3} = \frac{1}{2}$ and $2^0 = 2^{(3-3)} = \frac{2^3}{2^3} = 1$. Introduce fractional indices by relating them to roots (of positive integers), e.g. $x^{1/2} \times x^{1/2} = x^1$, so that $x^{1/2} = \sqrt{x}$. Use the rules of indices to show how values such as $16^{3/4}$ can be simplified. **Class activity:** By writing an integer as the product of primes investigate how expressions involving square roots can be simplified. For example, the expression $\sqrt{20} + \sqrt{45}$ can be written as $5\sqrt{5}$. (This is not on the syllabus but it will broaden candidates mathematical knowledge by introducing surds) Solve simple exponential equations, e.g. $5^x = 25$, $3^{(x+1)} = 27$, $2^{-x} = 8$, etc.	
5	Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, >, <, \geq, \leq$.	Use a number line to describe simple inequalities and ranges of values e.g. $x \geq 3$, $-2 \leq x < 5$, etc. **Class activity:** Given a list of quantities (e.g. a list of	

		fractions and decimals) order them by magnitude using inequality signs.	
6 13	Use the standard form $A \times 10^n$ where n is a positive or negative integer, and $1 \leq A < 10$. Use current units of mass, length, area, volume, and capacity in practical situations and express quantities in terms of larger or smaller units.	Use a range of examples to show how to write numbers in standard form and vice-versa. **Class activity:** Use the four rules of calculation with numbers in standard form. Use practical examples to illustrate how to convert between: millimetres, centimetres, metres and kilometres; grams, kilograms and tonnes; millilitres, centilitres and litres. Use standard form where appropriate.	
8 12	Make estimates of numbers, quantities and lengths, give approximations to specified numbers of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem. Use an electronic calculator efficiently; apply appropriate checks of accuracy.	Revise rounding numbers to the nearest 10, 100, 1000, etc., or to a set number of decimal places. Explain carefully how to round a number to a given number of significant figures. Use rounding to 1sf or 2sf to estimate the answer to a calculation. Check answers with a calculator. **Class activity:** Investigate the percentage error produced by rounding in calculations using addition/subtraction and multiplication/division. (Percentage error will need to be discussed beforehand)	Revision of estimating and rounding at http://www.math.com/school/subject1/lessons/S1U1L3GL.html
9	Give appropriate upper and lower bounds for data given to a specified accuracy (e.g. measured lengths).	Use straightforward examples to determine upper and lower bounds for data. For example, a length, l, measured as 3cm to the nearest millimetre has lower bound 2.95cm and upper bound 3.05cm. Show how this information can be written using inequality signs e.g. $2.95\text{cm} \leq l < 3.05\text{cm}$. **Class activity:** Investigate upper and lower bounds for quantities calculated from given formulae by specifying the accuracy of the input data.	
10	Demonstrate an understanding of the elementary ideas and notion of ratio direct and inverse proportion; divide a quantity in a given ratio Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities; increase and decrease a quantity by a given ratio. Demonstrate an understanding of common measures of rate; use scales in practical situations, calculate average speed.	Draw a graph to determine whether two quantities (y and x or y and x^2, etc.) are in proportion. Solve problems involving direct or inverse proportion using the notation $y \propto x \Rightarrow y = kx$ and $y \propto 1/x \Rightarrow y = k/x$, where k is a constant. Use straightforward examples to illustrate how a quantity can be increased or decreased in a given ratio, e.g. enlarging a photograph. The idea of similar shapes can be introduced here. **Class activity:** Investigate the ratio of the length of one side of an A5 sheet of paper to that of the corresponding side of an A4 sheet of paper. Draw and use straight line graphs to convert between different units e.g. between metric and imperial units or between different currencies.	

15	Calculate using money and convert from one currency to another.	Solve straightforward problems involving exchange rates. Up to date information from a daily newspaper is useful	Exchange rates can be found at http://cnnfn.cnn.com/markets/currencies/
19	Calculate the gradient of a straight line from the coordinates of two points on it; calculate the length and the coordinates of the midpoint of a straight line segment from the coordinates of its end points; interpret and obtain the equation of a straight line graph in the form $y = mx + c$. determine the equation of a straight line parallel to a given line	**Class activity:** Revise drawing a graph of $y = mx + c$ from a table of values. Starting with a straight line graph show how its equation ($y = mx + c$) can be obtained. Using examples which illustrate both positive and negative gradients, show how to calculate the gradient of a straight line given only the coordinates of two points on it.	
17	demonstrate famililiarity with cartesian co-ordinates in two dimensions, Interpret and use graphs in practical situations including travel graphs and conversion graphs, draw graphs from given data. Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs, acceleration and deceleration; calculate distance travelled as area under a linear speed-time graph.	Introduce the formula relating speed, distance and time. Solve simple numerical problems (which should involve converting between units e.g. find speed in m/s given distance in kilometres and time in hours). Solve straightforward problems using compound measures e.g. problems involving rate of flow. Draw and use distance-time graphs to calculate average speed (link to calculating gradients in Unit 1). Interpret information shown in travel graphs. Draw travel graphs from given data. **Class activity:** Draw a travel graph for the journey to and from school. Answer a set of questions about the journey, e.g. what is the average speed on the journey to school? Revise how to calculate the area of a rectangle and the area of a right angled triangle [Further work on this is completed in Unit 4]. Draw and use speed-time graphs to calculate acceleration and deceleration. Use straightforward examples to show that the area under a linear speed-time graph is equivalent to the distance travelled.	Information on speed, distance and time at http://www.mathforum.org/dr.math/faq/faq.distance.html
16	Use given data to solve problems on personal and household finance involving earnings, simple interest and compound interest, discount, profit and loss; extract data from tables and charts.	Solve simple problems using practical examples where possible, taking information from published tables or advertisements. (It is worth introducing a range of simple words and concepts here to describe different aspects of finance, e.g. tax, percentage profit, deposit, loan, etc.) Use the formula $I = PRT$ to solve a variety of problems involving simple interest. **Class activity:** Use newspapers to research the cost of borrowing money from different banks (or money lenders).	Information about interest rates can be found from most banks. They usually have their own web site in the format http://www.*bank name*.com/

11	Carry out calculations involving reverse percentages, e.g. finding the cost price given the selling price and the percentage profit.	**Revise:** Work covered on percentages in Unit 1. Use simple examples to show how to calculate the original value of something before a percentage increase or decrease took place.	
1	Use language, notation and Venn diagrams to describe sets and represent relationships between sets as follows: Definition of sets, e.g. A = {x: x is a natural number} B = {(x,y): $y = mx + c$} C = {x: $a \le x \le b$} D = {a, b, c, ...} Notation: number of elements in set A — n(A) "…. is an element of …." — $\in$ "…. is not an element of …." — $\notin$ Complement of the set A — A' The empty set — $\varnothing$ Universal set — ξ A is a subset of B — $A \subseteq B$ A is a proper subset of B — $A \subset B$ A is not a subset of B — $A \not\subseteq B$ A is not a proper subset of B — $A \not\subset B$ Union of A and B — $A \cup B$ Intersection of A and B — $A \cap B$	**Revise:** Properties of numbers covered in Unit 1. Give examples from work already covered to illustrate the language and notation of sets. Distinguish between a subset and a proper subset. Draw Venn diagrams and shade the regions which represent the sets $A \cup B$, $A \cap B$, $A' \cup B$, $A \cup B'$, $A' \cap B$, $A \cap B'$, $A' \cup B'$ and $A' \cap B'$. Show that $(A \cup B)'$ is the same as $A' \cap B'$ and that $(A \cap B)'$ is the same as $A' \cup B'$. Use Venn diagrams to solve problems involving sets.	Information and references to activities for teachers at http://www.mathworld.wolfram.com/VennDiagram.html
34	Calculate the probability of simple combined events, using possibility diagrams and tree diagrams where appropriate (in possibility diagrams outcomes will be represented by points on a grid and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of the branches).	**Revise:** Simple probabilities covered in Unit 1. Use simple examples to illustrate how possibility diagrams and tree diagrams can help to organise data. Use possibility diagrams and tree diagrams to help calculate probabilities of simple combined events, paying particular attention to how diagrams are labelled. Solve straightforward problems involving independent and dependent events, e.g. picking counters from a bag with and without replacement.	Various problems involving probability at http://nrich.maths.org/public/leg.php