GCSE BITESIZE examinations

General Certificate of Secondary Education

Specimen paper

MATHEMATICS
HIGHER TIER

Paper 1 Non-calculator

Time allowed: 2 hours

You must not use a calculator.

Answer all questions in the space provided.

Mark allocations are shown in brackets.
The maximum mark for this paper is 99.

Show clearly how you work out your answer.

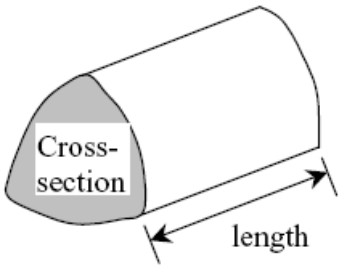
In addition to this paper, you will require:

- mathematical instruments

Formula sheet: Higher Tier

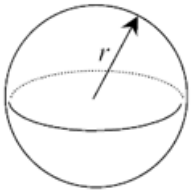
You may use the following formulas:

Volume of prism = area of cross section × length



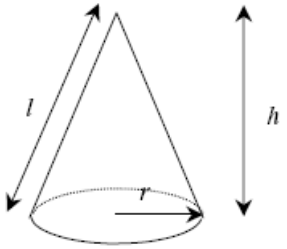
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



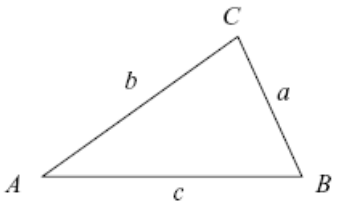
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl



In any triangle *ABC*

Area of triangle = $\frac{1}{2}ab \sin C$



Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

The quadratic equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Answer **all** questions in the spaces provided.

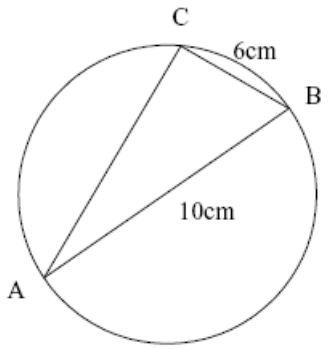
1. (a) Give prime factorisations of 432 and 522.

432 = ……
522 = …… **(1 mark)**

(b) Hence, or otherwise, find the Highest Common Factor of 432 and 522'

HCF = …… **(2 marks)**

2. The diameter AB of the circle is 10cm. The length of BC is 6cm. Calculate the length of AC.



AC = ……cm **(2 marks)**

3. (a) State the *n*th term of each of the following sequences:
(i) 3, 7, 11, 15, 19, ….

Answer:…… **(1 mark)**

(ii) 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, **...**

Answer:…… **(1 mark)**

(iii) 4, 7, 12, 19, 28……

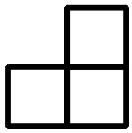
Answer:…….. **(1 mark)**

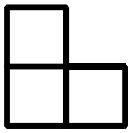
(b) Given that $u_n = 5u_{n-1} + 1$ and that $u_1 = 3$, find the value of u_4

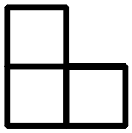
Answer:…….... . **(2 marks)**

4. (a) Sketch the net of a triangular-based pyramid. **(2 marks)**

(b) Here are the plan, front elevation and side elevation of a 3-D shape:

Plan 

Front elevation 

 Side elevation

Draw a sketch of the 3-D shape. **(2 marks)**

5. (a) Write $\frac{3}{8}$ as a decimal.

Answer:…….…..... . **(1 mark)**

(b) Write $0.\dot{2}\dot{4}$ as a fraction in its lowest terms.
Show **all** your working.

Answer:…….…..... . **(3 marks)**

6. (a) Factorise fully $3a^3b + 12a^2b^2 + 9a^5b^3$

…… **(1 mark)**

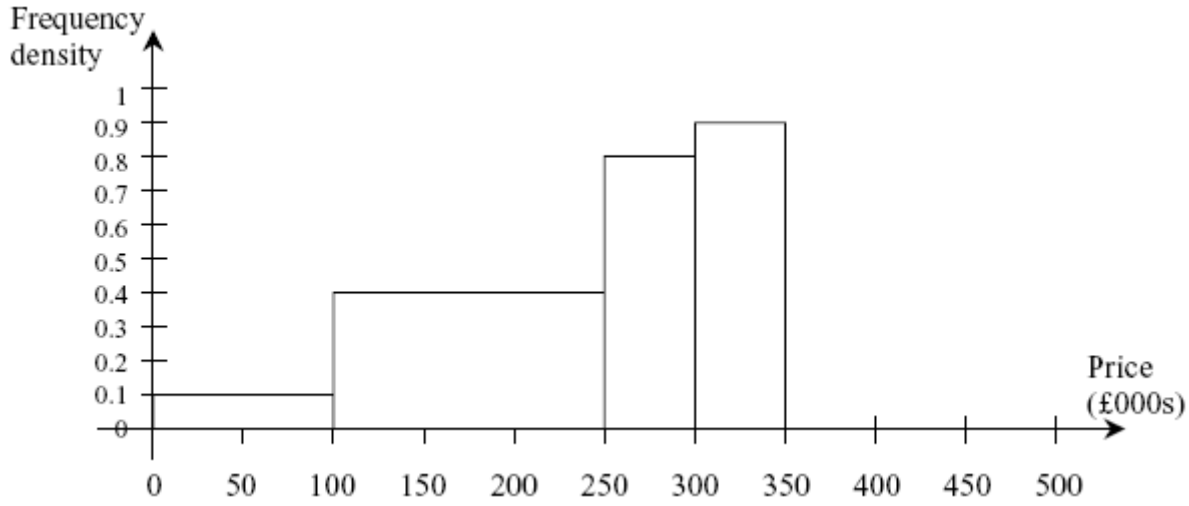
(b) Give the value of x when $2x^2 - x - 6 = 0$

…… **(2 marks)**

(c) Solve the equation $\frac{3x+2}{x-1} + 3 = 4$

…… **(2 marks)**

7. The histogram shows the price distribution of houses in an area of Manchester. Prices are given in thousands of pounds (to the nearest thousand).



Price £(x)000s	0 ≤ x < 100	100 ≤ x <250	250 ≤ x <300	300 ≤ x <350	350 ≤ x <500
Frequency					60

(a) Add a bar to the histogram showing the frequency density for the interval 350-499. **(2 marks)**

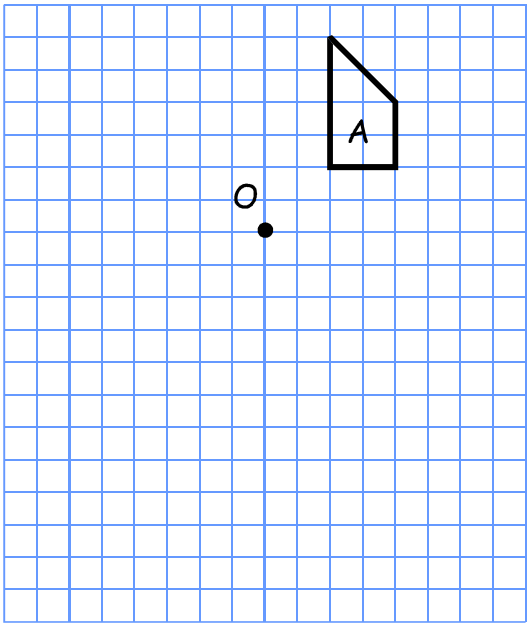
(b) Complete the table above, showing the frequencies for each interval. **(3 marks)**

8. Using a ruler and compasses only, and making sure you leave all construction lines visible:

(a) Construct a triangle of side lengths 4cm, 5cm and 6cm **(2 marks)**

(b). Construct a square of side length 5cm. **(3 marks)**

9. Enlarge shape A with a scale factor of -½, centre O. **(2 marks)**



10. (a) Solve the inequality $5x + 3 \leq 3x - 6$

…… **(1 mark)**

(b) Given that x is an integer and $-3 < x + 1 \leq 4$ list the possible values of x.

…… **(1 mark)**

(c) Find all possible integer values of *y* that satisfy the inequality:

$$-2 \leq \frac{3 - y}{2} < 3$$

…… **(2 marks)**

11. (a) Calculate $4\frac{3}{5} - 2\frac{1}{3}$

Give your answer as a mixed number.

…… **(3 marks)**

(b) Calculate $2\frac{1}{4} \div \frac{3}{5}$

Give your answer as a mixed number.

…… **(3 marks)**

12. Two dice are thrown. The first is a four-sided die numbered 1 to 4, the second a six-sided die numbered 1 to 6.

Ali throws the dice and scores the **product** of the two dice.

(a) (i) Complete the following table which shows the outcomes when Ali throws the dice:

1st die

2nd die \ product	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4				
3						
4						

(2 marks)

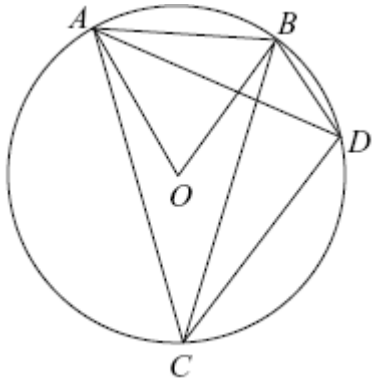
(ii) What is the probability that Ali scores 4?

Answer …….. **(2 marks)**

(b) Sanita throws the two dice and scores the **sum** of the two dice.
What is the probability that Sanita scores more than 6?

Answer …….. **(3 marks)**

13. Given that O is the centre of the circle and that $\angle AOB = 75^o$, $\angle CBD = 62^o$, $\angle BAD = 30^o$ calculate



(a) Angle ACB

Answer……° **(1 mark)**

(b) Angle BDA

Answer……° **(2 marks)**

(c) Angle ABD

Answer……° **(2 marks)**

14. A campaign group is designing a survey to investigate possible opposition to the building of a new road. The new road bypasses a small town, but comes close to two small villages. In one of the villages, a small construction firm has recently gone out of business.

(a) Suggest a possible main question.

…… **(2 marks)**

(b) Suggest three considerations in constructing a sample.
(i)

……
(ii)

……
(iii)

…… **(3 marks)**

(c) The total number of affected people is 4800. The group take a representative sample of 160. From this group, 107 say that they are opposed to the bypass. Approximately how many of the whole group would be expected to be opposed?

…… **(1 mark)**

15. (a) Find the value of $49^{\frac{-1}{2}}$

Answer:…….... **(1 mark)**

(b) Simplify $(2^3)^4$

Answer:…….... **(1 mark)**

(c) Evaluate $\left(\frac{7^3 \times 7^5}{7^{10}}\right)^{-1}$

Answer:…….... **(1 mark)**

16. A cuboid has sides such that the longest side is two units more than the shortest side, and the middle length side is one unit longer than the shortest side. The total surface area of the cuboid is 52 units^2.

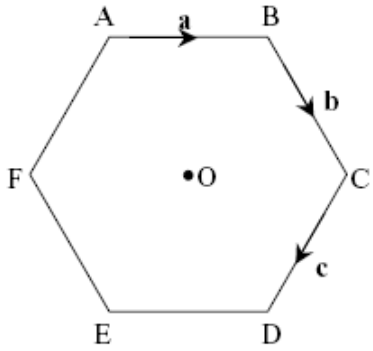
(a) Construct an equation to calculate the surface area.

Answer:…….. ……..... **(3 marks)**

(b) Use the equation to calculate the length of the shortest side.

Answer:…….. units **(3 marks)**

17. The diagram shows a regular hexagon with vertices labelled as shown. O is the centre of the hexagon. The vectors **a**, **b** and **c** are marked on the diagram. Express the following vectors in terms of **a**, **b** and **c**, simplified where possible:



(a)

$\overrightarrow{EF}$ = ……..**(1 mark)**

(b)

$\overrightarrow{DB}$ = ……..**(1 mark)**

(c)

$\overrightarrow{FD}$ = ……..**(1 mark)**

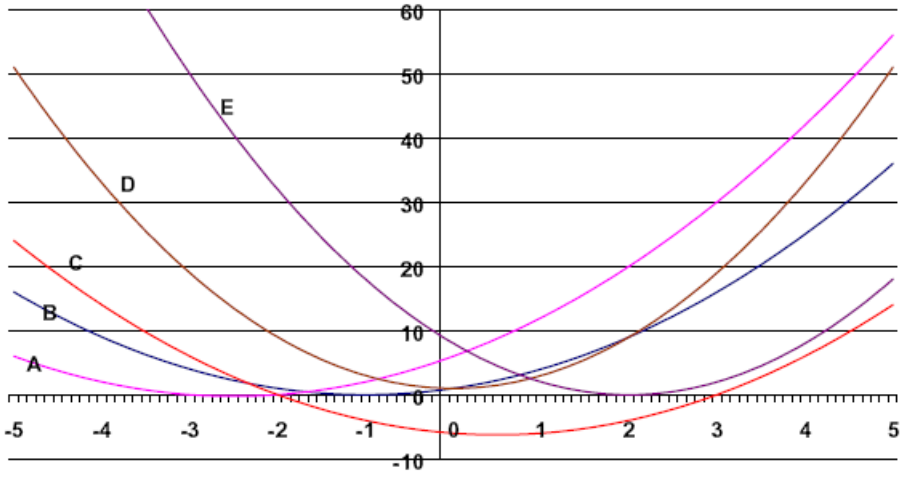
(d) Try to give two alternative answers.

$\overrightarrow{AO}$ = ……..**(1 mark)**
$\overrightarrow{AO}$ = ……..**(1 mark)**

18. Solve $x^2 + 2x - 4 = 0$, leaving your answer in simplest surd form.

Solutions $x =$ …… or …… **(4 marks)**

19. Match the functions to the graphs. Fill in the table with the letter corresponding to the function in each case. **(4 marks)**



Function	Graph
$y = (x+1)^2$	
$y = x^2 + 5x + 6$	
$y = 2x^2 + 1$	
$y = x^2 - x - 6$	
$y = 2(x-2)^2$	

20. (a) Write the product of the first five prime numbers in standard form.

Answer:…….. **(2 marks)**

(b) Write $\frac{3!}{5!}$ exactly in standard form

Answer:…….. **(3 marks)**

(c) Calculate $(5 \times 10^{3}) \div (2 \times 10^{-2})$. Give your answer as a whole number.

Answer:…….. **(2 marks)**

21. (a) The length of an arc in a circle of radius 12cm is 4π cm. Find the size of the angle which describes the arc.

Answer:……..…....° **(2 marks)**

(b) The curved surface area of a right cone with base radius 2cm is 5π cm². Find the slant height of the cone.

Answer:……..…..... . **(2 marks)**

22. The circle *c* has equation $x^2 + y^2 = 1$. The line *l* has gradient 3 and intercepts the *y* axis at the point (0, 1).
c and *l* intersect at two points.
Find the co-ordinates of these points.

Solutions (……,…….) (……,…….) **(3 marks)**