بسم الله الرحمن الرحيم

 مقابل هذا الجهد أرجو منكم الدعاء لي بالملفورة ولابنائي الهداية والنجاح

 والموافقة

 أرجو أن بساعد هذا المجهد على مساعدتنا إبداعنا طلبتي للكتابة في أحسن وآفضل الأدبيات وأعلى الدرجات إنشاء الله.

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 أبو أحمد

# Mathematics

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</table>
International General Certificate of Secondary Education

Instructions to candidates:
You should answer all the questions on the separate sheets of paper provided.
Show all your working on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
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Electronic calculators should be used.
Three figure accuracy is required in your answers except where stated otherwise.
The total of the marks for this paper is 130.
The number of marks available is shown in brackets [ ] at the end of each question or part question.

This Question Paper consists of 7 printed pages and 1 blank page.
Electronic calculators should be used.
Three figure accuracy is required in your answers except where stated otherwise.
The total of the marks for this paper is 75.
The intended marks for questions or parts of questions are given in brackets [ ].

[In this question take \( \pi \) to be 3.142. 1 hectare = 10,000 m\(^2\).]

On a still day, a helicopter hovers at a height of 200 m and sprays the ground with
fertilizer. The shaded part of the diagram shows the circular area sprayed.

(a) If the "angle of spray" is 32°, calculate the sprayed area in square metres. Give your
answer correct to three significant figures.

(b) The farmer wants to spray a circular area of 3 hectares from the same height. What
angle of spray" should he use?

2 Answer the whole of this question on a sheet of plain paper.

Draw a circle of radius 4 cm, whose centre \( O \) is 6 cm from a straight line \( l \).

(a) Construct

(i) the locus of points which are 3 cm from the line \( l \),

(ii) the locus of points which are 7 cm from \( O \).

(b) Hence construct a circle of radius 3 cm which touches the line \( l \) and also touches the
circle of radius 4 cm externally.
3. (a) Juma buys 3 large ice-cream cornets and 2 small ice-cream cornets. They cost him $3.35.
   Susan buys 2 large ones and 3 small ones. They cost her $3.15.
   Use algebra to find the cost of one large cornet, and the cost of one small cornet. [5]

(b) Points $M$ and $N$ are marked on the sides $AB$, $AD$ of a square $ABCD$ of side 10 cm.
    $AM = AN = x$ cm.
    The area of the shaded triangle is 30 cm$^2$.
    (i) By considering areas, show that:
    
    \[
    x^2 - 20x + 60 = 0.
    \]  [3]
    
    (ii) Solve the equation \(x^2 - 20x + 60 = 0\). Hence find the value of $x$, correct to two
    decimal places. [4]

4. (a) Solve the equation
    
    \[15 - 2x = 7.\]  [2]

(b) Find the range of values of $x$ for which
    
    \[15 - 2x \geq 7.\]  [1]

(c) Find \(\{x : 15 - 2x \geq 7\} \cap \{x : 5x - 2 > 10 - x\}\). [4]

5. Answer the whole of this question on a sheet of graph paper.
   Take 1 cm to represent 1 unit on each axis, and mark each axis from $-7$ to $+7$.
   (a) Draw and label triangle $A$, with vertices $(1, 2), (1, 3)$ and $(4, 2)$. [2]
   (b) Triangle $A$ is mapped on to triangle $B$ by the transformation represented by the matrix
   
   \[
   R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
   \]
   
   Draw and label triangle $B$ on your diagram. [3]
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(c) Another transformation is represented by the matrix

\[ \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \]

(i) Express HR as a single matrix.
(ii) Given that HR (\(d\)) = \(\hat{C}\), draw and label triangle \(C\) on your diagram.

(d) Describe, in full,
(i) the transformation represented by the matrix \(H\),
(ii) the transformation represented by the matrix \(II\).

6 Answer the whole of this question on a sheet of graph paper.

(a) Given that \(y = \frac{5}{x}\), copy and complete the following table, in which values of \(y\) are correct to two decimal places.

<table>
<thead>
<tr>
<th>(x)</th>
<th>0.5</th>
<th>0.7</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>10</td>
<td>7.14</td>
<td>5</td>
<td>3.33</td>
<td>2.5</td>
<td>2.0</td>
<td>1.67</td>
<td>0.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of \(y = \frac{5}{x}\) for \(0.5 \leq x \leq 8\).

(c) By drawing a tangent, find the gradient of the curve \(y = \frac{5}{x}\) at the point where \(x = 2\).

(d) On the same axes, draw the graph of \(2x + y = 9\).

(e) Write down the equation which is satisfied by the \(x\)-coordinates of the points of intersection of the two graphs.

Express your answer in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\) and \(c\) are integral constants.
Aristotle Jones wants to sail his yacht from $P$ to $Q$. In order to reach $T$ from $P$, he has to sail along $PS$ (vector $a$) and then along $ST$ (vector $b$). This is called a "tack".

(a) Write the vector $\overrightarrow{PT}$ in terms of $a$ and $b$. [1]

(b) $\begin{pmatrix} x \\ y \end{pmatrix}$ is a vector with components $x$ km East and $y$ km North.

If $a = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, find

(i) the components of $\overrightarrow{PT}$,

(ii) the length of $\overrightarrow{PT}$. [2]

(c) The distance from $P$ to $Q$ is 15 km.

(i) How many tacks does he need to take to reach $Q$? [2]

(ii) Write the vector $\overrightarrow{PQ}$ in terms of $a$ and $b$. [1]

(d) Find the bearing of $Q$ from $P$. [3]
International General Certificate of Secondary Education

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The number of marks available is shown in brackets ( ) at the end of each question or part question.

This Question Paper consists of 7 printed pages and 1 blank page.
It is given that

\[ y = \frac{3x^2}{4-x} \]

(a) Find the value of \( y \) when \( x = 3 \) and \( z = 2 \). [2]
(b) Find the value of \( z \) when \( x = 2 \) and \( y = 3 \). [2]
(c) Find both possible values of \( x \) when \( y = -8 \) and \( z = 10 \). [2]
(d) The value of \( x \) is doubled, and \( z \) remains unchanged. What effect does this have upon the value of \( y \)? [2]
(e) Make \( z \) the subject of the formula \( y = \frac{3x^2}{4-z} \). [3]

A glass paperweight consists of a cone mounted on a hemisphere. The common radius \( r \) is 4 centimetres; the height of the cone \( h \) is 5 centimetres. You are given that:

- The volume of a cone is \( \frac{1}{3} \pi r^2 h \);
- The volume of a sphere is \( \frac{4}{3} \pi r^3 \);
- The curved surface area of a cone is \( \pi r l \) (slant height \( l \));
- The surface area of a sphere is \( 4\pi r^2 \).

Take \( \pi \) to be 3.142.

(a) Calculate
(i) the volume of the paperweight, [4]
(ii) the surface area of the paperweight. [5]

(b) The mass of the paperweight is \( \frac{1}{2} \) kg. Calculate the density of the glass, in grams per cubic centimetre. [3]
3 Answer the whole of this question on a sheet of graph paper.

Using a scale of 1 centimetre to represent 1 unit on each axis, draw a pair of axes for $0 \leq x \leq 16$ and $0 \leq y \leq 10$.

(a) On your axes:
   (i) draw the line $y = x$; [1]
   (ii) mark the two points $A(10, 0)$ and $B(14, 3)$; [1]
   (iii) draw the locus of points which are equidistant from the points $A$ and $B$; [2]
   (iv) draw the locus of points which are equidistant from the line $y = x$ and the $x$-axis; [2]
   (v) draw the circle which touches the $x$-axis at $A$, and which passes through $B$. [3]

(b) Which other line, already drawn, does the circle touch? [1]

4 (a) In triangle $ABC$, $AB = 7$ cm, $BC = 5$ cm and $\angle ABC = 136^\circ$.

   \[ \text{NOT TO SCALE} \]

   (i) Calculate the length of $AC$. [3]
   (ii) Calculate the area of the triangle $ABC$. [3]

(b) The Piazza San Marco in Venice is a rectangle 135 metres long and 75 metres wide. The Campanile tower stands in one corner, and it is 97 metres high.

   \[ \text{NOT TO SCALE} \]

   Calculate the angle of elevation of the top of the tower
   (i) from $P$, [2]
   (ii) from $Q$. [4]
In the diagram, opposite sides of the hexagon are parallel and are in the ratio 1:2.

Given that $\overrightarrow{UV} = a$, $\overrightarrow{WX} = b$ and $\overrightarrow{YZ} = c$,

(a) (i) write down the vector representing $\overrightarrow{XV}$; [1]
(ii) hence show that $\overrightarrow{XY} = 2a - b$; [2]
(iii) use similar methods to write down $\overrightarrow{UY}$ and $\overrightarrow{UW}$; [2]

(b) Write down, in terms of $a$, $b$ and $c$,
$$\overrightarrow{XY} + \overrightarrow{UW} + \overrightarrow{UV},$$
expressing your answer in its simplest form. [2]

(c) Write down a vector equation which follows from the result of part (b). [2]

6

The rectangle has length $(3x + 4)$ cm and width $(x - 2)$ cm.

(a) Write down and simplify an expression for the perimeter of the rectangle. [2]

(b) Write down an expression for the area of the rectangle. [1]

(c) If the area of the rectangle is $57$ cm$^2$, show that
$$3x^2 - 2x - 65 = 0.$$ [3]

(d) Solve the quadratic equation $3x^2 - 2x - 65 = 0$. [3]

(e) Write down the length and width of the rectangle when its area is $57$ cm$^2$. [2]
7 (a) On each of two short holes on his golf course, Mr. A. Rabbit can take 3, 4, 5, 6, 7 or 8 strokes. All outcomes are equally likely.

Consider these two holes only.

(i) What is the probability that he takes a total of 6 strokes? [1]
(ii) What is the probability that he takes a total of 13 strokes? [2]
(iii) What is his most likely total? [1]

(b) If the weather is fine today, the probability that it will be fine tomorrow is 0.7. This and the other probabilities are shown in the following matrix.

<table>
<thead>
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<th></th>
<th>Fine</th>
<th>Wet</th>
</tr>
</thead>
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<td>TODAY</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>TODAY</td>
<td>(0.4</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Given also that the probability of the weather being fine on any one day is 0.6, copy and complete the tree diagram below, to represent all this information.

```
Day 1          Day 2
            0.6            0.7
          /       \         /       /
Fine      Wet    Fine     Wet
          \       /         \     /  
         Wet    Fine    Wet   Wet
```

Calculate the probability of

(i) two fine days, [1]
(ii) a wet day followed by a fine day, [1]
(iii) one fine day and one wet day, in either order. [2]
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question.

This Question Paper consists of 7 printed pages and 1 blank page.
1. A firm which manufactures golf balls is experimenting with the packaging of its product. 3 golf balls, each of radius 2.15 centimetres, are packaged in a rectangular box, a cross section of which is shown in the diagram below. The box is 12.9 centimetres long, 4.3 centimetres wide and 4.3 centimetres high.

\[ \text{NOT TO SCALE} \]

12.9 cm

4.3 cm

(a) Given that the volume of a sphere of radius \( r \) is \( \frac{4}{3}\pi r^3 \), calculate the amount of space within the box which is unfilled. [5]

The marketing department suggests that an equilateral triangular box of side 11.75 centimetres and height 4.3 centimetres might be more attractive. The diagrams show a plan and side view of the new box.

\[ \text{NOT TO SCALE} \]

11.75 cm

11.75 cm

4.3 cm

(b) Calculate the amount of space within this new box which is unfilled. [6]

\([\pi \approx 3.142] \)
In the diagram above, DEF is parallel to the diameter, AB, of the circle, centre O. Points C, D and E lie on the circumference of the circle.

Given that $\overline{AB}C = 40^\circ$ and $\overline{CD}E = 75^\circ$ calculate,

(a) $\overline{ABC}$,  
(b) $\overline{ABC}$,  
(c) $\overline{OBE}$,  
(d) $\overline{BEF}$,  
(e) $\overline{CDA}$.  

[2]  
[2]  
[2]  
[2]  
[2]
3 Graph paper must be used for the whole of this question.

<table>
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<th>t</th>
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<td>4</td>
<td>6.2</td>
<td>7.1</td>
<td>6.8</td>
<td>6.1</td>
<td>5.2</td>
<td>4.7</td>
</tr>
</tbody>
</table>

The table above shows the speed of a car, v metres per second, at time t seconds.

(o) Draw a graph of v against t, using a scale of 2 centimetres to represent 1 second horizontally and 2 centimetres to represent 1 metre per second vertically. [3]

(b) Use your graph to find
   (i) the maximum speed attained during this period of time, [1]
   (ii) the acceleration when t = 2, [3]
   (iii) the total distance travelled between t = 4 and t = 6, [3]
   (iv) the average speed between t = 4 and t = 6. [2]

(c) Find the range of values of t for which the gradient is negative. Explain what the negative gradient means in terms of the motion of the car. [2]

4 Einstein's formula

\[ E = mc^2 \]

states that E units of energy are produced when a decrease in mass of m kilograms occurs.

The velocity of light, c, is \(3 \times 10^8\) metres per second.

There are 1 million milligrams in a kilogram.

(o) Write 2 milligrams in kilograms, using standard form. [2]

(b) Use Einstein's formula to work out the number of units of energy produced by a decrease in mass of 2 milligrams. [2]

(c) An electric light bulb uses 100 units of energy each second. How many units does it use in 1 hour? [1]

(d) Find how many electric light bulbs could be lit for 1 hour by a decrease in mass of 2 milligrams. [2]
In the triangle $OAB$, $M$ is the mid-point of $AB$ and $N$ is the mid-point of $OA$.

(a) Given that $O\!M = a$ and $O\!N = b$,

express, in terms of $a$ and $b$, the vectors

(i) $\overrightarrow{AB}$,
(ii) $\overrightarrow{AM}$,
(iii) $\overrightarrow{OM}$,
(iv) $\overrightarrow{BN}$.

[6]

(b) $P$ lies on $OM$ such that $OP = \frac{2}{3} OM$. Express $\overrightarrow{BP}$ in terms of $a$ and $b$.

[3]

(c) Express $\overrightarrow{BP}$ in terms of $\overrightarrow{BN}$.

Explain the geometrical significance of this relationship.

[3]
The diagram above shows the square $OABC$, of area 1 square unit, and the parallelogram $OAB_1C_1$.

(a) Describe fully the single transformation which maps $OABC$ onto $OAB_1C_1$. [2]

(b) Given that $P = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ represents this transformation, find the value of $x$. [2]

(c) (i) Find the area of $OAB_1C_1$. [1]
(ii) State what effect the transformation represented by $P$ has on the area of any shape. [1]

(d) The matrix $Q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents a single transformation. Describe fully this transformation. [2]

(e) Using your value of $x$ from part (b), express $PQ$ as a single matrix. [2]

(f) (i) Find the inverse of $Q$. [1]
(ii) Describe in words the transformation represented by the inverse of $Q$. [2]

7 A triangle $ABC$ has sides of length

\[ AB = 2x - 3y + 14, \]
\[ BC = 5y - 4x, \]
\[ CA = 4x - 6. \]

(a) In any triangle, the sum of the lengths of any two sides is greater than the length of the third side. For example

\[ AB + BC > CA. \]

Use this inequality to show that

\[ 3x - y - 10 < 0. \] [2]

(b) Deduce a second inequality involving $x$ and $y$ from the statement

\[ CA + AB > BC, \]

simplifying your answer as far as possible. [2]

(c) Using these inequalities, and given that $y = \cdots$, find the range of possible values of $x$. [4]
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This Question Paper consists of 7 printed pages and 1 blank page.
(a) A firm manufactures personal stereo radios and has calculated that, by selling them at $40 each, a profit of 25% would be made on the cost of manufacture.

Calculate the cost of manufacture of a radio. [2]

(b) 2000 of these radios were manufactured, 23 of them were found to be imperfect because of small scratches on their cases. The perfect ones were still sold for $40, whilst the imperfect ones were sold for $12.

Calculate the total profit made by the firm assuming that all 2000 radios were sold. [5]

(c) Express the total profit as a percentage of the total cost of manufacture. [3]
In the triangle $XYZ$, angle $XY = 60^\circ$, $XY = 9.5$ cm, $XZ = 8$ cm and $YZ = x$ cm.

(a) (i) Use the cosine rule to show that $x$ satisfies the equation
$$4x^3 - 32x - 105 = 0.$$ [4]

(ii) By solving this quadratic equation, find the length of $YZ$. [3]

(b) Calculate angle $XYZ$. [3]

Two tangents are drawn from the point $T$ meeting the circle, centre $O$ and radius 8 cm, at the points $A$ and $B$. Angle $BTA = 20^\circ$.

(a) Calculate the length of $OT$. [3]

(b) The lines $OT$ and $AB$ meet at $E$. Calculate the length of $OE$. [3]

(c) Chord $AC$, with mid-point $F$, is the same length as $AB$.
   (i) State the length of $OF$. [1]
   (ii) Calculate angle $ACB$. [2]
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4 Answer the whole of this question on a sheet of graph paper.

(a) Given that \( y = x + \frac{6}{x} \), copy and complete the following table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{6}{x} )</td>
<td></td>
<td></td>
<td>2.4</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>4.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

[3]

(b) Using a scale of 2 centimetres to represent 1 unit on each axis draw the graph of \( y = x + \frac{6}{x} \) for \( 1 \leq x \leq 6 \).

[3]

(c) Use your graph to solve the equation

\[ x + \frac{6}{x} = 5.5. \]

[3]

(d) Show clearly how the equations of the curve \( y = x + \frac{6}{x} \) and the straight line \( y = 6 \) can be combined to give the equation

\[ x^2 - 6x + 6 = 0. \]

[3]

5

NOT TO SCALE

The diagram above shows a new logo, designed in red, white and green, for a car company.

The radius of the circle is 5 centimetres.

The three red sectors each have an angle of 30° at the centre of the circle.

The three white triangles each have a right angle at the centre of the circle.
6 A teacher has an unfortunate habit of oversleeping in the morning. The probability that he oversleeps is 0.3.

When he oversleeps there is a probability of 0.6 that he misses breakfast.
Even when he does not oversleep there is a probability of 0.2 that he misses breakfast.

(a) Copy and complete the tree diagram below to show this information, marking clearly on the dotted lines on each branch the probability of that outcome.

(b) What is the probability that he
    (i) oversleeps and misses breakfast,
    (ii) misses breakfast?

(c) What is the probability that he misses breakfast two days in succession?
Answer the whole of this question on a sheet of graph paper.

The vertices of the parallelogram $OPQR$ are $O(0, 0)$, $P(2, 0)$, $Q(3, 1)$ and $R(1, 1)$.

(a) (i) Using a scale of 2 cm to represent 1 unit on each axis, draw $x$ and $y$ axes for $-4 \leq x \leq 4$ and $-4 \leq y \leq 4$.

Draw and label the parallelogram $OPQR$.  

(ii) A rotation through $90^\circ$ anticlockwise about $O$ maps $OPQR$ onto $OP_1Q_1R_1$.

Draw $OP_1Q_1R_1$.  

(iii) A reflection in the $x$-axis maps $OP_1Q_1R_1$ onto $OP_2Q_2R_2$.

Draw $OP_2Q_2R_2$.  

(iv) Write down the single transformation which would map $OP_2Q_2R_2$ back onto the original parallelogram $OPQR$.

(b) A shear, with the $x$-axis invariant and with $(3, 1)$ mapped onto $(2, 1)$, maps the parallelogram $OPQR$ onto $OP_3Q_3R_3$.

(i) What special name is given to the shape $OP_3Q_3R_3$?  

(ii) Find the matrix which represents this transformation.
International General Certificate of Secondary Education

Instructions to candidates:

You should answer all the questions on the separate sheets of paper provided.

Show all your working on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

Write your name and examination number on each separate piece of writing paper or graph paper you use. If you use more than one sheet of paper for your answers, all answer sheets should be placed in correct order and fastened together.

Electronic calculators should be used.

Three figure accuracy is required in your answers except where stated otherwise.

The total of the marks for this paper is 130.

The intended marks for questions or parts of questions are given in brackets [ ].
A gardener has 357 tulip bulbs to plant.

(a) If she planted a rectangle of 15 rows, with 23 bulbs in each row, how many bulbs would be left over? [2]

(b) How many bulbs would there be in the largest square that she could plant? [2]

(c) (i) If she plants $x$ rows, with $y$ bulbs in each row, write down a formula for the number of bulbs left over. [2]

(ii) If $10 < x < 20$ and $y > 20$, find the value of $x$ and the value of $y$ such that no bulbs are left over. [2]

---

A(-3, 4), B(5, -2) and C(2, -6) are three vertices of a parallelogram $ABCD$.

(a) Write down the vector $\vec{BA}$ in the form \( \begin{pmatrix} p \\ q \end{pmatrix} \) [2]

(b) Find the coordinates of the vertex $D$. [2]

(c) Calculate the lengths of the line segments $AB$, $BC$ and $AC$. [3]

(d) Use your answers in part (c) to show that the parallelogram $ABCD$ is a rectangle. [2]

(e) Calculate the area of $ABCD$. [2]

(f) The equation of the line through $A$ and $B$ is

\[ y = \frac{3}{4}x + \frac{7}{4}. \]

(i) What is the gradient of this line? [2]

(ii) Write down the coordinates of the point at which this line cuts the $y$-axis. [2]
The diagram shows the speed-time graph of a boy cycling from home to school.

(a) How long does the journey take him? [1]
(b) After how many seconds does he first reach the speed of 6 metres per second? [1]
(c) What is his maximum speed? [1]
(d) Describe his journey, in as much detail as possible. [4]
(e) Calculate, in metres, the distance between his home and the school. [4]

In the diagram, P, Q, R and S are points on the circle, centre O.

Angle \(PQ'R\) = 135° and \(PQ = QR = 4\) cm.

Calculate

(a) angle \(PSR\), [2]
(b) the marked angle \(POR\), [2]
(c) the length of the chord \(PR\), [3]
(d) the radius of the circle. [4]
5 Answer the whole of this question on a sheet of graph paper.

The graph of

\[ f(x) = x - \frac{4}{x} \]

consists of two separate branches. One of these branches is shown in the sketch graph.

(c) Copy and complete this table of values for \( f(x) = x - \frac{4}{x} \).

<table>
<thead>
<tr>
<th>x</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>-0.8</th>
<th>0.8</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>-4.2</td>
<td>-3</td>
<td>-1.7</td>
<td>3</td>
<td>4.2</td>
<td>-4.2</td>
<td>-3</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>4.2</td>
<td></td>
</tr>
</tbody>
</table>

[2]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of the function.

[4]

(c) Describe the symmetry of the graph.

[2]

(d) Use your graph to find both solutions of the equation \( x - \frac{4}{x} = -1.6 \).

[2]

(e) Draw the tangent to the curve at the point where \( x = 3 \). Hence estimate the gradient of the curve at that point.

[3]

6 (a) Solve the equation

\[ 2x^2 - 3x - 6 = 0 \]

giving your answers correct to two decimal places.

[5]

(b) (i) A French loaf is 72 cm long. It is cut into slices, each \( x \) cm long, so that none of it is left over.

Write down an expression for the number of slices obtained.

[1]

(ii) A second loaf is also 72 cm long. It is also cut up completely into equal slices. Each slice is 1 cm longer than each slice of the first loaf.

Write down an expression for the number of slices obtained from the second loaf.

[1]

(iii) If there are six more slices from the first loaf than from the second, form an equation in terms of \( x \). Show that it reduces to

\[ x^2 + x - 12 = 0 \].

[3]

(iv) Solve this equation, and hence state the length of the slices from each loaf.

[3]
7 (a) (i) A scale of $1:350\,000$ is the same as $1\text{ cm} : N\text{ km}$. Find the value of $N$. [2]
(ii) On a map, drawn to a scale of $1:350\,000$, the approximate area of Ascension Island is $7\text{ cm}^2$.

Use your answer to part (i) to find the approximate area of Ascension Island, in square kilometres. [3]

(b) [Diagram]

(i) A classroom globe is a sphere of radius $0.6\text{ m}$. Calculate the volume of the globe in cubic metres. [2]

[Volume of a sphere of radius $r$ is $\frac{4}{3}\pi r^3$.] $\pi$ is approximately $3.142$.

(ii) The earth is a sphere of radius $6000\text{ km}$ approximately. Express this radius in metres. [1]

(iii) Find the ratio

Radius of earth : Radius of classroom globe,

giving your answer in the form $10^n : 1$. [2]

(iv) By what number (again as a power of 10) should you multiply your answer to (b) (i) in order to obtain an approximate value, in cubic metres, of the volume of the earth? [2]
8 Answer the whole of this question on a sheet of graph paper.

(a) The graph below shows the cumulative frequency of live births (in thousands) in Scotland in 1936, plotted against mother's age.

Using the graph,

(i) find the total number of live births in Scotland in 1936, [1]

(ii) find the number of live births in which the mother's age was less than 30 years, [1]

(iii) estimate the median age. [1]
(b) The table below shows the number of live births in Scotland in 1986.

<table>
<thead>
<tr>
<th>Mother's age (in years)</th>
<th>Frequency (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 ≤ x &lt; 20</td>
<td>5</td>
</tr>
<tr>
<td>20 ≤ x &lt; 25</td>
<td>18</td>
</tr>
<tr>
<td>25 ≤ x &lt; 30</td>
<td>22</td>
</tr>
<tr>
<td>30 ≤ x &lt; 35</td>
<td>15</td>
</tr>
<tr>
<td>35 ≤ x &lt; 40</td>
<td>3.5</td>
</tr>
<tr>
<td>40 ≤ x &lt; 45</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(i) Copy and complete the cumulative frequency table below for this data.

<table>
<thead>
<tr>
<th>Mother's age (in years)</th>
<th>Cumulative frequency (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than 20</td>
<td></td>
</tr>
<tr>
<td>less than 25</td>
<td></td>
</tr>
<tr>
<td>less than 30</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Draw the cumulative frequency diagram for this data.
   Use the same scales as for the graph in part (a).
   (iii) Use your graph to estimate the median age.
   (iv) Use your graph to estimate the interquartile range.

(c) Write a comment comparing the ages at which Scottish women became mothers in 1936 and 1986.

9 In the pentagon ABCDE, interior angles A, B, C, and D are all equal to 120°.
   (a) Calculate angle E.
   (b) Draw a sketch of the pentagon, and explain why AB is parallel to ED.
   (c) Which other two sides are parallel?
   (d) In addition, AB = BC = CD = 3 cm. Draw the pentagon accurately.
   (e) Draw and label the line of symmetry in your diagram.
   (f) By taking suitable measurements from your diagram, or otherwise, calculate the area of the pentagon. You must show your method clearly.
(a) Describe fully the single transformation that will map triangle $A$ onto triangle $B$. [2]

(b) Find the matrix $X$ of the transformation that you have described in part (a). [2]

(c) Describe fully the transformation represented by the matrix $Y$, where $Y = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. [2]

(d) Find $YX$. [2]

(e) What single transformation does $YX$ represent? [2]

11 (a) Work out

(i) $26 \times 93$ and $62 \times 39$, [2]

(ii) $36 \times 42$ and $63 \times 24$.

(b) Find another pair of multiplications with the same property.

For the remainder of the question $pq, rs, qp$ and $sr$ each represent a 2 digit number.

(c) In parts (a) and (b),

\[ pq \text{ times } rs = qp \text{ times } sr. \]

State a relationship between $p, q, r$ and $s$. [3]

(d) 26 can be written ($10 \times 2 + 6$).

\[ pq \text{ can be written } (10p + q). \]

Therefore, the multiplication of $pq$ and $rs$ can be written $(10p + q)(10r + s)$.

Use this idea to prove your statement in part (c). [4]
International General Certificate of Secondary Education

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Electronic calculators should be used.

Three figure accuracy is required in your answers except where stated otherwise.

The total of the marks for this paper is 130.

The intended marks for questions or parts of questions are given in brackets [ ].
Describe fully the transformations of the shaded triangle on to triangles \( A, B, C \) and \( D \). [9]

2 A ball is thrown vertically upwards.
After \( t \) seconds the height, \( h \) metres, of the ball is given by the formula
\[
h = 14t - 5t^2.
\]

(a) Find the height of the ball after 1 second. [1]

(b) Find the height of the ball after 2\( \frac{1}{2} \) seconds. [2]

(c) The answer to (b) is less than the answer to (a).

Explain why this is so. [2]

(d) When \( h = 8 \), find the two possible values of \( t \). [4]

(e) Calculate the two times when the ball is at a height of 5 metres. Give your answers correct to two decimal places. [5]
The diagram shows the rectangular cross-section of a canal. It is $7\frac{1}{2}$ metres wide and the water is 2 metres deep.

(a) Calculate the area of the cross-section of the water in the canal, in square metres. [1]

(b) The water flows at the rate of $\frac{3}{4}$ km/h.

   Calculate the volume of water passing a fixed point on the canal bank in one minute. Give your answer in cubic metres. [5]

(c) The mass of one cubic centimetre of water is 1 gram.

   Find the mass, in kilograms, of one cubic metre of water. [3]

(d) Calculate the mass of water passing a fixed point on the bank every minute. [2]

---

4

(a) \[ X = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 5 \\ -4 & -2 \end{pmatrix} \]

Find

(i) $2X$,

(ii) $X + Y$,

(iii) the determinant of $X$,

(iv) $X^{-1}$. [5]

(b) \[
\begin{bmatrix}
  a & 0 & 0 \\
 -c & 3 & -1 \\
  b & 2 & 4
\end{bmatrix} = \begin{bmatrix} 9 & 11 & 15 \end{bmatrix}
\]

(i) Multiply out the matrices on the left hand side and hence write down three equations. [3]

(ii) $a$, $b$ and $c$ all represent positive integers.

   By solving your equations, or otherwise, find the value of $a$, of $b$ and of $c$. [4]
The shape $ABCDEF$ consists of a trapezium $ACDF$ and a minor segment $ABC$ of a circle, centre $O$. The lines $FA$ and $DC$ are tangents to the circle at $A$ and $C$ respectively.

The radius of the circle is $2\, m$.

$AC = 3.6\, m$ and $AF = CD = 5\, m$.

$\pi$ is approximately $3.142$.

(a) Show that angle $AOC$ is $128.3^\circ$, correct to one decimal place. [2]

(b) Calculate the area of the sector $OABC$. [2]

(c) Calculate the area of the triangle $OAC$ and hence the area of the minor segment $ABC$. [1]

(d) Show that the perpendicular distance between $AC$ and $FD$ is $4.5\, m$. [2]

(e) Find the area of the trapezium $ACDF$ and hence the area of the whole shape $ABCDEF$. [4]

---

6 A farmer makes a sheep pen, in the shape of a quadrilateral, out of four pieces of fencing. Each side of the quadrilateral is $3$ metres long and one of the angles is $60^\circ$.

(a) Using a scale of 1 to 30, make an accurate drawing of the quadrilateral. [3]

(b) Mark in its axes of symmetry with broken lines (- - - - -) and describe how they cut each other. [3]

(c) What is the special geometrical name of this shape? [1]

(d) Calculate the area enclosed by the sheep pen, giving your answer in square metres. [3]

(e) By changing the angles (but leaving the lengths of the sides unchanged), the area enclosed by the sheep pen can be varied. What is the greatest possible area that can be enclosed? [2]
7. The whole of this question should be answered on a sheet of graph paper.

The table gives some values of the function

\[ f(x) = x^3 - 11x + 12. \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3.5</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-16.9</td>
<td>0</td>
<td>18</td>
<td>20</td>
<td>12</td>
<td>0</td>
<td>-10</td>
<td>-12</td>
<td>0</td>
<td>13.1</td>
</tr>
</tbody>
</table>

(a) Using a scale of 2 cm to represent 1 unit on the \( x \)-axis and 2 cm to represent 5 units on the \( f(x) \)-axis, plot the given points and hence draw the graph of the function for values of \( x \) in the range \(-3.5 \leq x \leq 4.5\) [4]

(b) (i) Calculate \( f(5) \).

(ii) Hence describe how the graph continues when \( x > 4.5 \) [2]

(c) By drawing a suitable tangent, estimate the gradient of the curve at the point where \( x = 4 \) [3]

(d) State the coordinates of the two points on the curve at which the gradient is zero. [3]

---

8. In order to sail round a small island, a fisherman steers his boat for 2 km on a bearing 075° from \( A \) to \( B \) and then for 3 km on a bearing 130° from \( B \) to \( C \).

(a) Show that angle \( ABC \) is 125° [2]

(b) Calculate the direct distance from \( A \) to \( C \) [4]

(c) Calculate the bearing of \( C \) from \( A \) [3]
9 The whole of this question should be answered on a sheet of graph paper.

Rapid Delivery Services have to deliver 1350 parcels to the next town. Their lorries can take 150 parcels at a time and their vans can take 90 parcels at a time.

(a) If x lorries and y vans are used, show that

\[ 5x + 3y \geq 45. \]  

[2]

(b) Only twelve drivers are available, so twelve vehicles at most can be used. Write down another inequality which must be satisfied by x and y.  

[2]

(c) At least four vans must be used. Write this as a third inequality.  

[2]

(d) Using a scale of 2 cm to represent 2 vehicles on each axis, draw x and y axes and number each of them from 0 to 16. Represent the three inequalities on your graph. Indicate clearly, by shading the unwanted regions, the region within which \((x, y)\) must lie.  

[6]

(e) The cost of one lorry journey is \$30 and the cost of one van journey is \$20. Use your graph to find how many lorries and how many vans will be needed to deliver the parcels at the least cost. What is that least cost?  

[3]

10 The whole of this question should be answered on a sheet of graph paper.

A survey of the age distribution of the population of Great Britain was made in 1989. The data obtained was expressed in two ways, as shown in the following tables.

<table>
<thead>
<tr>
<th>Age, x, in years</th>
<th>0 ( \leq ) x &lt; 10</th>
<th>10 ( \leq ) x &lt; 20</th>
<th>20 ( \leq ) x &lt; 30</th>
<th>30 ( \leq ) x &lt; 40</th>
<th>40 ( \leq ) x &lt; 50</th>
<th>50 ( \leq ) x &lt; 60</th>
<th>60 ( \leq ) x &lt; 70</th>
<th>70 ( \leq ) x &lt; 80</th>
<th>80 ( \leq ) x &lt; 90</th>
<th>90 ( \leq ) x &lt; 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (millions)</td>
<td>6.5</td>
<td>6.5</td>
<td>8</td>
<td>7</td>
<td>6.5</td>
<td>5.5</td>
<td>5</td>
<td>3.5</td>
<td>1.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Age, x, in years</th>
<th>x &lt; 10</th>
<th>x &lt; 20</th>
<th>x &lt; 30</th>
<th>x &lt; 40</th>
<th>x &lt; 50</th>
<th>x &lt; 60</th>
<th>x &lt; 70</th>
<th>x &lt; 80</th>
<th>x &lt; 90</th>
<th>x &lt; 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative frequency (millions)</td>
<td>6.5</td>
<td>13</td>
<td>21</td>
<td>A</td>
<td>34.5</td>
<td>B</td>
<td>45</td>
<td>48.5</td>
<td>C</td>
<td>50.2</td>
</tr>
</tbody>
</table>

(a) Find the value of A, of B and of C.  

[2]

(b) Using a scale of 1 cm to represent 10 years horizontally and 2 cm to represent 5 millions vertically, draw a cumulative frequency diagram to represent this distribution.  

[5]

(c) Use your diagram to estimate

(i) the median,

(ii) the lower quartile,

(iii) the inter-quartile range.  

[3]

(d) Showing your working clearly, calculate the mean age of the population as accurately as you can from the data given.  

[6]
Sarah Jane has a set of holiday photographs (fewer than a hundred) which she is going to put into a photograph album.

If she puts 2 photographs on each page, she will have 1 photograph left over.
If she puts 3 photographs on each page, she will have 2 photographs left over.
If she puts 4 photographs on each page, she will have 3 photographs left over.
If she puts 5 photographs on each page, she will have 4 photographs left over.
How many photographs does she have altogether?
You must show how you arrive at your answer.
Instructions to candidates:

Answer all the questions on the separate sheets of paper provided.

Show all your working on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

Write your name and examination number on each separate place of writing paper or graph paper you use. If you use more than one sheet of paper for your answers, all answer sheets should be placed in correct order and fastened together.

Electronic calculators should be used.

Three figure accuracy is required in your answers except where stated otherwise.

The total of the marks for this paper is 130.

The number of marks available is shown in brackets [ ] after each question or part question.

This Question Paper consists of 7 printed pages and 1 blank page.
Hazel Green sells 2½ litre tins of paint in her shop for $14.30 each.

(a) (i) Express this price in dollars per litre.  
(ii) At this price, how many millilitres of paint do you get for $1?  

(b) Hazel makes a profit of 30% on the cost price of the tin of paint.  
Calculate the cost price.  

(c) In a sale, she reduces the price of the tin of paint to $11.44.  
Calculate  
(i) the percentage reduction in the selling price of the tin of paint,  
(ii) the percentage profit she now makes on the cost price.  

(d) A 5 litre tin of paint is priced in the sale at $21.12.  
Calculate the price of this tin in pounds sterling, given that £1 = $1.65.

---

2 The formula

\[ A = 180 - \frac{360}{n} \]

gives the size of each interior angle, \( A \), of a regular polygon with \( n \) sides.

(a) Find the value of \( A \) when \( n \) equals  
(i) 180,  
(ii) 360,  
(iii) 720,  
(iv) 7200.  

(b) As \( n \) becomes very large,  
(i) what value does \( A \) approach,  
(ii) what shape does the polygon approach?  

(c) Find the value of \( n \) when \( A = 162 \).  

(d) Make \( n \) the subject of the formula.

(e) Three regular polygons, two of which are octagons, meet at a point so that they fit together without any gaps.  
Showing all your working, identify the third polygon.
A glass is in the shape of an inverted cone of radius 6 cm and height 18 cm.

(a) Calculate the capacity of the glass.

[The volume of a cone of radius \( r \) and height \( h \) is \( \frac{1}{3} \pi r^2 h \), and \( \pi \) is approximately 3.142]

(b) Milk is poured into the glass to a height of 9 cm. Calculate the volume of milk in the glass.

(c) If the height of milk in the glass is \( x \) cm.

(i) find the radius of its surface, in terms of \( x \),

(ii) find a formula for the volume of milk, in terms of \( x \) and \( \pi \),

(iii) show that, when \( x = 9 \), your formula gives the same answer as in part (b).

4 The whole of this question should be answered on a sheet of graph paper.

A farmer keeps \( x \) goats and \( y \) cows. Each goat costs $2 a day to feed and each cow costs $4 a day to feed. The farmer can only afford to spend $32 a day on animal food.

(a) Show that \( x + 2y \leq 16 \).

(b) The farmer has room for no more than 12 animals.

He wants to keep at least 6 goats and at least 3 cows.

Write down three more inequalities.

(c) Using a scale of 1 cm to represent 1 unit on each axis, represent the four inequalities on a graph.

(d) One possible combination which satisfies all the inequalities is 6 goats and 4 cows. Write down all the other possible combinations.

(e) If he makes a profit of $50 on each goat and $80 on each cow, which combination will give him the greatest profit? Calculate the profit in this case.

5 (a) Find the solution set of the inequality

\[ 3 - 2x < 11. \]

(b) (i) Solve the equation

\[ x^2 + x - 30 = 0. \]

(ii) Solve the equation

\[ x^2 + x - 15 = 0, \]

giving your answers correct to two decimal places.

(c) Simplify, as far as possible,

\[ \frac{a + b}{a^2 + ab - 2b^2} - \frac{2}{3a - 3b}. \]
6. When the arrow on spinner $X$ is spun it is equally likely to stop on any of the numbers 1, 2, 3, 4, 5 or 6.

Similarly the arrow on spinner $Y$ is equally likely to stop on 1, 2, 3 or 4.

(a) Copy and complete the possibility diagram below, showing the possible totals when both arrows are spun.

```
(a) Copy and complete the possibility diagram below, showing the possible totals when both arrows are spun.

<table>
<thead>
<tr>
<th>SPINNER $X'$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
```

(b) What is the probability of a total of

(i) 2,
(ii) 7,
(iii) 12?

(c) Which totals are the most probable?

(d) The sectors on the two spinners are dotted, white or striped.

What is the probability of

(i) both arrows pointing to a dotted sector,

(ii) one arrow pointing to a dotted sector, and one arrow pointing to a white sector?
7 The whole of this question should be answered on a sheet of graph paper.

(a) Copy and complete the given table of values for the function \( y = 3^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td>5.2</td>
<td>9</td>
<td>13.6</td>
<td>27</td>
<td>46.8</td>
<td></td>
</tr>
</tbody>
</table>

(Where appropriate, \( y \) values are given correct to one place of decimals.)

(b) Using a scale of 2 cm to represent 1 unit on the \( x \)-axis, and 2 cm to represent 10 units on the \( y \)-axis, draw the graph of \( y = 3^x \) for values of \( x \) from \(-1\) to 4 inclusive.

(c) What happens to \( y \) when \( x \) is both negative and very large?

(d) Use your graph to find the value of

(i) \( y \) when \( x = 1.2 \),

(ii) \( x \) when \( y = 20 \).

(e) By drawing a suitable tangent, estimate the gradient of the graph when \( x = 2 \).

The diagram represents an artificial ski slope.

The surface of the slope, \( PQRS \), is a rectangle.

\( T \) is a point vertically below \( R \), and \( U \) is vertically below \( Q \), so that \( PSTRU \) is a horizontal rectangle.

\( M \) is the midpoint of \( RS \).

\( PQ = 80 \text{ m}, \ QR = 100 \text{ m} \) and \( RT = 45 \text{ m} \).

(a) Calculate angle \( RST \).

(b) Carol skis down the slope along the line \( RP \).

Calculate (i) the length of \( RP \),

(ii) the angle that her path makes with the horizontal.

(c) She returns to the top of the slope by walking from \( P \) to \( M \) and then from \( M \) to \( Q \).

Find (i) the distance that she has to walk,

(ii) the angle of depression of \( M \) from \( Q \).
The whole of this question should be answered on a sheet of graph paper.

In a standards testing survey, the mass of cornflakes in 200 packets was found, and the following results were obtained.

<table>
<thead>
<tr>
<th>Mass ((m)) of cornflakes in grams</th>
<th>Number of packets (Frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(746 \leq m &lt; 748)</td>
<td>5</td>
</tr>
<tr>
<td>(748 \leq m &lt; 750)</td>
<td>11</td>
</tr>
<tr>
<td>(750 \leq m &lt; 752)</td>
<td>22</td>
</tr>
<tr>
<td>(752 \leq m &lt; 754)</td>
<td>52</td>
</tr>
<tr>
<td>(754 \leq m &lt; 756)</td>
<td>68</td>
</tr>
<tr>
<td>(756 \leq m &lt; 758)</td>
<td>36</td>
</tr>
<tr>
<td>(758 \leq m &lt; 760)</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) State the modal class. [1]

(b) Copy and complete the following cumulative frequency table.

<table>
<thead>
<tr>
<th>Mass ((m)) in grams</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m &lt; 748)</td>
<td>5</td>
</tr>
<tr>
<td>(m &lt; 750)</td>
<td>16</td>
</tr>
<tr>
<td>(m &lt; 752)</td>
<td>38</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>(m &lt; 760)</td>
<td>200</td>
</tr>
</tbody>
</table>

(c) Using a scale of 2 cm to represent 2 grams horizontally, and 2 cm to represent 20 packets vertically, draw a cumulative frequency diagram. [4]

(d) Use your diagram to estimate

(i) the median mass,

(ii) the interquartile range. [3]

(e) To avoid complaints, all packets whose contents had a mass of less than 751 grams were rejected.

What percentage of the packets was rejected? [2]
(a) Using ruler and compasses only, construct the shape in the diagram. The three shorter sides are each 3 cm long, and the six longer sides are each 5 cm long.

(b) What special name is given to
   (i) $\triangle XYZ$.
   (ii) $\triangle AYZ$?

(c) From your drawing, measure and write down the size of
   (i) $\angle ZXY$.
   (ii) $\angle BAC$.

(d) Check your answer to (c)(i) by using trigonometry.

(e) Describe the symmetry of the shape as fully as possible.

(f) Name the three-dimensional solid which would be formed by folding the shape along $XY$, $YZ$, and $ZX$, so that $A$, $B$ and $C$ coincide.

(g) How many (i) vertices,
    (ii) edges would the solid have?

---

11 (a) Prove, for all non-zero values of $p$ and $q$, that

$\left(1 + \frac{p}{q}\right)\left(1 + \frac{2}{p}\right) = \left(1 + \frac{p}{q}\right) + \left(1 + \frac{2}{p}\right)$.

(b) (i) By substituting $p = 3$ and $q = 2$ show that the above identity reduces to

$2\frac{1}{2} \times 1\frac{1}{2} = 2\frac{1}{2} + 1\frac{1}{2}$.

(ii) By evaluating $2\frac{1}{2} \times 1\frac{1}{2}$ and $2\frac{1}{2} + 1\frac{1}{2}$ prove that the statement in (b)(i) is true.

(c) By choosing other integer values of $p$ and $q$, where $p$ and $q$ are different, write down four other statements similar to that in part (b)(i). Include at least one statement in which the value of $p$ is negative.
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

Instructions to candidates:

You should answer all the questions on the separate sheets of paper provided.

Show all your working on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

Write your name and examination number on each separate piece of writing paper or graph paper you use. If you use more than one sheet of paper for your answers, all answer sheets should be placed in correct order and fastened together.

Electronic calculators should be used.

Three figure accuracy is required in your answers except where stated otherwise.

The total of the marks for this paper is 130.

The number of marks available is shown in brackets [ ] at the end of each question or part question.

This Question Paper consists of 7 printed pages and 1 blank page.
1 (a) The front page of a newspaper is rectangular and measures 60 cm by 40 cm, correct to the nearest centimetre.

Between what limits does the area of the front page lie?

Write your answer in the form \( \ldots \leq \text{area} \leq \ldots \). [3]

(b) The diagram is an accurate plan of a garden.

(i) Measure the sides of the diagram and the diagonal \( BD \).

Using the given scale, express the five lengths in metres. [2]

(ii) Correct each of the five lengths to the nearest 5 metres. [2]

(iii) Draw another plan of the garden, using the approximated measurements and the same scale. [2]

(iv) Write down the single word which completes the following statement.

"After approximating the measurements to the nearest 5 metres, the shape of the garden becomes a _________." [1]

2 The volume of a regular hexagonal prism is given by the formula

\[ V = 2.6 s^3, \]

where \( s \) is the length of each edge of the prism.

(a) Find \( V \) if \( s = 3.3 \) cm. [2]

(b) Make \( s \) the subject of the formula. [2]

(c) (i) Use trigonometry to obtain an expression for the area of the (shaded) hexagon, in terms of \( s \). [4]

(ii) Hence show that the original formula is approximately correct. [1]
In 1772, a German astronomer named Bode gave this table for the distance of each of the planets from the Sun.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Bode’s number</th>
<th>Actual distance (in Bode units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>4</td>
<td>3.9</td>
</tr>
<tr>
<td>Venus</td>
<td>7 (3 + 4)</td>
<td>7.2</td>
</tr>
<tr>
<td>Earth</td>
<td>10 (6 + 4)</td>
<td>10.0</td>
</tr>
<tr>
<td>Mars</td>
<td>16 (12 + 4)</td>
<td>15.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jupiter</td>
<td>52 (15 + 4)</td>
<td>52.0</td>
</tr>
<tr>
<td>Saturn</td>
<td>100 (96 + 4)</td>
<td>95.4</td>
</tr>
</tbody>
</table>

(a) What is the missing Bode’s number? (This led to the discovery of the Asteroid Belt.) [3]

(b) Bode’s numbers were used to forecast the position of Uranus, the next planet further out than Saturn, and it was discovered in 1781.

(i) What was Bode’s number for Uranus? [1]

(ii) The actual distance of Uranus from the Sun was 193 units.

Express the error in Bode’s number as a percentage of the actual distance. [2]

(c) The distance of the Earth from the Sun, given as 10.0 in the table above, is $1.49 \times 10^8$ kilometres.

Calculate the distance from the Sun of

(i) Mercury;

(ii) Jupiter,

giving your answers in kilometres, in standard form. [4]
5 In the quadrilateral $PQRS$, $\vec{PQ} = a$, $\vec{QR} = b$ and $\vec{SR} = 2a$.

(a) Write down two things that this tells you about the line segments $PQ$ and $SR$. [2]

(b) Express $\vec{PR}$ and $\vec{SP}$ in terms of $a$ and $b$. [3]

(c) If $a = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ and $b = \begin{pmatrix} -3 \\ -2 \\ 7 \end{pmatrix}$.

(i) Make an accurate drawing of the quadrilateral $PQRS$, using a scale of 1 cm to represent 1 unit. [3]

(ii) Calculate the area of triangle $PQR$. [3]

6 A game warden is standing 100 metres due East of a look-out tower.

(a) The angle of elevation of the top of the tower, from where he stands, is $9^\circ$.

Calculate the height of the tower. [2]

(b) A tourist, at the top of the tower, sights a rhinoceros at a distance of 150 metres from the foot of the tower, and on a bearing of $220^\circ$.

(i) Draw a sketch showing the positions of the foot of the tower, the game warden and the rhinoceros. [2]

(ii) Calculate the distance between the game warden and the rhinoceros. [3]

(c) Calculate the angle of depression of the rhinoceros from the top of the tower. [2]
7. Six hundred students were asked if they smoked. The results were recorded and probabilities were calculated. Some of the probabilities are shown on the tree diagram below.

\[
\begin{array}{c}
\text{Male} \\
\frac{7}{10} \\
\frac{3}{10} \\
\text{Female}
\end{array}
\begin{array}{c}
\text{Smoke} \\
\frac{3}{5} \\
\frac{2}{5} \\
\text{Do not smoke}
\end{array}
\]

(a) Copy and complete the tree diagram. 

(b) How many of the students who were asked were male?

(c) One of the students is selected at random. What is the probability that the person selected is
   (i) a male who smokes.
   (ii) a non-smoker?

---

8. Answer the whole of this question on a sheet of graph paper.

<table>
<thead>
<tr>
<th>(x)</th>
<th>0.6</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td></td>
<td>-5.9</td>
<td>-3.7</td>
<td>-2.3</td>
<td>-1.1</td>
<td>0.3</td>
<td>1.9</td>
<td>3.8</td>
<td>(q)</td>
<td>(r)</td>
</tr>
</tbody>
</table>

Some of the values for the function

\[ y = \frac{x^3 - 6}{12 - x} \]

are shown in the table above. Values of \(y\) are given correct to one decimal place.

(a) Find the values of \(p\), \(q\) and \(r\).

(b) Using a scale of 2 cm to represent 1 unit on the \(x\)-axis, and 1 cm to represent 1 unit on the \(y\)-axis, draw the graph of \(y = \frac{x^3 - 6}{12 - x}\) for \(0.6 \leq x \leq 5\).

(c) Find, from your graph, correct to 1 decimal place, the value of \(x\) for which \(\frac{x^3 - 6}{12 - x} = 0\).

(d) Draw the tangent to the curve at the point where \(x = 1\), and hence estimate the gradient of the curve at that point.
9 (a) Using ruler and compasses only construct,
(i) triangle $PQR$ with $PQ = 10$ cm, $QR = 9$ cm and $RP = 7$ cm.,
(ii) the perpendicular bisectors of $PQ$ and $QR$,
(iii) the circle, with its centre at the point where the perpendicular bisectors meet, passing through $P$, $Q$ and $R$.
(This is the circumscribed circle of triangle $PQR$.)

(b) A formula for the area ($A$) of triangle $ABC$ is

$$A = \frac{1}{2}ch \sin A.$$  

It is also given that

$$\frac{h}{\sin A} = 2R,$$

where $R$ is the radius of the circumscribed circle.

(i) Combine these two formulae to show that

$$A = \frac{abc}{4R}.$$  

(ii) Measure the radius of the circumscribed circle you have drawn in part (a), and hence calculate the area of triangle $PQR$.

10 The table shows the distribution of emergency admissions to a hospital per day over a period of two months.

<table>
<thead>
<tr>
<th>Number of Admissions</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>10–19</td>
<td>25</td>
</tr>
<tr>
<td>20–29</td>
<td>15</td>
</tr>
<tr>
<td>30–39</td>
<td>8</td>
</tr>
<tr>
<td>40–49</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) State the modal class of the distribution.

(b) (i) Write down the mid-interval value in the class interval 10–19.

(ii) Calculate an estimate of the mean number of admissions per day.

(c) Calculate an estimate of the median number of admissions per day.

(d) Construct a histogram to represent this information.
11 (a) Factorise, if possible.
   (i) $9a^2 - 4b^2$.
   (ii) $9a^2 + 4b^2$.
   (iii) $2x^2 + 7x - 4$.

(b) Factorise completely
   
   

(c) Solve the quadratic equation
   
   \[(x - 3)^2 = 5,\]
   giving your answers correct to two decimal places.

(d) The sum $S$ of the first $n$ positive integers is given by the formula
   
   \[S = \frac{1}{2}n(n + 1)\].

(i) Use the formula to find the value of
   
   

(ii) Find $n$ when $S = 465$.

12 Answer the whole of this question on a sheet of graph paper.
The vertices of a rectangle $OPQR$ are $O(0, 0), P(2, 0), Q(2, 5)$ and $R(0, 5)$.

(a) Taking 1 cm to represent 1 unit on each axis and marking each axis from $-6$ to $+6$, draw and label the rectangle $OPQR$.

(b) The rectangle $OPQR$ is mapped onto rectangle $OP_1Q_1R_1$ by the transformation represented by the matrix $L$, where

   \[L = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.\]

   Draw and label the rectangle $OP_1Q_1R_1$ on your diagram, and describe the transformation fully in geometrical terms.

(c) Reflect the original rectangle $OPQR$ in the $y$-axis.
   Label the new rectangle $OP_2Q_2R_2$.
   Write down the matrix $M$ which represents this transformation.

(d) The rectangle $OP_1Q_1R_1$ can be mapped onto the rectangle $OP_2Q_2R_2$ by a single transformation represented by the matrix $N$.

   (i) Describe this transformation fully in geometrical terms.

   (ii) Write down the matrix $N$, which represents this transformation.

   (iii) State a relationship between the matrices $L, M$ and $N$. 

1617
0580/4 IGCSE JUNE
0581/4 MATHEMATICS
PAPER 4

Wednesday 10 JUNE 1992 Morning 2 h 30 min

Additional materials provided by the Syndicate:
1. Mathematical tables
2. 3 sheets of graph paper
3. Electronic calculator
4. Geometrical instruments
5. Answer paper

UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
INTERNATIONAL EXAMINATIONS

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Write your name and examination number on each separate piece of writing paper or graph paper you use. If you use more than one sheet of paper for your answers, all answer sheets should be placed in correct order and fastened together.
Electronic calculators should be used.
Three figure accuracy is required in your answers except where stated otherwise.
The total of the marks for this paper is 130.
The number of marks available is shown in brackets [ ] after each question or part question.

This Question Paper consists of 7 printed pages and 1 blank page.

2055 © U.C.L.E.S. 1992 [Turn over
1 \( (a) \)

The diagram represents a regular pentagon \( ABCDE \) inscribed in a circle, centre \( O \).

The tangents at \( A \) and \( B \) meet at \( W \).

Calculate

(i) angle \( BCD \), \( [2] \)
(ii) angle \( CBD \), \( [2] \)
(iii) angle \( OAB \), \( [1] \)
(iv) angle \( WAB \), \( [1] \)
(v) angle \( AWB \). \( [2] \)

(b) The angles of a hexagon are in the ratio \( 3:4:4:4:4:5 \).

Calculate the size of the smallest angle. \( [4] \)

2 \( (a) \)

(i) Calculate the circumference of a bicycle wheel of diameter \( 0.64 \) m. \( \pi \) is approximately 3.142. \( [2] \)

(ii) Calculate the number of complete turns the wheel makes when the bicycle travels \( 700 \) m. \( [2] \)

(b) A rectangular field measures \( 350 \) m by \( 200 \) m, each measured to the nearest \( 10 \) m.

Calculate the limits between which the area of the field must lie. \( [4] \)

(c) \( A, B \) and \( C \) are three similar containers.

Their heights are \( 40 \) cm, \( 30 \) cm and \( 15 \) cm respectively.

The container \( C \) has a surface area of \( 450 \) cm\(^2\) and has a capacity of \( 0.8 \) litres.

Calculate

(i) the surface area of container \( A \), \( [3] \)
(ii) the capacity of container \( B \). \( [3] \)
The diagram shows some angles and some direct distances between four towns in Brazil.

(a) Calculate the direct distance between Brasília and São Paulo. [4]
(b) Calculate the direct distance between Rio de Janeiro and São Paulo. [4]
(c) Calculate the area of the quadrilateral BHRS. [4]
4. Answer the whole of this question on a sheet of graph paper.

A new breed of wheat is being developed and, in an experiment, the heights of 100 plants are measured. The results are shown in the following table.

<table>
<thead>
<tr>
<th>Height ((h)) in centimetres</th>
<th>(0 &lt; h &lt; 3)</th>
<th>(3 &lt; h &lt; 6)</th>
<th>(6 &lt; h &lt; 9)</th>
<th>(9 &lt; h &lt; 12)</th>
<th>(12 &lt; h &lt; 15)</th>
<th>(15 &lt; h &lt; 18)</th>
<th>(18 &lt; h &lt; 21)</th>
<th>(21 &lt; h &lt; 24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of plants</td>
<td>5</td>
<td>7</td>
<td>12</td>
<td>20</td>
<td>23</td>
<td>18</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Calculate the mean height of the plants.

(b) The following table shows the cumulative frequencies of the same data.

<table>
<thead>
<tr>
<th>Height ((h)) in centimetres</th>
<th>(h \leq 3)</th>
<th>(h \leq 6)</th>
<th>(h \leq 9)</th>
<th>(h \leq 12)</th>
<th>(h \leq 15)</th>
<th>(h \leq 18)</th>
<th>(h \leq 21)</th>
<th>(h \leq 24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative frequency</td>
<td>5</td>
<td>(p)</td>
<td>24</td>
<td>44</td>
<td>67</td>
<td>85</td>
<td>(q)</td>
<td>100</td>
</tr>
</tbody>
</table>

(i) Find the value of \(p\) and the value of \(q\).

(ii) Using your values for \(p\) and \(q\) and the information given in the cumulative frequency table, draw a cumulative frequency diagram. Use a scale of 1 cm to represent 2 cm of height and 1 cm to represent 10 plants.

(c) From your graph, find

(i) the median height,

(ii) the interquartile range of the heights,

(iii) an estimate of the number of plants with a height greater than 10 cm.

5.

The diagram shows the speed–time graph of a car during the first 28 seconds of its motion.

(a) Calculate the acceleration during the first 10 seconds.

(b) Describe the motion taking place between 10 and 20 seconds.

(c) Find the speed at 23 seconds.

(d) Calculate the distance travelled during the 28 seconds.
$T_1$ and $T_2$ are transmitters 200 km apart. $T_2$ is due East of $T_1$.

The signals from $T_1$ can reach a distance of 150 km and those from $T_2$ can reach a distance of 120 km.

(a) Using a scale of 1 cm to represent 20 km, make an accurate drawing to represent the transmitters and the area where signals from both transmitters can reach. [3]

(b) A ship is sailing on a bearing of 330° and passes through the point exactly half-way between the two transmitters. On the same drawing, show accurately the path of the ship. [3]

(c) Use your drawing to find the distance the ship sails whilst receiving signals from both transmitters. [2]

(d) Given that the speed of the ship is 25 km/h, calculate the length of time during which the ship can receive signals from both transmitters. [2]

7 (a) (i) If £1 = 9.80 French francs, calculate how much 100 francs are worth in pounds (£), giving your answer correct to two decimal places. [2]

(ii) If £1 = $x$ francs, write down an expression, in terms of $x$, for the value in pounds of 100 francs. [1]

(b) A French holidaymaker toured Britain in 1989 and in 1990. In 1990, the exchange rate was £1 = $x$ francs.

In 1989, it was £1 = ($x + 1$) francs.

The holidaymaker found that, for 100 francs, she received £1 more in 1990 than in 1989. Write down an equation in $x$ and show that it reduces to

\[x^2 + x - 100 = 0.\] [4]

(ii) Use the above equation to calculate the value of $x$, giving your answer correct to two decimal places. [4]

(iii) Use your answer to (b) (ii) to find the value, in pounds, of 100 francs in the year 1990. Give your answer correct to two decimal places. [1]
8  Answer the whole of this question on a sheet of graph paper.
   (a) The tables of values are for the graphs of \( y = x^2 \) and \( y = 2^x \).

   \[
   \begin{array}{c|ccccc}
   x & -2 & -1 & 0 & 1 & 2 & 3 \\
   \hline
   y = x^2 & 4 & \rho & 0 & 1 & 4 & \gamma \\
   \end{array}
   \]

   \[
   \begin{array}{c|ccccc}
   x & -2 & -1 & 0 & 1 & 2 & 3 \\
   \hline
   y = 2^x & r & 0.5 & i & s & 2 & 4 & 9 \\
   \end{array}
   \]

   (i) Calculate the values of \( \rho, \gamma, r, s, \) and \( i \). \[3\]
   (ii) On the same axes and using a scale of 2 cm to represent 1 unit on both the \( x \) and \( y \) axes, draw the graphs of \( y = x^2 \) and \( y = 2^x \), for \(-2 \leq x \leq 3\). \[6\]
   (iii) From your graphs, and the two solutions, in the range \(-2 \leq x \leq 3\), of the equation \( x^2 = 2^x \), \[2\]

   (b) (i) On the same axes, draw the graph of \( x + y = 1 \). \[2\]
   (ii) Write down the \( x \)-coordinates of the points of intersection of the graphs of \( y = x^2 \) and \( x + y = 1 \). \[2\]
   (iii) Write down the quadratic equation in \( x \) satisfied by these values. \[1\]

9  Answer the whole of this question on a sheet of graph paper.
   (a) Draw \( x \) and \( y \) axes from \(-6 \) to \(+6\), using a scale of 1 cm to represent 1 unit.

   Draw and label triangle \( ABC \) with \( A (4,3) \), \( B (1,3) \) and \( C (4,4) \). \[2\]

   (b) The transformation \( T_1 \) is represented by the matrix
   \[
   M_1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}
   \]

   (i) Draw the image of triangle \( ABC \) under \( T_1 \), labelling it \( A_1B_1C_1 \). \[3\]
   (ii) Describe fully the single transformation \( T_1 \). \[2\]

   (c) The transformation \( T_2 \) is represented by the matrix
   \[
   M_2 = \begin{pmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{pmatrix}
   \]

   (i) Draw the image of triangle \( ABC \) under \( T_2 \), labelling it \( A_2B_2C_2 \). \[3\]
   (ii) Describe fully the single transformation \( T_2 \). \[2\]
10  (a) \( \frac{2x + 1}{3} \leq 2 \) and \( x \) is a positive integer.

Find all the possible values of \( x \). [2]

(b) A company which carries goods has two types of carton, small ones which hold 20 kg and large ones which hold 20 kg.

For a particular job, the company uses \( x \) small cartons and \( y \) large cartons.

On the diagram above, the unshaded region, \( R \), represents the requirements of this job.

(i) Write down three inequalities (in addition to \( x \geq 0 \)) in \( x \) and \( y \) which represent these requirements. [5]

(ii) Find the largest number of cartons which could be carried. [1]

(iii) Find the minimum mass which must be carried. [2]

(iv) A small carton costs $1 and a large carton costs $3. Use the diagram to find the cheapest possible cost which satisfies the requirements of the job. [2]

11 Give all your answers to this question as fractions.

(a) An ordinary die, with six faces numbered 1 to 6, is rolled once.

State the probability that

(i) the uppermost face is a 6, [1]

(ii) the uppermost face is not a 6. [1]

(b) In an experiment a student rolls the die until the uppermost face is a 6.

(i) Calculate the probability that the first roll is not a 6 and the second roll is a 6. [2]

(ii) Calculate the probability that the first roll is not a 6, the second roll is not a 6 and the third roll is a 6. [2]

(iii) Without evaluating your answer, find an expression for the probability that the student first gets a 6 with his or her twelfth roll. [2]
International General Certificate of Secondary Education

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Show all your working on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

Write your name and examination number on each separate piece of writing paper or graph paper you use. If you use more than one sheet of paper for your answers, all answer sheets should be placed in correct order and fastened together.

Electronic calculators should be used.

Three figure accuracy is required in your answers except where stated otherwise.

The total of the marks for this paper is 130.

The number of marks available is shown in brackets [ ] after each question or part question.

This Question Paper consists of 8 printed pages.

2546
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1. \( P \) is the set of cars which use unleaded petrol.
\( S \) is the set of cars which have a sun-roof.

A survey of 100 cars is taken.
68 use unleaded petrol \( (P) \) and 35 have a sun-roof \( (S) \). 10 are in neither set.

(a) Draw a Venn diagram to illustrate this information.
(b) Find \( n(P \cap S) \), the number of cars in the survey which use unleaded petrol and also have a sun-roof.
(c) Find \( n(P' \cap S) \), where \( P' \) is the complement of \( P \).
(d) Shade, in your Venn diagram, \( (P \cup S)' \).
(e) Express, as briefly as possible in set notation, the following statement.
"\( x \) belongs to the set of cars using unleaded petrol, but not to the set of cars which have a sun-roof."

2. An aeroplane takes off from town \( A \) and flies to \( B \), where it stops for 1 hour. It then flies on to \( C \).

(a) If the aeroplane departs from \( A \) at 13:35, and its total time in the air is 1.5 hours, at what time does the aeroplane land at \( C \)?

\( N \)

\( A \)

\( N \)

(b) The diagram above represents the aeroplane's journey. The bearing of \( B \) from \( A \) is \( 063^\circ \), angle \( ABC = 120^\circ \), \( AB = 200 \text{ km} \) and \( BC = 250 \text{ km} \).

(i) Show, by calculation, that \( AC = 391 \text{ km} \), to the nearest kilometre.
(ii) Calculate angle \( ACB \), to the nearest degree.
(iii) Calculate the bearing of \( C \) from \( A \).
3 Scientists test two types of seed, A and B, in a country where the probability of good rainfall in any given year is 0.4. Type A seed is found to have a probability of growing of 0.9 when the rainfall is good, but only 0.3 when it is poor.

(a) (i) Copy and complete the tree diagram above for type A seed. [2]
(ii) Calculate the probability that there is good rainfall and type A seed grows. [2]
(iii) Calculate the probability that type A seed grows, whatever the rainfall. [2]

(b) Type B seed is tested in the same country and the probability of good rainfall is again 0.4. Type B seed is found to have a probability of growing of 0.8 when the rainfall is good, and 0.5 when it is poor.

(i) Draw a tree diagram for type B seed, giving the probabilities for each branch. [2]
(ii) Calculate the probability that type B seed grows, whatever the rainfall. [2]

(c) Which type of seed, A or B, would you advise a farmer in this country to use? Give a reason for your answer. [2]

4 Answer the whole of this question on a sheet of graph paper.

The following table gives the values of \( y = 1 + 4x - x^2 \), for \(-1 \leq x \leq 5\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>-4</td>
</tr>
</tbody>
</table>

(a) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of \( y = 1 + 4x - x^2 \) for \(-1 \leq x \leq 5\). [4]

(b) Draw the tangent to the graph at the point (3, 4) and hence find the gradient of the curve at this point. [4]

(c) Use your graph to estimate, to 1 decimal place, the solutions of the equation \( 1 + 4x - x^2 = 0 \). [2]

(d) By using the quadratic formula, or otherwise, solve the equation \( x^2 - 4x - 1 = 0 \), giving your answers correct to 2 decimal places. [5]
The diagram shows a field in which \( AB = 35 \text{ m}, AD = 70 \text{ m}, BC = 32 \text{ m}, \) angle \( ABC = 120^\circ \) and angle \( BAD = 80^\circ \).

(a) Using a scale of 1 cm to represent 5 m, construct an accurate plan of the field. Label \( A, B, C \) and \( D \) on your plan. [4]

(b) A post \( P \) is situated in the field, so that it is equidistant from the sides \( CD \) and \( CB \), and also equidistant from the points \( A \) and \( B \).

On your diagram construct, using ruler and compasses only,
(i) the locus of points which are equidistant from \( CD \) and \( CB \),
(ii) the locus of points which are equidistant from the points \( A \) and \( B \).

Label, with the letter \( P \), the point which represents the position of the post. [5]

(c) A goat is tied to the post \( P \) by a rope of length 20 metres.
Shade the part of the field which the goat cannot reach. [3]

6. A train usually completes a journey of 360 km at an average speed of \( v \) km/h.
One day engine trouble causes the average speed to be 10 km/h less than usual.

(a) Write down, in terms of \( v \), an expression for the time taken, in hours, for the usual journey. [1]

(b) Write down a similar expression for the time taken for the slower journey. [2]

(c) The time for the slower journey was 30 minutes more than the time for the usual journey. Write down an equation in \( v \) using your answers to (a) and (b), and hence show that
\[
v^2 - 10v - 7200 = 0.
\] [5]

(d) Solve the equation \( v^2 - 10v - 7200 = 0 \), by factorising or otherwise. Write down the usual average speed of the train. [5]
50 boys take a mathematics examination in which the highest possible mark is 100.

The cumulative frequency diagram for the boys' marks is shown above.

(i) What is the boys' median mark? [1]
(ii) If 70% of the boys pass, how many boys fail? [2]
(iii) What is the pass mark? (It must be an integer.) [2]

(b) (i) 80 girls take the same examination and their median mark is 60. Their upper quartile mark is 76 and the interquartile range is 24. What is Q, the girls' lower quartile mark? [2]
(ii) The top 5 girls score more than 90 marks and the lowest 5 marks are in the interval 20 < x ≤ 40.

Using this and all the information in (b) (i), copy and complete the following cumulative frequency table for the girls' marks, replacing the letter Q in the table by your answer to (b) (i).

<table>
<thead>
<tr>
<th>Mark (x)</th>
<th>&lt;=20</th>
<th>&lt;=40</th>
<th>&lt;=Q</th>
<th>&lt;=60</th>
<th>&lt;=76</th>
<th>&lt;=90</th>
<th>&lt;=100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>80</td>
</tr>
</tbody>
</table>
8 Answer the whole of this question on a sheet of graph paper.

(a) (i) Copy the above diagram accurately on your graph paper. [1]
(ii) Draw the reflection of \( F \) in the line \( x = 3 \). Label it \( A \). [2]
(iii) Draw the reflection of \( A \) in the line \( y = 3 \). Label it \( B \). [2]
(iv) Draw the rotation of \( F \) through 180° about the origin. Label it \( C \). [2]
(v) Describe fully the single transformation which maps \( B \) onto \( C \). [2]

(b) (i) Find the matrix \( M \) of the transformation described in (a) (iv). [3]
(ii) Calculate the matrix product \( NM \) where \( N = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \). [2]
(iii) Describe fully the single transformation represented by \( NM \). [2]
One central circle, of radius 3 cm and centre $O$, is completely surrounded by other circles which touch it and touch each other, as shown in the diagram. These outer circles are identical to each other.

(a) If the radius of each outer circle is $x$ cm, write down the following lengths in terms of $x$.
   (i) $OA$, [1]
   (ii) $OB$, [1]
   (iii) $AB$, [1]

(b) On one occasion there are 6 circles completely surrounding the central circle.
   (i) Calculate angle $AOB$. [1]
   (ii) What special type of triangle is $AOB$ in this case? [1]
   (iii) Use your previous answers to find $x$. [2]

(c) On another occasion there are 20 small circles completely surrounding the central circle.
   (i) Calculate angle $AOB$. [1]
   (ii) $M$ is the mid-point of $AB$. Consider the triangle $MAO$ and write down an equation involving $x$ and a trigonometric ratio. [3]
   (iii) Solve this equation to find $x$ correct to 2 decimal places. [3]
<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^2$</td>
<td>1</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n^4$</td>
<td>1</td>
<td></td>
<td>81</td>
<td></td>
</tr>
</tbody>
</table>

(a) Copy and complete the table of values above.

(b) In the table below,

\[ p = 1^2 + 2^2, \]
\[ q = 1^2 + 2^2 + 3^2 + 4^2, \]
\[ r = 3(2^2) + 3(2) - 1, \]
\[ s = 3(3^2) + 3(3) - 1, \]
\[ t = 1^4 + 2^4 + 3^4, \]
\[ u = 1^4 + 2^4 + 3^4 + 4^4. \]

Calculate the values of $p$, $q$, $r$, $s$, $t$ and $u$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>\ldots</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row $X$</td>
<td>1</td>
<td>$p$</td>
<td>14</td>
<td>$q$</td>
<td>\ldots</td>
<td></td>
</tr>
<tr>
<td>Row $Y$</td>
<td>5</td>
<td>$r$</td>
<td>$s$</td>
<td>59</td>
<td>\ldots</td>
<td></td>
</tr>
<tr>
<td>Row $Z$</td>
<td>1</td>
<td>17</td>
<td>$t$</td>
<td>$u$</td>
<td>\ldots</td>
<td></td>
</tr>
</tbody>
</table>

(c) For the first four values of $n$ in the table, consider the $(\text{Row } X \text{ value}) \times (\text{Row } Y \text{ value})$ and the Row $Z$ value. Find a formula which connects Row $X$ and Row $Y$ with Row $Z$.

(d) (i) The value in Row $X$ for $n = 20$ can be found by putting $n = 20$ into the formula

\[ X = \frac{n(n + 1)(2n + 1)}{6}. \]

Find this value of $X$.

(ii) The value in Row $Y$ for $n = 20$ can be found by putting $n = 20$ into the formula

\[ Y = 3n^2 + 3n - 1. \]

Find this value of $Y$ exactly.

(e) Use your answers to parts (c) and (d) to find the exact value of

\[ 1^4 + 2^4 + 3^4 + \ldots + 19^4 + 20^4. \]
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

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This Question Paper consists of 8 printed pages.
Copy the above diagram twice.

(i) In your first diagram, shade the region which represents
(A ∪ B)'.

(ii) In your second diagram, shade the region which represents
A ∩ B'.

Describe, in set notation, the shaded region in the diagram.

\[ \mathcal{S} = \{ \text{students in an international school} \} \]

\[ G = \{ \text{girls} \} \]

\[ L = \{ \text{students who speak more than one language} \} \]

The Venn diagram shows the number of students in each subset, in a school of 150 students.

(i) How many girls speak only one language?  
(ii) How many boys are there in the school?

Give your answers to parts (iii) to (vi) as fractions in their lowest terms.

(iii) A student is selected at random. What is the probability that this student speaks more than one language?

(iv) A girl is selected at random. What is the probability that she speaks more than one language?

(v) A student who speaks more than one language is selected at random. What is the probability that this student is a girl?

(vi) Two students are selected at random. What is the probability that they are both boys?
The diagram shows the plan for a new school. 

\( ABX \) is a straight line. The angles at \( A, X \) and \( Y \) are each 90°. Angle \( ADB = 50^\circ \), angle \( BDC = 50^\circ \) and angle \( DCB = 100^\circ \). \( DB = 150 \text{ m} \), \( BX = 71 \text{ m} \) and \( XY = CY = 110 \text{ m} \).

(a) Calculate, correct to three significant figures,

(i) the area of \( BXYC \),

(ii) the length of \( AD \),

(iii) the length of \( AB \),

(iv) the length of \( DC \). 

(b) Using your answers to part (a), calculate the total area of the school grounds, \( AXYCD \). Give your answer correct to two significant figures.

---

3 (a) A water tank is in the shape of a cuboid, measuring 150 cm by 100 cm by 80 cm.

(i) How many litres of water would the tank contain when full? 

(ii) The tank is initially empty and water flows into it from a pipe. The cross-sectional area of the pipe is 2.1 cm\(^2\) and the water flows along the pipe at a rate of 35 cm/s.

By calculating the volume of water flowing from the pipe in 1 second, find the time taken to fill the tank. Give your answer in hours and minutes, correct to the nearest minute.

(b) The diagram shows a hemisphere of radius \( r \), attached to a cone of base radius \( r \) and height \( h \).

The total volume, \( V \), of the solid is given by the formula:

\[ V = \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3. \]

(i) Calculate the volume when \( r = 8 \text{ cm} \) and \( h = 10 \text{ cm} \). [\( \pi \) is approximately 3.142.]

(ii) Find a formula for \( h \) in terms of \( \pi \), \( r \) and \( V \).

Give your answer as a single fraction, in its simplest form.
4 Answer the whole of this question on a sheet of graph paper.

(a) $\begin{array}{|c|c|c|c|c|c|c|}
\hline
x & 1 & 1.5 & 2 & 3 & 4 & 5 & 6 \\
\hline
y = \frac{6}{x} & p & 4 & q & 1.5 & r & 1 \\
\hline
\end{array}$

The above table is for the function

$\Gamma : x \rightarrow \frac{6}{x}$.

(i) Calculate the values of $p$, $q$ and $r$. [2]

(ii) Using a scale of 2 cm to represent 1 unit on each axis, draw a pair of axes for $0 \leq x \leq 6$ and $0 \leq y \leq 8$.

Draw the graph of $y = \frac{6}{x}$ for $1 \leq x \leq 6$. [3]

(b) By drawing a suitable tangent, estimate the gradient of the curve $y = \frac{6}{x}$ at the point $(2, 3)$. [3]

(c) $\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
x & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
y = \frac{x^2}{5} & 0 & k & 0.8 & l & 3.2 & m & 7.2 \\
\hline
\end{array}$

The above table is for the function

$g : x \rightarrow \frac{x^2}{5}$.

(i) Calculate the values of $k$, $l$ and $m$. [2]

(ii) Draw, on the axes already used for part (a) (ii), the graph of

$y = \frac{x^2}{5}$ for $0 \leq x \leq 6$. [3]

(d) (i) Write down the $x$-coordinate of the point of intersection of the curves

$y = \frac{6}{x}$ and $y = \frac{x^2}{5}$. [1]

(ii) Use the equations of the two curves to show that the exact value of $x$ is $30^4$. [2]
City A and city B each have a population of 100,000 people.

The above cumulative frequency graphs show the number of people less than x years old. Use the graphs to answer the following questions.

(a) What is the median age in each city? [2]

(b) What is the interquartile range of the ages of the people in city A? [3]

(c) How many people are less than 20 years old
   (i) in city A, [1]
   (ii) in city B? [1]

(d) How many people are at least 70 years old
   (i) in city A, [1]
   (ii) in city B? [1]

(e) In ten years time, which city would you expect to have the larger population? Give one reason for your answer. [2]
(a) In the diagram, \( SR \) is parallel to \( PQ \).
\( SR = 4 \text{ cm}, \ SX = 2 \text{ cm}, \ RX = 3 \text{ cm} \) and \( PQ = 7 \text{ cm} \).

(i) Explain why the triangles \( RSX \) and \( PQX \) are similar. [2]

(ii) Calculate the length of \( PX \) and the length of \( QX \). [3]

(iii) It is also given that the area of triangle \( RSX \) is 2.90 cm\(^2\).
Calculate the area of triangle \( PQX \), correct to two significant figures. [3]

(iv) Use trigonometry to calculate the size of angle \( SRX \), to the nearest degree. [4]

(b) In the diagram, the points \( B, C, D \) and \( E \) lie on the circle. \( ABC \) and \( AED \) are straight lines.

Angle \( ABE = 65^\circ \), angle \( BAE = 19^\circ \) and angle \( DBE = 80^\circ \).
Calculate

(i) angle \( CDE \), [2]

(ii) angle \( CDB \), [2]

(iii) angle \( BCE \). [2]
7 (a) The lengths, in centimetres, of the sides of a right-angled triangle are 
\[x, x + 2\] and \[x + 5\].

(i) Use Pythagoras' theorem to write down an equation in \(x\). Show that it simplifies to 
\[x^2 - 6x - 21 = 0\].

(ii) Solve the equation \(x^2 - 6x - 21 = 0\), giving your answers correct to two decimal places.

(iii) Write down the length of the hypotenuse of the triangle.

(b) A student walks \(y\) kilometres at 3 km/h.

(i) Write down an expression for the time, in hours, that he takes.

He then walks a further \(y + 3\) kilometres at 4 km/h.

The total walking time is 4 hours 50 minutes.

(ii) Write down an equation in \(y\) and solve it.

(iii) Find the total distance walked by the student.

8 Answer the whole of this question on a sheet of graph paper.

(a) Draw and label \(x\) and \(y\) axes from \(-6\) to \(+6\) using a scale of 1 cm to represent 1 unit on each axis.

Draw the triangle whose vertices are \(A\) (2, 2), \(B\) (5, 2) and \(C\) (5, 3).

(b) \(M\) is the matrix \(\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\) which represents the transformation \(T\).

Draw accurately the image of triangle \(ABC\) under the transformation \(T\), labelling it \(PQR\).

(c) \(N\) is the matrix \(\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\) which represents the transformation \(U\).

Draw accurately the image of triangle \(ABC\) under the transformation \(U\), labelling it \(XYZ\).

(d) (i) Describe fully the single transformation which maps triangle \(PQR\) onto triangle \(XYZ\).

(ii) Find the matrix which represents this transformation.

(e) (i) Calculate the matrix \(NM\).

(ii) This matrix represents the transformation \(\mathcal{V}\).

Draw accurately the image of triangle \(ABC\) under the transformation \(\mathcal{V}\), labelling it \(FGH\).

(iii) State whether the transformation \(\mathcal{V}\) is equivalent to "transformation \(T\) followed by transformation \(U\)" or to "transformation \(U\) followed by transformation \(T\)".

Question 9 is printed on the back page
9 In this question, you are asked to look at what happens to given numbers when you repeat a set of instructions several times.

In each case, when the instructions are repeated many times, a certain number is approached. This number is called the \textit{limit}.

(a) 

\[
\begin{array}{c}
\text{Divide by 2} \\
\xrightarrow{} \\
\text{Add 1} \\
\xrightarrow{} \\
\text{Write down number}
\end{array}
\]

(i) Starting with the number 7, the following numbers are calculated by repeating the instructions in the diagram.

\[
\begin{align*}
7 & \rightarrow 4.5 \\
4.5 & \rightarrow 3.25 \\
3.25 & \rightarrow 2.625 \\
2.625 & \rightarrow 2.3125 \\
2.3125 & \rightarrow p \\
p & \rightarrow q
\end{align*}
\]

Calculate the exact values of \( p \) and \( q \).

(ii) Start with the number 0.5 and repeat the above instructions six times, setting out your working as in part (i).

(iii) In parts (i) and (ii) the \textit{limit} is the same whole number. Suggest the value of this \textit{limit}.

(b) 

\[
\begin{array}{c}
\text{Divide by 5} \\
\xrightarrow{} \\
\text{Add 1} \\
\xrightarrow{} \\
\text{Write down number}
\end{array}
\]

Start with the number 8 and repeat the instructions in the diagram several times.

The \textit{limit} in this case is an exact fraction. Find this \textit{limit}.

(c) 

\[
\begin{array}{c}
\text{Divide by 4} \\
\xrightarrow{} \\
\text{Add 1} \\
\xrightarrow{} \\
\text{Write down number}
\end{array}
\]

Start with any number and repeat the instructions in the diagram several times. The \textit{limit} is another exact fraction. Find this \textit{limit}.

(d) 

\[
\begin{array}{c}
\text{Divide by } n \\
\xrightarrow{} \\
\text{Add 1} \\
\xrightarrow{} \\
\text{Write down number}
\end{array}
\]

Use your answers to (a), (b) and (c) to find the \textit{limit}, in terms of \( n \), in this case.
International General Certificate of Secondary Education

Instructions to candidates:

You should answer all the questions on the separate sheets of paper provided.

Show all your working on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

Write your name and examination number on each separate piece of writing paper or graph paper you use. If you use more than one sheet of paper for your answers, all answer sheets should be placed in correct order and fastened together.

Electronic calculators should be used.

Three figure accuracy is required in your answers except where stated otherwise.

The total of the marks for this paper is 130.

The number of marks available is shown in brackets [ ] at the end of each question or part question.

This Question Paper consists of 8 printed pages.
1. Klaus and Heidi plan a holiday in the U.S.A. In August.

(a) Klaus decides to change 800 Deutschmarks (DM) into dollars in January when the exchange rate is $1 = DM1.68. A bank charge of 1% is then deducted. Calculate how much he receives, to the nearest dollar. [3]

(b) (i) Heidi invests her DM800 in a bank at an annual rate of 9% simple interest. Calculate the amount she has after 6 months. [3]

(ii) She now changes this amount into dollars. The exchange rate is $1 = DM1.87, but this time there is no bank charge. Calculate how much Heidi receives, to the nearest dollar. [2]

(c) Who made the better decision? [1]

(d) They bring a total of $120 back with them and exchange it for Deutschmarks at a rate of $1 = DM1.72 with no bank charge. Calculate how much they receive, to the nearest Deutschmark. [2]

---

The diagram represents a garden ABCDE. AB = 2.5 m, AE = 10 m, ED = 5.2 m and DC = 6.9 m.

Angle EAB = 120°, angle DEA = 90° and angle EDC = 110°.

(a) (i) Using a scale of 1 cm to represent 1 m, construct an accurate plan of the garden. [4]

(ii) Construct the locus of points equidistant from CD and CB. [2]

(iii) Construct the locus of points 6 metres from A. [2]

A fountain is to be placed nearer to CD than to CB and no more than 6 metres from A.

(b) (i) Shade, and label R, the region within which the fountain could be placed in the garden. [1]

(ii) Construct the locus of points in the garden 3.4 metres from AE. [2]

(iii) Is it possible for the fountain to be 3.4 metres from AE and in the region R? [1]
(a) How many lines of symmetry does the shape in the diagram possess?

(b) State the vector which translates $AB$ onto $HG$.

(c) Find the equation of the line in which $AB$ is the reflection of $HG$.

(d) Describe fully the transformation which maps $A$ onto $K$ and $B$ onto $J$.

(e) Describe fully the transformation which maps $A$ onto $D$ and $B$ onto $E$.

(f) The matrix \[
\begin{pmatrix}
1 & 3 \\
-1 & 7
\end{pmatrix}
\] transforms $F(3, 1)$ onto another point on the diagram. Calculate the coordinates of this point and state its letter name.
The diagram shows the relative positions of Osaka (O), Tokyo (T) and Sapporo (S) in Japan. $ST = 850 \text{ km}$, $TO = 400 \text{ km}$ and angle $STO = 110^\circ$.

(a) Calculate $OS$, the distance from Osaka to Sapporo.

(b) Calculate the angle $SOT$, to the nearest degree.

(c) The bearing of Sapporo from Osaka is $030^\circ$. Find the bearing of Osaka from Tokyo.

(d) A plane flew from Sapporo to Tokyo at an average speed of $500 \text{ km/h}$. It left Sapporo at 09:30. At what time did it arrive in Tokyo?
5 In a football stadium, ticket prices are $15 for a seat and £8 for a standing place. Initially the stadium has 10000 seats and 20000 standing places.

(a) Calculate the amount of money taken when the stadium is full.

It is possible to replace some of the standing places with extra seats:
Each extra seat takes away two standing places.

(b) Extra seats are put in until only 4000 standing places remain.
   (i) Find the number of seats now in the stadium.
   (ii) Find the total amount of money taken for a full stadium.
   (iii) If all 4000 standing places are full, find the number of seats which must be sold for a total amount of $200000 to be taken.

(c) On another occasion, x standing places remain.
   (i) Write down and simplify an expression in terms of x for the number of seats that there are now.
   (ii) Given also that there are twice as many seats as standing places, form an equation in x and solve it.

Hence find the maximum number of spectators the stadium can hold in this case.

6

The diagram shows an archery target, consisting of a central circle called the "bull", an "inner" ring and an "outer" ring. These are formed by three concentric circles, radii 10 cm, 20 cm and 30 cm respectively.

(a) Show that the ratio of the areas of bull : inner : outer is 1 : 3 : 5.

(b) If an arrow is equally likely to land anywhere on the target, what is the probability that it hits the bull?

(c) Alexander is a good archer, and the probabilities that he hits the bull, the inner and the outer are 3/4, 1/2 and 1/5 respectively.

Using a tree diagram, or otherwise, answer the following questions.
   (i) He shoots 2 arrows. What is the probability that they both hit the outer?
   (ii) There is a $10 prize for hitting the bull, $2 for hitting the inner, but no prize for hitting the outer. What is the probability that Alexander wins exactly $12 with 2 shots?
   (iii) Alexander takes 3 shots. What is the probability that he wins $30?
The diagram shows a vertical cross-section of a solid sphere and a hollow cone, both resting on a horizontal table. The sphere, centre $O$, touches the cone at $S$ and $T$. Angle $\angle VTO = \angle PVB = 20^\circ$.

(a) (i) Explain why angle $VTO = 90^\circ$. [1]
(ii) Calculate angle $TOS$. [1]
(iii) Calculate angle $TPS$. [2]

(b) The radius of the sphere, $r$, is 10 cm.
(i) Calculate $VO$. [3]
(ii) Show that the height of the cone, $h$, is 39.2 cm. [1]
(iii) Show that the base radius of the cone, $R$, is 14.3 cm. [2]

(c) [The volume of a cone is $\frac{1}{3}\pi R^2 h$. The volume of a sphere is $\frac{4}{3}\pi r^3$. $\pi$ is approximately 3.142.]
Using the values for $r$, $h$ and $R$ given in part (b), calculate
(i) the volume of the cone, [2]
(ii) the volume of the sphere, [2]
(iii) the volume of the empty space inside the cone, as a percentage of the volume of the cone. [3]
Anna throws a ball from a point $A$, one metre above the ground, towards a wall. The ball travels along the arrowed path from $A$ to $B$, given by the equation $y = 4 + 2x - x^2$, where the $x$-axis represents the horizontal ground and the $y$-axis represents the wall.

(a) Show that the $x$-coordinate of $A$ satisfies the equation $x^2 - 2x - 3 = 0$.  

(b) Solve the equation $x^2 - 2x - 3 = 0$ and state the $x$-coordinate of $A$.  

(c) The ball rebounds from the wall at $B$ to the ground at $G$. The equation of its path $BG$ is $y = 4 - 2x - x^2$.

Solve the equation $x^2 + 2x - 4 = 0$, and hence state the $x$-coordinate of $G$ to 2 decimal places.  

Answer the rest of this question on a sheet of graph paper.

(d) Draw axes on a sheet of graph paper, using a scale of 4 cm to represent 1 m on both $x$ and $y$ axes. Use the tables below, together with your answers to (b) and (c), to draw the path of the ball from $A$ to $B$ and then $B$ to $G$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>4.75</td>
<td>5</td>
<td>4.75</td>
<td>4</td>
<td>2.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>2.75</td>
<td>1</td>
</tr>
</tbody>
</table>

(e) How far from the wall is the ball when it is only 0.5 m above the ground?  

(f) Draw the tangent to your graph at $T(2, 4)$.

Calculate the gradient of the curve at this point.
9. (a) Show that

\[
\frac{1}{n} - \frac{1}{n + 1} = \frac{1}{n(n + 1)}.
\]

(b) Copy the following table, completing the rows for \(n = 2, 3, 4, 99\) and \(100\).

<table>
<thead>
<tr>
<th>(n)</th>
<th>(\frac{1}{n} - \frac{1}{n + 1})</th>
<th>(\frac{1}{n(n + 1)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{1}{1} - \frac{1}{2})</td>
<td>(\frac{1}{1 \times 2})</td>
</tr>
<tr>
<td>2</td>
<td>(\ldots - \ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>3</td>
<td>(\ldots - \ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>4</td>
<td>(\ldots - \ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>99</td>
<td>(\ldots - \ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>100</td>
<td>(\ldots - \ldots)</td>
<td>(\frac{1}{100 \times 101})</td>
</tr>
</tbody>
</table>

(c) Use part (a) and your table to find another expression for

\[
\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \ldots + \frac{1}{100 \times 101}.
\]

Write your answer as a single fraction.
International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICS
PAPER 4
Wednesday 8 JUNE 1994 Morning 2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.
Answer all questions.
Write your answers and working on the separate answer paper provided.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

INFORMATION FOR CANDIDATES
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.
Electronic calculators should be used.
Three figure accuracy is required in your answers except where stated otherwise.
1
(a) Show that an interior angle of a regular pentagon is 108°.

(b) (i) Draw accurately a regular pentagon $ABCDE$ whose sides are of length 8 cm.

(ii) Using straight edge and compasses only, construct

(a) the perpendicular bisector of $DE$,

(b) the bisector of angle $A$.

(iii) $X$ is the point which is equidistant from $D$ and $E$ and equidistant from $AB$ and $AE$. Mark the point $X$ on your diagram. Measure and write down the length of $AX$.

(iv) Shade the region inside the pentagon which contains the points which are nearer to $E$ than $D$ and nearer to $AE$ than $AB$.

2

In a fitness exercise, students run across a field from $A$ to $B$, then from $B$ to $C$ and then from $C$ to $A$.

(a) A student runs from $A$ to $B$ in 10 seconds.

Calculate his speed in

(i) metres/second,

(ii) kilometres/hour.

(b) Another student runs from $A$ to $B$ in 10.5 seconds, from $B$ to $C$ in 13 seconds and from $C$ to $A$ at a speed of 8.5 m/s.

Calculate her overall average speed in metres/second.

(c) Showing all your working, calculate angle $BAC$.

(d) The bearing of $B$ from $A$ is 062°.

Calculate

(i) the bearing of $C$ from $A$,

(ii) the bearing of $A$ from $C$. 
3

\[ f(x) = 3x^2 - 2x - 4 \text{ and } g(x) = 4 - 3x. \]

(a) State the value of \( f(-2). \)

(b) Solve the equation \( f(x) = -3. \)

(c) Solve the equation \( f(x) = 0, \) giving your answers correct to 2 decimal places.

(d) Solve for \( x \) the equation \( g(x) = 2g(x) - 1.\)

(e) Find \( g^{-1}(x). \)

(f) Study the six sketches \( A, B, C, D, E \) and \( F. \) Which one could be the graph of

(i) \( y = f(x), \)

(ii) \( y = g(x)? \)

4

Answer the whole of this question on a sheet of graph paper.

In a school gardening project, teachers and students carry earth to a vegetable plot.

A teacher can carry 24 kg and a student can carry 20 kg.

Each person makes one trip.

Altogether at least 240 kg of earth must be carried.

There are \( x \) teachers and \( y \) students.

(a) Show that \( 6x + 5y \geq 60. \)

(b) There must not be more than 13 people carrying earth, and there must be at least 4 teachers and at least 3 students.

Write down three more inequalities in \( x \) and/or \( y. \)

(c) (i) Draw \( x \) and \( y \) axes from 0 to 14, using 1 cm to represent 1 unit of \( x \) and \( y. \)

(ii) On your grid, represent the information in parts (a) and (b). Shade the unwanted regions.

(d) From your graph, find

(i) the least number of people required,

(ii) the greatest amount of earth which can be carried.
The diagram shows two circles, centres $O$ and $C$, of radii 12 cm and 5 cm, which cut each other at right angles.

(a) Show, by calculation, that the distance between the centres is 13 cm. [2]

(b) State the radius of the circle which passes through $O$, $A$ and $C$. [1]

(c) Calculate angle $AOC$. [2]

(d) $CO$ is extended to meet the larger circle at $P$.

What is the size of angle $APC$? [2]

(e) $Q$ is the point on the larger circle such that the line $AQ$ is parallel to the line $CO$.

What is the size of angle $AOQ$? [2]

(f) What is the size of angle $APQ$? [1]

(g) $R$ is a point on the minor arc $AQ$ of the larger circle.

What is the size of angle $QRA$? [1]
6 Answer the whole of this question on a sheet of graph paper.

(a) (i) Draw $x$ and $y$ axes from $-6$ to $+6$, using $1$ cm to represent $1$ unit of $x$ and $y$.

(ii) Draw the triangle $ABC$ with $A(2, 1)$, $B(5, 1)$ and $C(5, 5)$.

(b) (i) Draw the image of triangle $ABC$ under the transformation represented by the matrix

\[
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
\]

and label it $A'B'C'$.

(ii) Describe this single transformation.

(c) (i) Draw the image of triangle $ABC$ under a reflection in the line $y = -x$ and label it $A''B''C''$.

(ii) Find the matrix which represents this transformation.

(d) (i) Describe fully the single transformation which maps triangle $A'B'C'$ onto triangle $A''B''C''$.

(ii) Find the matrix which represents this transformation.
The histogram represents the frequencies of heights of flowers as measured during an experiment.

(a) Copy and complete the table.

<table>
<thead>
<tr>
<th>Height (h) cm</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; h ≤ 5</td>
<td>20</td>
</tr>
<tr>
<td>5 &lt; h ≤ 10</td>
<td>40</td>
</tr>
<tr>
<td>10 &lt; h ≤ 15</td>
<td></td>
</tr>
<tr>
<td>15 &lt; h ≤ 25</td>
<td></td>
</tr>
<tr>
<td>25 &lt; h ≤ 50</td>
<td></td>
</tr>
</tbody>
</table>

(b) Calculate an estimate of the mean height of the flowers.

(c) Copy and complete the table.

<table>
<thead>
<tr>
<th>Height (h) cm</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>h ≤ 5</td>
<td>20</td>
</tr>
<tr>
<td>h ≤ 10</td>
<td></td>
</tr>
<tr>
<td>h ≤ 15</td>
<td></td>
</tr>
<tr>
<td>h ≤ 25</td>
<td></td>
</tr>
<tr>
<td>h ≤ 50</td>
<td>250</td>
</tr>
</tbody>
</table>

(d) (i) Which class interval contains the median height?

(ii) Calculate an estimate of the median height of the flowers, correct to the nearest centimetre.

(e) A flower is picked at random. State the probability that it has a height greater than 10 cm. Give your answer as a fraction.

(f) Two flowers are picked at random from the 250 flowers.

Calculate the probability that both flowers have a height greater than 10 cm. Give your answer as a decimal.
The diagram shows a prism of cross-sectional area 0.42 cm$^2$ and volume 7.56 cm$^3$.

(a) Calculate the length of the prism. [2]

(b) The prism is made of wood and 1 cm$^3$ of this wood has a mass of 0.88 g. Calculate the mass of the prism. [2]

(c) The prisms are made from a block of wood of volume 0.5 m$^3$. It is known that 25% of the wood is wasted. Calculate the number of prisms which can be made, giving your answer to the nearest thousand. [5]

(d) The cross-section of the prism is a regular hexagon of area 0.42 cm$^2$.

(i) State the area of triangle $OAB$. [1]

(ii) What special type of triangle is triangle $OAB$? [1]

(iii) Given that the length of $AB$ is $x$ cm, find an expression for the area of triangle $OAB$ in terms of $x$. Hence find the length of $AB$ correct to the nearest millimetre. [4]
9 (a) Calculate the gradient of the straight line joining the points (3, 18) and (3.5, 24.5).

(b) Find an expression for the length of QR in terms of c and h, and simplify your answer.

(i) P is the point (c, d). Write down d in terms of c.

(ii) Q is the point (c + h, e). Write down e in terms of c and h.

(iii) Write down the length of PR.

(iv) Show that the gradient of the line PQ is

\[ 4c + 2h. \]

(v) If P is the point (3, 18) and Q is the point (3.5, 24.5), state the value of c and the value of h, and use these values to show that (b) (iv) gives the same answer as (a).

(vi) If P is the point (3, 18) and Q is the point (3.1, 19.22) state the value of c and the value of h, and use (b) (iv) to find the gradient of the line PQ.

(vii) If P is the point (3, 18) and Q gets closer and closer to P, what happens

(a) to the value of h,

(b) to the value of the gradient of the line PQ?
International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICS
PAPER 4
18 NOVEMBER 1994  2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables

TIME  2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces provided on the answer paper(answer booklet.
Answer all questions.
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INFORMATION FOR CANDIDATES
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.
Electronic calculators should be used.
Three-figure accuracy is required in your answers except where stated otherwise.
1. Alexis, Biatrix and Carlos are business partners.

(a) 60% of each week’s income is used for the business.

The rest is divided between Alexis, Biatrix and Carlos in the ratio 5:3:1.

(i) Calculate how much they each receive in a week when the income is $9000.

(ii) Calculate the income in a week when Carlos receives $420.

(b) Alexis buys Carlos’ share of the business for $16 000, which he borrows from the bank at a rate of 12% simple interest per year.

How much interest will he have to repay in 6 months?

2. Mahmoud enjoys flying his kite. On any given day, the probability that there is a good wind is \( \frac{1}{2} \).

If there is a good wind, the probability that the kite will fly is \( \frac{1}{4} \).

If there is not a good wind, the probability that the kite will fly is \( \frac{1}{2} \).

(a) (i)

```
good wind
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>kite flies</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
not a good wind     |
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>kite flies</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>kite does not fly</td>
</tr>
</tbody>
</table>
```

(b) Copy the given tree diagram.

Write on your diagram the probability for each branch.

(ii) What is the probability of a good wind and the kite flying?

(iii) Find the probability that, whatever the wind, the kite does not fly.

(b) If the kite flies, the probability that it sticks in a tree is \( \frac{1}{3} \).

Calculate the probability that, whatever the wind, the kite sticks in a tree.

(c)

<table>
<thead>
<tr>
<th>Wind strength</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of days</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

The table shows the wind strength measured on each of 50 days.

(i) State the mode and find the median wind strength.

(ii) Calculate the mean wind strength.

(iii) A ‘good wind’ has strength \( x \) such that \( 3 \leq x \leq 7 \).

Estimate the probability of a good wind from this data.
An aeroplane leaves its base $H$ and flies to $A$, $B$ and $C$ before returning to $H$. The bearing of $A$ from $H$ is $037^\circ$ and angle $HAB$ is $83^\circ$.

(a) Calculate the bearing of
   (i) $B$ from $A$, [2]
   (ii) $C$ from $B$. [2]

(b) $HA = 120$ km and $AB = 100$ km. Using a scale of $1$ cm to represent $10$ km, construct a scale drawing of the quadrilateral $ABCD$.

   Hence find the distance, in kilometres, from $C$ to $H$. [4]

4 Answer the whole of this question on a sheet of graph paper.

A farmer keeps $x$ cows and $y$ sheep, where $x \geq 4$ and $y \geq 10$.

(a) On your graph paper, draw axes from $0$ to $60$, using a scale of $2$ cm to represent $10$ units on each axis.
   Draw and label the lines $x = 4$ and $y = 10$. [3]

(b) The total number of cows and sheep must not be more than $49$.
   Write this as an inequality and draw the appropriate line on your graph. [2]

(c) Shade the unwanted regions of the graph. [1]

(d) The farmer makes $\$100$ profit per cow and $\$50$ per sheep.
   What is his maximum profit? [2]
5 (a) Factorise completely
   (i) \( 3xu + 6xh - 9xr, \) [2]
   (ii) \( x^2 - 10x - 24, \) [2]
   (iii) \( 10x^2 - 7x + 1, \) [2]

(b) \( y = \frac{a}{x} + bx. \)
   (i) When \( x = 1, y = 2 \) and when \( x = 2, y = -5. \)
       Find the value of \( a \) and the value of \( b. \) [4]
   (ii) When \( y = 16, \) use your values of \( a \) and \( b \) to show that the equation \( y = \frac{a}{x} + bx \) becomes \( 2x^2 + 8x - 3 = 0. \) [2]
       Solve this equation, giving your answers correct to 2 decimal places. [5]

6 In triangle \( ABC, \) \( AB = 7 \) cm, \( AC = 9 \) cm and angle \( BAC = 120^\circ. \)

(a) Calculate (i) the length \( BC, \) [4]
       (ii) the angle \( ABC. \) [4]

(b) The three sides of triangle \( ABC \) are tangents to a circle, centre \( O, \) radius \( r \) cm.

The circle touches \( AB \) at \( S. \)

(i) Find the size of angle \( OAS \) and the size of angle \( OBS. \) [2]
(ii) Use trigonometry in the triangle \( OAS \) to write \( AS \) in terms of \( r. \) [2]
(iii) Use trigonometry in the triangle \( OBS \) to write \( BS \) in terms of \( r. \) [2]
(iv) Use the fact that \( AB = 7 \) cm to form an equation in \( r, \) and solve it. [3]
A spherical ball, radius $r$, diameter $AB$, is floating in water with its centre $O$ at a depth $h$ below the surface.

$CD$ is a diameter of the circular cross-section formed at the surface. [$\pi$ is approximately 3.142]

If $r = 13$ cm and $h = 5$ cm, calculate

(a) (i) the length of $CD$,
(ii) the angle $COD$,
(iii) the length of the arc $CBD$,
(iv) the distance from $C$ to $D$ round the semicircle, diameter $CD$.

(b) (i) The area of surface above the water level is given by the formula $2\pi r(r - h)$.

Find the area above the water level.

(ii) The total surface area of a sphere is $4\pi r^2$.

Find the area above the water level as a percentage of the total surface area of the sphere.
8. Answer the whole of this question on a sheet of graph paper.

(a) In a chemical reaction, the mass $M$ grams of a chemical is given by the formula

$$M = 160 \times 2^{-t},$$

where $t$ is the time in minutes, after the start.

A table of values for $t$ and $M$ is given below.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>80</td>
<td>40</td>
<td>20</td>
<td>$q$</td>
<td>5</td>
<td>$r$</td>
<td>1.25</td>
<td></td>
</tr>
</tbody>
</table>

(i) Find $p$, $q$ and $r$.

(ii) Draw the graph of $M$ against $t$ for $0 \leq t \leq 7$.

Use a scale of 2 cm to represent 1 minute on the horizontal $t$-axis and 1 cm to represent 10 grams on the vertical $M$-axis.

(iii) Draw a suitable tangent to your graph and use it to estimate the rate of change of mass when $t = 2$.

(b) The other chemical in the same reaction has mass $m$ grams which is given by

$$m = 160 - M.$$ 

(i) For what value of $t$ do the two chemicals have equal mass?

(ii) State a single transformation which would give the graph for $m$ from the graph for $M$. 

(a) In each case describe fully the single transformation which maps $A$ onto

(i) $B$,
(ii) $C$,
(iii) $D$,
(iv) $E$,
(v) $F$.

(b) State which shapes have an area equal to that of $A$.

(c) Find the matrix which transforms $A$ onto $E$.

(d) The matrix which transforms $A$ onto $F$ is $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$.

Find the matrix which transforms $F$ onto $A$. 

[2]

[2]

[2]

[2]

[2]
In an Olympic diving competition, 7 judges each award a mark between 0 and 10 for a dive. The final score for the dive is found by following the instructions below.

(a) 3 competitors obtained marks for their first dive as shown in the table below.

<table>
<thead>
<tr>
<th>Competitor</th>
<th>Marks</th>
<th>Mean</th>
<th>'Difficulty factor'</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Claus</td>
<td>6.8 8.0 7.0 7.3 7.0 8.0 7.7</td>
<td>7.4</td>
<td>1.5</td>
<td>11.1</td>
</tr>
<tr>
<td>Erik</td>
<td>7.2 6.9 7.3 6.8 7.1 6.7 7.0</td>
<td>a</td>
<td>1.8</td>
<td>b</td>
</tr>
<tr>
<td>Jawed</td>
<td>4.9 4.3 4.7 5.2 5.1 5.1 5.3</td>
<td>c</td>
<td>2.3</td>
<td>d</td>
</tr>
</tbody>
</table>

The score for Claus has been worked out.

Calculate the values of a, b, c and d. [4]

(b) Miguel performed a dive with 'difficulty factor' 2.2.

The marks from the judges were 8.0, 7.0, 6.5, 7.3, 7.6, 8.2 and x.

The lowest and highest marks were 6.5 and 8.2.

The score for the dive was 16.5.

Calculate the value of x. [4]

(c) Tarik's marks for a dive were 7.0, 7.1, 7.1, 7.1, 7.1, y and z.

When the highest and lowest marks were deleted, the mean of the remaining 5 marks was 7.2.

Find a possible pair of values for y and z. [2]
International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICS
PAPER 4
Wednesday 7 JUNE 1995 Morning 2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical Instruments
Graph paper (2 sheets)
Mathematical tables (optional)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.
Answer all questions.
Write your answers and working on the separate answer paper provided.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer.
Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
If you use more than one sheet of paper, staple the sheets together.

INFORMATION FOR CANDIDATES
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.
Electronic calculators should be used.
Three figure accuracy is required in your answers except where stated otherwise.
1 Answer the whole of this question on a sheet of graph paper.

The diagram shows triangle $A$, with vertices (2, 1), (3, 3) and (4, 3).

(a) Using a scale of 1 cm to represent 1 unit, draw on your graph paper an $x$-axis for $-6 \leq x \leq 8$ and a $y$-axis for $-6 \leq y \leq 8$. Draw triangle $A$. [1]

(b) Draw the enlargement of triangle $A$, centre (0, 0), scale factor 2. Label it $B$. [2]

(c) Draw the rotation of triangle $A$, through 90° anticlockwise about (0, 0). Label it $C$. [2]

(d) Draw the reflection of triangle $A$ in the line $y = -1$. Label it $D$. [2]

(e) Draw the translation of triangle $A$ by the vector \( \begin{pmatrix} -2 \\ -4 \end{pmatrix} \). Label it $E$. [2]

(f) (i) A transformation is represented by the matrix \( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \). Draw the image of triangle $A$ under this transformation. Label it $F$. [2]

(ii) Describe fully the single transformation which maps $A$ onto $F$. [2]

(g) (i) Describe fully the single transformation which maps $F$ onto $C$. [2]

(ii) Find the matrix for this transformation. [2]
$BCA$ is the diameter of a circle, centre $C$, radius $r$. $DAE$ is a tangent to the circle at $A$. $DE = 3r$ and angle $DCA = 30^\circ$.

(a)  
(i) Draw the diagram accurately when $r = 3$ cm.  
(ii) Measure and write down the length of $BE$ in your diagram.  
(iii) Calculate the length of the semicircular arc $BA$ when $r = 3$ cm.  

[π is approximately 3.142.]

(b) In the case when $r = 10$ cm, calculate, to 2 decimal places,

(i) the length of $DA$,  
(ii) the length of $AE$,  
(iii) the length of $BE$,  
(iv) the length of the semicircular arc $BA$.  

(c) Comment on the relationship between the length of $BE$ and the length of the semicircular arc $BA$.  

[1]
3 Ahmed earns $20,000 each year.

(a) In 1991, he paid no tax on the first $3000 of his earnings.
He paid 25% of the rest as tax.
Show that he paid $4250 as tax. [2]

(b) In 1992, he paid no tax on the first $4000 of his earnings.
He paid 30% of the rest as tax.
Calculate how much he paid as tax. [2]

(c) In 1993, he paid no tax on the first $x$ of his earnings.
He paid 30% of the rest as tax.
(i) Find an expression in terms of $x$ for the amount of tax he paid. [2]
(ii) Calculate the value of $x$ if he paid $4950 as tax. [3]

4 Answer the whole of this question on a sheet of graph paper.
A table of values for $y = \frac{6}{x^3}$ is given below.
(The values of $y$ are correct to 1 decimal place.)

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.8</th>
<th>0.8</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$p$</td>
<td>0.7</td>
<td>1.5</td>
<td>2.7</td>
<td>9</td>
<td>$\omega$</td>
<td>$\omega$</td>
<td>$\omega$</td>
<td>$\omega$</td>
<td>2.7</td>
<td>1.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

(a) Calculate the values of $p, q$ and $r$. [3]

(b) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 1 cm to represent 1 unit on the $y$-axis, draw the graph of $y = \frac{6}{x^3}$ for $-4 \leq x \leq -0.8$ and $0.8 \leq x \leq 4$. [5]

(c) Draw the line $y = 2x + 7$ on your graph. [3]

(d) The graphs meet when $2x + 7 = \frac{6}{x^3}$.
Write down the three solutions of this equation, giving your answers correct to 1 decimal place. [3]

(e) By drawing a suitable tangent to your curve, estimate the gradient of the curve when $x = 2$. [4]
In this question, give all your probabilities as fractions in their lowest terms.

(a)

Six chairs are placed in a row.

Alain is equally likely to sit on any one of the chairs.

(i) What is the probability that Alain sits on one of the end chairs?

(ii) After Alain has sat down, Bernard chooses any chair at random.

What is the probability that Bernard sits next to Alain,

(a) if Alain is sitting at an end,

(b) if Alain is not sitting at an end?

(iii) Copy the tree diagram and write the probabilities on each branch.

(iv) Find the probability that Bernard sits next to Alain, wherever Alain sits.

(b)

The six chairs are placed in a circle and Alain and Bernard sit down.

What is the probability that Bernard sits next to Alain?

(c) There are $n$ chairs in a circle and Alain and Bernard sit down.

The probability that Bernard sits next to Alain is $\frac{1}{4}$.

Find the value of $n$. 

[1] [1] [1] [3] [3] [1] [2]
6

(i) The diagram shows a hollow cone with base radius \( AC = 3 \text{ cm} \) and edge \( OA = 18 \text{ cm} \).

Calculate (a) the height \( OC \), \([2]\)
(b) angle \( AOC \), \([2]\)
(c) the circumference of the base. \([1]\)

\( \pi \) is approximately 3.142.

(ii) The cone is cut along the line \( OA \) and opened out to form the sector \( AOA' \).

Calculate (a) the circumference of a circle of radius 18 cm, \([1]\)
(b) angle \( AOA' \). \([3]\)

The top part of a solid cone is removed.

The height of the remaining solid is half the height of the original cone.

(i) Write down, in the form \( 1 : \pi \), the ratios

(a) the base radius of the cone removed : the base radius of the original cone, \([1]\)
(b) the curved surface area of the cone removed : the curved surface area of the original cone, \([1]\)
(c) the volume of the cone removed : the volume of the original cone. \([1]\)

(ii) The curved surface area of the original cone was \( 24\pi \text{ cm}^2 \).

Calculate, in terms of \( \pi \), the curved surface area of the remaining solid. \([2]\)

(iii) The volume of the original cone was \( V \text{ cm}^3 \).

Give the volume of the remaining solid in terms of \( V \). \([2]\)
7 (a) The masses, in kilograms, of the 11 members of a soccer team are given below.

74, 70, 68, 69, 74, 62, 61, 65, 67, 73, 64.

Find (i) the median,
(ii) the mode,
(iii) the mean.

(b) 5 numbers have a median of 17, a mode of 19 and a mean of 14. Find a possible set of 5 positive integers which satisfy these conditions.

(c) The heights of trees were estimated to the nearest metre and the results recorded in the table below.

<table>
<thead>
<tr>
<th>Height in metres</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–4</td>
<td>8</td>
</tr>
<tr>
<td>5–8</td>
<td>7</td>
</tr>
<tr>
<td>9–12</td>
<td>5</td>
</tr>
</tbody>
</table>

Calculate an estimate of the mean height.

(d) Extra information was later found about the frequency of trees of height 13–20 metres. It is shown in the histogram below.

(i) How many trees are of height 13–20 metres?
(ii) In which class interval does the median now lie?
8 (a) 

In the right angled triangle $PQR$, $PR = x\text{ cm}$ and $QR = (x + 1)\text{ cm}$. 

The area of the triangle $PQR$ is $5\text{ cm}^2$. 

(1) Show that $x^2 + x - 10 = 0$. 

(2) Solve the equation $x^2 + x - 10 = 0$, giving your answers correct to 1 decimal place. Hence write down the length of $PR$. 

(b) 

In triangle $ABC$, angle $ACB = \ldots^\circ$, $AC = y\text{ cm}$ and $BC = (y + 2)\text{ cm}$. 

(1) Use the cosine rule to find an expression for $AB^2$ in terms of $y$. 

(2) When $AB = 7\text{ cm}$, show that $y^2 + 2y - 15 = 0$. 

(3) Factorise $y^2 + 2y - 15$. 

(4) Solve the equation $y^2 + 2y - 15 = 0$. 

Hence write down the lengths of $AC$ and $CB$. 

9 Give exact answers to each part of this question. 

It is given that $1^2 + 2^2 + 3^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$. 

(a) Substitute $k = 100$ in the formula above to find the value of $1^2 + 2^2 + 3^2 + \ldots + 100^2$. 

(b) $2^2 + 4^2 + 6^2 + \ldots + 100^2 = 2^2(1^2 + 2^2 + 3^2 + \ldots + n^2)$. 

(i) Write down the value of $n$. 

(ii) Hence find the value of $2^2 + 4^2 + 6^2 + \ldots + 100^2$. 

(c) Use your answers to parts (a) and (b) (ii) to find the value of $1^2 + 3^2 + 5^2 + \ldots + 99^2$. 

(d) Use some of your previous answers to find the value of 

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots + 99^2 - 100^2.$$ 


INSTRUCTIONS TO CANDIDATES

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Answer all questions.

Write your answers and working on the separate answer paper provided.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

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Electronic calculators should be used.

Three figure accuracy is required in your answers except where stated otherwise.
The diagram shows a trench which has been dug out of level ground so that a cylindrical water pipe can be laid.

The cross-section, ABCD, of the trench is a trapezium with horizontal sides of length 1.1 m and 0.8 m and height 0.7 m. The length of the trench is 500 m.

(a) Calculate the volume of earth removed. [3]

(b) If 1 m$^3$ of earth has a mass of 1.8 tonnes, calculate the mass of earth removed. [2]

(c) The diameter of the pipe is 0.5 m. After the pipe has been laid earth is replaced until the ground is again level. Calculate the percentage of the earth which is not replaced. [π is approximately 3.142] [4]

(d) If water flows through the pipe at 0.8 m/s, how many litres will flow through the pipe in 1 hour? [1 m$^3$ = 1000 litres.] [3]
The end, $A$, of a pendulum, $OA$, moves along the arc $AB$ as shown in the diagram. The length of the pendulum is $h$ metres and the time, $t$ seconds, taken to move from $A$ to $B$ is given by

$$t = \pi \sqrt{\frac{h}{9.81}}.$$

[$\pi$ is approximately 3.142]

(a) Find $t$ when $h = 1.6$.  

(b) Find the length of a pendulum which takes 1 second to move from $A$ to $B$.  

(c) Write $h$ in terms of $\pi$ and $t$.  

(d) If $h = 1$ and the arc length, $AB$, is 1 m, calculate

(i) angle $AOB$,  

(ii) the area of the sector $AOB$.  

[Turn over]
3 Answer the whole of this question on a sheet of graph paper.

\[ f(x) = 6 - x - x^2 \quad \text{and} \quad g(x) = x^3. \]

(a) The table shows some values of \( f(x) \) for \(-4 \leq x \leq 3\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>( p )</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>( q )</td>
</tr>
</tbody>
</table>

Calculate the values of \( p \) and \( q \). \[2\]

(b) The table shows some values of \( g(x) \) for \(-2 \leq x \leq 2\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>-8</td>
<td>-3.4</td>
<td>( r )</td>
<td>-0.1</td>
<td>0</td>
<td>0.1</td>
<td>1</td>
<td>3.4</td>
<td>( s )</td>
</tr>
</tbody>
</table>

Calculate the values of \( r \) and \( s \). \[1\]

(c) Using a scale of 2 cm to represent 1 unit, draw an \( x \)-axis from \(-4 \) to \( 3 \) and, using a scale of 1 cm to represent 1 unit, draw a \( y \)-axis from \(-8 \) to \( 8 \). \[1\]

(d) On the same grid draw the graphs of

(i) \( y = f(x) \) for \(-4 \leq x \leq 3\), \[4\]

(ii) \( y = g(x) \) for \(-2 \leq x \leq 2\). \[4\]

(e) Use your graphs

(i) to solve the equation \( 6 - x - x^2 = 2 \), \[2\]

(ii) to write down the coordinates of the point of intersection of \( y = f(x) \) and \( y = g(x) \), giving your answers correct to 1 decimal place. \[1\]

(f) By drawing a suitable straight line, estimate the gradient of the graph of \( y = f(x) \) at the point \((-2, 4)\). \[3\]

(ii) Deduce an estimate of the gradient of the graph \( y = f(x) \) at the point \((1, 4)\). \[1\]
The diagram represents three straight roads which surround a village. The bearing of $A$ from $C$ is $021^\circ$. Angle $ACB = 41^\circ$. The lengths of the roads $CA$ and $CB$ are $450$ m and $600$ m respectively.

(a) Calculate the bearing of (i) $B$ from $C$, [1]
(ii) $C$ from $A$. [2]

(b) Calculate how far $A$ is north of $C$. [3]

(c) Calculate the length of the road $AB$. [4]

(d) The area $ABC$ contains homes for 374 people. Calculate the average number of people per hectare in the area. (1 hectare = 10 000 m$^2$.) [5]

5 Answer the whole of this question on a sheet of graph paper.

(a) Draw $x$ and $y$ axes from $-5$ to $5$ using a scale of $1$ cm to represent $1$ unit on each axis. Draw triangle $ABC$ with $A(1, 1)$, $B(4, 1)$ and $C(4, 2)$. [2]

(b) (i) Draw the image of triangle $ABC$ when it is rotated $90^\circ$ anticlockwise about the origin. Label this image $A_1B_1C_1$. [2]

(ii) Triangle $A_1B_1C_1$ is translated by the vector $(3, 1)$. Draw and label this image $A_2B_2C_2$. [2]

(iii) Describe fully the single transformation which maps triangle $ABC$ onto triangle $A_2B_2C_2$. [3]

(c) (i) Draw the image of triangle $ABC$ under the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$. Label this image $A_3B_3C_3$. [4]

(ii) Describe fully the single transformation which maps triangle $ABC$ onto triangle $A_2B_2C_2$. [2]
6 In the triangle \(ABC\), \(AB = x\) cm. The side \(AC\) is 3 cm shorter than \(AB\) and the side \(BC\) is 5 cm shorter than \(AB\).

(a) (i) Show that the perimeter of the triangle, \(p\) cm, is given by

\[ p = 3x - 8. \]

(ii) The perimeter is \(2\frac{1}{4}\) times the length of \(AB\).

Find the length of \(AB\). \[2\]

(iii) Given that angle \(ACB = 83.2^\circ\), calculate the smallest angle of the triangle, giving your answer correct to the nearest degree. \[4\]

(b) (i) If, instead, the triangle \(ABC\) is right angled, show that

\[ x^2 - 16x + 34 = 0. \]

(ii) Solve the equation

\[ x^2 - 16x + 34 = 0, \]
giving your answers correct to 2 decimal places. \[5\]

(iii) Hence find the lengths of the sides of the right-angled triangle. \[2\]

7 Answer the whole of this question on a sheet of graph paper.

720 runners take part in a half-marathon race and their times are given in the table.

<table>
<thead>
<tr>
<th>Time ((t)) minutes</th>
<th>(60 &lt; t \leq 90)</th>
<th>(90 &lt; t \leq 105)</th>
<th>(105 &lt; t \leq 120)</th>
<th>(120 &lt; t \leq 150)</th>
<th>(150 &lt; t \leq 210)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of runners</td>
<td>152</td>
<td>200</td>
<td>120</td>
<td>120</td>
<td>128</td>
</tr>
</tbody>
</table>

(a) A runner is selected at random. What is the probability that the runner took more than 2 hours to complete the race? Give your answer as a fraction in its lowest terms. \[2\]

(b) \(\frac{1}{4}\) of the runners have been sponsored for charity. A runner is selected at random. What is the probability that this runner was sponsored and did not take more than 2 hours to complete the race? Give your answer as a decimal. \[3\]

(c) Calculate an estimate of the mean running time. \[4\]

(d) Draw an accurate histogram to represent this information. Use a scale of 1 cm to represent 15 minutes on the \(t\)-axis. Use an area of 1 cm² to represent 10 runners. \[7\]
$O$ is the centre of the circle.

Angle $BOD = 132^\circ$.

The chords $AD$ and $BC$ meet at $P$.

(a) (i) Calculate angles $BAD$ and $BCD$. [2]

(ii) Explain why triangles $ABP$ and $CDP$ are similar. [1]

(iii) $AP = 6\, \text{cm}, \, PD = 8\, \text{cm}, \, CP = 3\, \text{cm}$ and $AB = 17.5\, \text{cm}$.

Calculate the lengths of $PB$ and $CD$. [4]

(iv) If the area of triangle $ABP$ is $n\, \text{cm}^2$, write down, in terms of $n$, the area of triangle $CPD$. [2]

(b) (i) The tangents at $B$ and $D$ meet at $T$.

Calculate angle $BTD$. [2]

(ii) Use $OB = 9.5\, \text{cm}$ to calculate the diameter of the circle which passes through $O$, $B$, $T$ and $D$, giving your answer to the nearest centimetre. [3]
In the diagram, $O$ is the origin, $ABC$ is a straight line and $M$ is the mid-point of $OA$.

\[ \overrightarrow{OA} = a, \quad \overrightarrow{OB} = b \quad \text{and} \quad \overrightarrow{AC} = 3\overrightarrow{AB}. \]

(a) Find, in terms of $a$ and/or $b$, in their simplest forms,

(i) $\overrightarrow{MA}$, \hspace{1cm} \text{[1]}

(ii) $\overrightarrow{AB}$, \hspace{1cm} \text{[1]}

(iii) $\overrightarrow{AC}$, \hspace{1cm} \text{[1]}

(iv) $\overrightarrow{MC}$, \hspace{1cm} \text{[1]}

(v) the position vector of $C$. \hspace{1cm} \text{[2]}

(b) It is also given that $\overrightarrow{MN} = \frac{1}{3}\overrightarrow{MC}$.

(i) Find $\overrightarrow{ON}$ in terms of $a$ and/or $b$. \hspace{1cm} \text{[2]}

(ii) Find the ratio $ON : NB$. \hspace{1cm} \text{[1]}
International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICS
PAPER 4
Wednesday 5 JUNE 1996 Morning 2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables (optional)

TIME 2 hours 30 minutes

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The total of the marks for this paper is 130.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

This question paper consists of 7 printed pages and 1 blank page.

[Turn over
1 Pierre won $12,000 and divided it in the ratio 3 : 2.

He used the larger amount to buy a car, and invested the remainder in a bank.

The bank paid simple interest at a rate of 8% per annum.

18 months later, he sold the car at a loss of 30%, and took his money and interest from the bank.

Calculate
(a) the amount Pierre paid for the car,
(b) the amount for which he sold the car,
(c) the total amount he took from the bank,
(d) the percentage of the $12,000 he then had left.

2

Two ships leave port P at the same time.

One ship sails 60 km on a bearing of 030° to position A.

The other ship sails 100 km on a bearing of 110° to position B.

(a) Calculate
(i) the distance AB,
(ii) angle PAB,
(iii) the bearing of B from A.

(b) Both ships took the same time, t hours, to reach their positions.

The speed of the faster ship was 20 km/h.

Write down
(i) the value of t,
(ii) the speed of the slower ship.
3 (a) Solve for \( x \) and \( y \)

\[
\begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix} \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}.
\]

(b) Find the inverse of the matrix \( \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix} \).

(c) Solve for \( r \) and \( u \)

\[
\begin{pmatrix} 3t & u \\ -t & 3u \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \end{pmatrix}.
\]

4 Give your answers to this question as fractions in their lowest terms.

There are 21 students in a class. 12 are boys and 9 are girls.
The teacher chooses two students at random.

(a) If the first student chosen is a boy, explain why the probability that the second student chosen is also a boy is \( \frac{11}{20} \).

(b) Copy the tree diagram below. Write the correct probability on each branch.

```
first student  second student
          \  /  \  \\
       boy  11/20  boy
\ /
boy  girl
\ /
       girl  boy
```

(c) What is the probability that
   (i) both students are boys,
   (ii) both students are girls,
   (iii) one is a boy and one is a girl?

(d) The teacher chooses a third student at random.
    What is the probability that
    (i) all three students are boys,
    (ii) at least one of the three students is a girl?
The diagram shows a pyramid with a square base $ABCD$. The vertex $E$ is vertically above the corner $A$. $AB = BC = CD = DA = 4\text{ cm}$ and $AE = 3\text{ cm}$.

(a) Calculate the length of
   (i) $EB$,  \hfill [1]
   (ii) $AC$, \hfill [2]
   (iii) $EC$.  \hfill [2]

(b) Construct an accurate net of this pyramid. \hfill [6]

6

A rectangle is $(x + 3)\text{ cm}$ long and $y\text{ cm}$ wide. The perimeter of the rectangle is $20\text{ cm}$.

(a) Show that $y = 7 - x$.  \hfill [2]

(b) The area of the rectangle is $19\text{ cm}^2$. Show that

$$x^2 - 4x - 2 = 0.$$  \hfill [3]

(c) Solve the equation $x^2 - 4x - 2 = 0$. Give your answers to 2 decimal places.  \hfill [4]

(d) Write down the length and the width of the rectangle.  \hfill [2]
The table shows the length of time of 100 telephone calls.

<table>
<thead>
<tr>
<th>Time (t minutes)</th>
<th>0 &lt; t ≤ 1</th>
<th>1 &lt; t ≤ 2</th>
<th>2 &lt; t ≤ 4</th>
<th>4 &lt; t ≤ 6</th>
<th>6 &lt; t ≤ 8</th>
<th>8 &lt; t ≤ 10</th>
<th>10 &lt; t ≤ 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of calls</td>
<td>12</td>
<td>14</td>
<td>20</td>
<td>14</td>
<td>12</td>
<td>18</td>
<td>10</td>
</tr>
</tbody>
</table>

(a)  (i) Calculate an estimate of the mean time, in minutes, of a telephone call. [4]

(ii) Write your answer in minutes and seconds, to the nearest second. [1]

(b) Make a cumulative frequency table for the 100 calls.

Start it like this.

<table>
<thead>
<tr>
<th>Time (t minutes)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>≤ 1</td>
<td>12</td>
</tr>
<tr>
<td>≤ 2</td>
<td>26</td>
</tr>
<tr>
<td>≤ 4</td>
<td>[3]</td>
</tr>
</tbody>
</table>

(c) Draw a cumulative frequency diagram on a sheet of graph paper. Use a scale of 1 cm to represent 1 unit on the horizontal t-axis and 2 cm to represent 10 units on the vertical axis. [4]

(d) Use your graph to find, correct to the nearest 0.1 minute,

(i) the median time, [1]

(ii) the upper quartile, [1]

(iii) the interquartile range. [2]
A circle, centre $C$, has radius 4 cm.
The length of the arc $ADB$ is $\frac{4\pi}{3}$ cm.

(i) Show that angle $ACB = 60^\circ$.

(ii) Calculate the area of sector $ACBD$.
[\pi \text{ is approximately } 3.142.]

(iii) Calculate the area of triangle $ACB$.

(iv) Find the area of the shaded segment $ADB$, correct to 2 decimal places.

A circle, centre $O$, has radius 10 cm.
The chord $AB = 4$ cm.

(i) Show that angle $AOB = 23.1^\circ$, to the nearest $0.1^\circ$.

(ii) Calculate the area of the shaded segment $AEB$.

The two circles in parts (a) and (b) are placed together so that they intersect at $A$ and $B$.
Write down the shaded area enclosed by the arcs $ADB$ and $AEB$. [1]
Answer the whole of this question on a sheet of graph paper.

The tables below give values of \( f(x) \) and \( g(x) \), correct to 1 decimal place.

\[
f(x) = \frac{12}{x} \quad \quad \quad g(x) = (x - 8)(2 - x)
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>( p )</td>
<td>1.7</td>
<td>( r )</td>
<td>1.3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>0</td>
<td>( s )</td>
<td>8</td>
<td>( t )</td>
<td>8</td>
<td>5</td>
<td>( 0 )</td>
<td>( u )</td>
</tr>
</tbody>
</table>

(a) Calculate the values of \( p, q, r, s, t \) and \( u \). [3]

(b) Using a scale of 2 cm to represent 1 unit, draw an \( x \)-axis for \( 0 \leq x \leq 9 \) and using a scale of 1 cm to represent 1 unit, draw a \( y \)-axis for \( -8 \leq y \leq 10 \).

Draw the graphs of \( y = f(x) \) and \( y = g(x) \) for \( 2 \leq x \leq 9 \) on the same grid. [7]

(c) (i) Show that the equation which is satisfied by the points of intersection of the graphs can be simplified to

\[
x^3 - 10x^2 + 16x + 12 = 0.
\]

(ii) Write down the two solutions to this equation which can be found from your graphs. Give your answers correct to 1 decimal place. [2]

(d) Draw the tangent to \( y = g(x) \) at the point \((7, 5)\).

Use this to estimate the gradient of the curve \( y = g(x) \) at this point. [4]

10 (a) (i) Write down the next 2 terms in the sequence

\[
\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \ldots, \ldots
\]

(ii) This sequence can be written in the form

\[
\frac{a}{b}, \frac{b}{a+b}, \frac{a+b}{a+2b}, \ldots, \ldots
\]

Write down the next two terms of the sequence in terms of \( a \) and \( b \). [2]

(b) A different sequence follows the pattern

\[
\frac{1}{x}, \frac{2}{x+1}, \frac{3}{x+2}, \frac{4}{x+3}, \ldots, \ldots
\]

(i) Write down the next two terms of this sequence. [1]

(ii) Write down the 100th term of this sequence. [1]

(iii) Find \( x \) if the tenth term equals \( \frac{1}{2} \). [2]

(c) Simplify \( \frac{2}{x+1} - \frac{3}{x+2} \). [3]
INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer all questions.

Write your answers and working on the separate answer paper provided.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [  ] at the end of each question or part question.

The total of the marks for this paper is 130.

Electronic calculators should be used.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place.
\(ABC\) is a tangent to a circle, centre \(O\).
\(BOD\) is a diameter, and \(DEC\) is a straight line.
Angle \(BOE = 50^\circ\).

(a) Calculate
   (i) angle \(BDE\),
   (ii) angle \(OEC\),
   (iii) angle \(BCE\).

(b) \(F\) is any point on the minor arc \(DE\). Calculate angle \(DFE\).

---

2 (a) The probability that a student is left-handed is 0.15.

(i) What is the probability that the student is right-handed? [1]

(ii) Two students are chosen at random. What is the probability that both are left-handed? [2]

(b) A family with three children could have a girl first, followed by a boy, followed by another boy. This could be written \(GBB\).

(i) Use the letters \(B\) and \(G\) to make a list of all possible combinations in families with three children. [3]

(ii) Assume that each of the combinations that you have listed is equally likely.

A family with three children is chosen at random.

Find the probability that
(a) it contains at least one girl,
(b) it contains exactly two girls,
(c) the oldest and the youngest are of the same sex. [3]
A glass is in the shape of a hemisphere of radius 3 cm and a cylinder of radius 3 cm and height 4 cm. The glass is filled to the top with water.

(a) Calculate the volume of water in the glass. [4]

[Volume of a sphere of radius $r$ is $\frac{4}{3} \pi r^3$]

[Volume of a cylinder of radius $r$ and height $h$ is $\pi r^2 h$. $\pi$ is approximately 3.142.]

(b) (i) How many identical glasses can be filled from a 0.7 litre bottle of water? [3]

(ii) How much water will be left in the bottle? [2]

4 Answer the whole of this question on a sheet of graph paper.

(a) Draw axes from $-5$ to $+5$, using a scale of 1 cm to represent 1 unit on each axis.

(i) Plot the points $A(5, 0)$, $B(1, 3)$ and $C(-1, 2)$ and draw triangle $ABC$. [1]

(ii) Plot the points $A'(3, 4)$, $B'(3, -1)$ and $C'(1, -2)$ and draw triangle $A'B'C'$. [1]

(b) (i) Draw and label the line $l$ in which triangle $A'B'C'$ is a reflection of triangle $ABC$. [2]

(ii) Write down the equation of the line $l$. [2]

(iii) Find the values of $p$, $q$, $r$ and $s$ such that

\[
\begin{pmatrix} A & B & C \\ A' & B' & C' \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} & \begin{pmatrix} 5 & 1 & -1 \\ 0 & 3 & 2 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 3 & 3 & 1 \\ 4 & -1 & -2 \end{pmatrix} \] [4]

(iv) What transformation does the matrix \( \begin{pmatrix} p & q \\ r & s \end{pmatrix} \) represent? [2]

(c) Reflect triangle $A'B'C'$ in the $y$-axis. Label the new triangle $A''B''C''$. [2]

(d) If triangle $ABC$ is rotated about the origin, it will map onto triangle $A''B''C''$. What is the angle of rotation? [2]
Three friends live in houses $A$, $B$ and $C$. House $C$ is 100 metres from $A$ and 120 metres from $B$.

(a) Calculate, showing all your working,

(i) angle $BAC$,

(ii) the bearing of $A$ from $C$,

(iii) the shortest distance of $C$ from the road.

(b) (i) With the line $AB$ near the centre of your page, use a scale of 1 cm to represent 20 m to make an accurate scale drawing of the diagram.

(ii) House $D$ is to be built on the opposite side of the road to $C$. It must be at least 80 metres from $A$ and at least 100 metres from $B$. It must not be more than 60 metres from the road. Shade the area on your diagram where house $D$ can be built.

6 In a set of three numbers, the first is a positive integer, the second is 3 more than the first and the third is the square of the second.

(a) The first number in the set is $x$.

Write the second and third numbers in terms of $x$.

(b) The sum of the three numbers is 77.

(i) Write down an equation in $x$.

(ii) Show that your equation simplifies to $x^2 + 8x - 65 = 0$.

(iii) Solve the equation $x^2 + 8x - 65 = 0$.

(iv) Write down the three numbers.
7 Answer the whole of this question on a sheet of graph paper.

Arnie and Bernie are tailors. They make \( x \) jackets and \( y \) suits each week. Arnie does all the cutting, and Bernie does all the sewing. To make a jacket takes 5 hours of cutting and 4 hours of sewing. To make a suit takes 6 hours of cutting and 10 hours of sewing. Neither tailor works for more than 60 hours a week.

(a) For the sewing, show that 
\[ 2x + 5y \leq 30. \]

(b) Write down another inequality in \( x \) and \( y \) for the cutting.

(c) They make at least 8 jackets each week. Write down another Inequality.

(d) (i) Draw axes from 0 to 16, using 1 cm to represent 1 unit on each axis.
(ii) On your grid, show the information in parts (a), (b) and (c). Shade the unwanted regions.

(e) The profit on a jacket is $20 and on a suit is $100. Calculate the maximum profit that Arnie and Bernie can make in a week.

8 Answer the whole of this question on a sheet of graph paper.

The table shows the amount of money, \( $x \), spent on books by a group of students.

<table>
<thead>
<tr>
<th>Amount spent ($x$)</th>
<th>( 0 &lt; x \leq 10 )</th>
<th>( 10 &lt; x \leq 20 )</th>
<th>( 20 &lt; x \leq 30 )</th>
<th>( 30 &lt; x \leq 40 )</th>
<th>( 40 &lt; x \leq 50 )</th>
<th>( 50 &lt; x \leq 60 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>11</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Calculate an estimate of the mean amount of money per student spent on books.

(b) Use the information in the table above to find the values of \( p \), \( q \) and \( r \) in the following cumulative frequency table.

<table>
<thead>
<tr>
<th>Amount spent ($x$)</th>
<th>( x \leq 10 )</th>
<th>( x \leq 20 )</th>
<th>( x \leq 30 )</th>
<th>( x \leq 40 )</th>
<th>( x \leq 50 )</th>
<th>( x \leq 60 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative frequency</td>
<td>0</td>
<td>4</td>
<td>( p )</td>
<td>( q )</td>
<td>( r )</td>
<td>40</td>
</tr>
</tbody>
</table>

(c) Using a scale of 2 cm to represent 10 units on each axis, draw a cumulative frequency diagram.

(d) Use your diagram
(i) to estimate the median amount spent,
(ii) to find the upper and lower quartiles, and the inter-quartile range.
9 (a) Write as a single fraction
\[
\frac{2x + 1}{3} - \frac{x - 1}{2}
\]
[4]

(b) (i) Factorise \(x^2 - 5x + 6\).
(ii) Simplify
\[
\frac{x^2 - 5x + 6}{x^2 + x - 6}
\]
[3]

(c) Solve the equation
\[3x^2 = 7x - 1.\]

Show all your working and give your answers correct to 2 decimal places. [5]

Diagram 1 shows a regular tetrahedron \(WXYZ\) with all sides of length 6 cm.
Diagram 2 shows the base \(XYZ\) of the tetrahedron.
'O' is the centre of the base, \(M\) is the mid-point of \(XZ\), and \(N\) is the mid-point of \(XY\).

(a) Write down the size of angle \(OZM\). [1]

(b) Show that, correct to 4 significant figures, the length of \(OZ\) is 3.464 cm. [3]

(c) Calculate the height, \(OW\), of the tetrahedron. [3]

(d) Calculate the volume of the tetrahedron.
[Volume of a tetrahedron = \(\frac{1}{3}\) base area \(\times\) height.]

(e) Calculate the angle between the edge \(WZ\) and the base \(XYZ\). [3]
11 (a) As the product of its prime factors, $1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$. Write 135, 210 and 1120 as the product of their prime factors.

(b) Copy this grid onto your answer paper:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = 1$</td>
<td>$b = $</td>
<td>$c = $</td>
</tr>
<tr>
<td></td>
<td>$d = $</td>
<td>$e = $</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f = $</td>
</tr>
<tr>
<td>$g = $</td>
<td>$h = $</td>
<td>$i = 8$</td>
</tr>
</tbody>
</table>

The nine digits $1, 2, 3, 4, 5, 6, 7, 8, 9$ are to be placed in your grid in such a way that the following four statements are all true.

\[
\begin{align*}
    a \times b \times d \times e &= 135. \\
    b \times c \times e \times f &= 1080. \\
    d \times e \times g \times h &= 210. \\
    e \times f \times h \times i &= 1120.
\end{align*}
\]

The digits 1 and 8 have already been placed for you.

Use your answers to part (a) to answer the following questions.

(i) Which is the only digit, other than 1, that is a factor of 135, 1080, 210 and 1120? [1]

(ii) Which is the only letter to appear in all four statements above? [1]

(iii) 7 is a factor of only two of the numbers 135, 1080, 210 and 1120. Which two? [1]

(c) Now complete the grid you have drawn on your answer paper. [3]
TIME  2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer all questions.

Write your answers and working on the separate answer paper provided.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 130.

Electronic calculators should be used.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
1 A cinema has 200 seats. Ticket prices are $5 for an adult and $2.50 for a child.
(a) One evening, 80% of the seats in the cinema are occupied.
Twenty of the people present are children.
Calculate the total money taken from the sale of tickets. [3]
(b) Another evening, $x$ children are present and all the seats are occupied.
The money taken for tickets is $905.
(i) Write down an equation in $x$. [2]
(ii) Calculate the value of $x$. [2]
(c) The money taken for tickets for a week is $10,800.
This sum is divided between costs, wages and profit in the ratio 2 : 3 : 7.
Calculate
(i) the profit for the week, [2]
(ii) the simple interest earned if this profit is invested at a rate of 5% per annum for 4 months. [3]

2

\[2.8\text{ cm} \quad C \quad 12.1\text{ cm}\]
\[
\begin{array}{c}
B \\
\hline
4.3\text{ cm} \quad 110^\circ \quad 8\text{ cm} \quad E \\
\hline
D
\end{array}
\]

In a pentagon $ABCDE$, $AB = 4.3\text{ cm}$, $BC = 2.8\text{ cm}$, $CD = 12.1\text{ cm}$, $DE = 6\text{ cm}$ and $AE = 8\text{ cm}$.
Angle $BAE = 110^\circ$ and the diagonal $BD$ is parallel to $AE$.
(a) Construct the pentagon accurately. Measure and write down the length of $BD$. [6]
(b) Measure any lengths you need from your diagram and use them to calculate the area of the pentagon. Show all your working clearly. [5]
A circular board is divided into twelve equal sectors numbered from 1 to 12. A dart is thrown and lands on the board. Assume that it is equally likely to land anywhere in the circle. The score is the number in the sector where the dart lands.

(a) When one dart is thrown, find the probability that the score is
   (i) a square number,
   (ii) a prime number or less than 6 or both.

(b) When two darts are thrown, the sum of the two scores is calculated.
   (i) List all the possible ways of scoring 21.
   (ii) Find the probability of scoring 21 with two darts. Give your answer as a fraction in its lowest terms.

(c) The table shows the scores when a player throws one dart 30 times.

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

For these scores, find
   (i) the mode,
   (ii) the median,
   (iii) the mean.

(d) To prevent damage to the wall, the board, which has a radius of 10 cm, is placed on a wooden square of side 30 cm. One dart is thrown by a beginner who is equally likely to hit anywhere within the square.

   (i) Calculate the area of the sector numbered 2. [π is approximately 3.142.]
   (ii) Calculate the probability that the beginner scores 2, giving your answer as a decimal.
Hussein travels 12 km from \( A \) to \( B \) on a bearing of 025°. He then travels due east for 14 km to \( C \).

(a) Show that angle \( ABC \) is 115°.  

(b) Calculate  

(i) the distance \( AC \),  
(ii) the angle \( BAC \),  
(iii) the bearing of \( A \) from \( C \).  

---

Triangles \( ABC \) and \( ADE \) are similar, with \( BC \) parallel to \( DE \). \( AC = 5 \), \( CE = 2x \), \( BC = x + 3 \) and \( DE = 4x + 1 \).

(i) Use the ratios of the sides of the similar triangles to write down an equation in \( x \).  
Hence show that \( 2x^2 - 9x + 10 = 0 \).  

(ii) Factorise \( 2x^2 - 9x + 10 \).  

(iii) Solve the equation \( 2x^2 - 9x + 10 = 0 \).  

(iv) When \( x \) is the larger of your two solutions, find the fraction \( \frac{\text{area of } \triangle ABC}{\text{area of } \triangle ADE} \) in its simplest form.

(b) \[ M = \begin{pmatrix} 2y + 1 & 3y - 4 \\ y & 2y + 3 \end{pmatrix}. \]

(i) Show that when the determinant of \( M \) is 10, then \( y^2 + 12y - 7 = 0 \).  

(ii) Solve this equation, showing all your working.  
Give your answers correct to 1 decimal place.
A child's toy consists of a cone inside a sphere. The radius of the sphere, $OA$, is 6 cm and the radius of the base of the cone, $AC$, is 3.6 cm.

[The volume of a sphere of radius $R$ is $\frac{4}{3} \pi R^3$.]
[The volume of a cone of base radius $r$ and height $h$ is $\frac{1}{3} \pi r^2 h$.]
[$\pi$ is approximately 3.142.]

(a) Show that $VOC$, the height of the cone, is 10.8 cm. [2]

(b) Calculate (i) the volume of the sphere, [2]
(ii) the volume of the cone, [2]
(iii) the percentage of the volume of the sphere not occupied by the cone. [3]

(c) The sphere rolls 3 metres across the floor in a straight line.

Calculate (i) the circumference of the sphere, [2]
(ii) the number of complete revolutions made by the sphere, [2]
(iii) the number of degrees through which the sphere must still turn in order to complete another revolution. [3]
7 Answer the whole of this question on a sheet of graph paper.

(a) \( f(x) = x^3 - 12x + 5 \).

The table below shows some values of \( x \) and \( f(x) \).

Find the value of \( a \) and the value of \( b \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3.5</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-11</td>
<td>4.1</td>
<td>14</td>
<td>19.4</td>
<td>21</td>
<td>( a )</td>
<td>5</td>
<td>-6</td>
<td>-11</td>
<td>-4</td>
<td>5.9</td>
</tr>
</tbody>
</table>

(b) Using 2 cm to represent 1 unit on the \( x \)-axis and 2 cm to represent 5 units on the \( y \)-axis, draw axes for \(-4 \leq x \leq 4 \) and \(-15 \leq y \leq 25 \).

Draw the graph of \( y = f(x) \) for \(-4 \leq x \leq 4 \).

(c) Use your graph to solve the equations
   (i) \( f(x) = 0 \),
   (ii) \( x^3 - 12x + 10 = 0 \).

(d) (i) Draw the tangent to your graph at \( x = -2.5 \).
    (ii) Estimate the gradient of the graph at \( x = -2.5 \).
    (iii) Write down another value of \( x \) where the graph has this gradient.
(a) Describe fully the single transformation which maps the pentagon $A$ onto
(i) $B$, [2]
(ii) $C$, [2]
(iii) $D$. [2]

(b) Find the matrix of the transformation which maps $A$ onto $D$. [2]

(c) Describe the single transformation which maps $D$ onto $C$. [2]

(d) Find the matrix of the transformation which maps $D$ onto $C$. [2]

(e) Find the equation of the line in which $B$ is reflected onto $C$. [2]

Q9 is printed on the back page.
Maria thinks of 3 possible savings schemes for her baby son.

Scheme A: save $10 on his 1st birthday, $20 on his 2nd birthday, $30 on his 3rd birthday, $40 on his 4th birthday, .......

Scheme B: save $1 on his 1st birthday, $2 on his 2nd birthday, $4 on his 3rd birthday, $8 on his 4th birthday, .......

Scheme C: save $1 on his 1st birthday, $4 on his 2nd birthday, $9 on his 3rd birthday, $16 on his 4th birthday, .......

She puts these ideas in a table.

<table>
<thead>
<tr>
<th>Birthday Scheme</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$10</td>
<td>$20</td>
<td>$30</td>
<td>$40</td>
</tr>
<tr>
<td>B</td>
<td>$1</td>
<td>$2</td>
<td>$4</td>
<td>$8</td>
</tr>
<tr>
<td>C</td>
<td>$1</td>
<td>$4</td>
<td>$9</td>
<td>$16</td>
</tr>
</tbody>
</table>

(a) Write down, for each of the Schemes A, B and C, the amount to be saved on
   (i) his 7th birthday,
   (ii) his nth birthday.

(b) The formulae for the total amount saved up to and including his nth birthday are as follows.
   Scheme A: total = $5n(n + 1).
   Scheme B: total = $(2^n - 1).
   Scheme C: total = $\frac{n(n + 1)(2n + 1)}{6}$.

(i) For each of the schemes A, B and C, find the total amount saved up to and including his 10th birthday.
(ii) Which scheme gives the smallest total amount of savings up to and including his 18th birthday?
(iii) Find the birthday when the scheme you have selected in part (b)(ii) first gives the smallest total amount of savings.
TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer all questions.

Write your answers and working on the separate answer paper provided.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 130.

Electronic calculators should be used.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place.
(a) A tin of soup is 11 centimetres high and has a diameter of 8 centimetres (Diagram 1).
Calculate the volume of the tin. \(\pi\) is approximately 3.142. \[2\]

(b) The tins are packed tightly in boxes of 12, seen from above in Diagram 2.
The height of each box is 11 centimetres.
(i) Write down the length and the width of the box. \[2\]
(ii) Calculate the percentage of the volume of the box which is not occupied by the tins. \[4\]

(c) A shopkeeper sells the tins of soup for $0.60 each.
By doing this he makes a profit of 25\% on the cost price.
Calculate the cost price of
(i) one tin of soup, \[2\]
(ii) a box of 12 tins. \[1\]

(d) The shopkeeper tries to increase sales by offering a box of 12 tins for $6.49.
At this price
(i) how much does a customer save by buying a box of 12 tins, \[1\]
(ii) what percentage profit does the shopkeeper make on each box of 12 tins? \[2\]
In triangle $XYZ$, $XY = 5$ cm, $XZ = 2$ cm and $YZ = 6$ cm.

(a) Triangle $XUV$ is the reflection, $M$, of triangle $XYZ$ in the line $l$. So $M(Y) = U$.

(i) Copy and complete the statement "$M(Z) = \ldots\ldots\ldots\ldots$". \hspace{1cm} [1]

(ii) Write down the length of $XU$ and the length of $XV$. \hspace{1cm} [2]

An enlargement, $E$, has centre $X$ and $E(XV) = XY$.

(iii) What is the scale factor of the enlargement? \hspace{1cm} [2]

(iv) $E(XU) = XW$. Calculate the length of $XW$ and the length of $YW$. \hspace{1cm} [2]

(v) Find two other angles in the diagram equal to angle $XYZ$. \hspace{1cm} [2]

(b) Using the triangle $XYZ$, calculate angle $YXZ$. \hspace{1cm} [4]
(a) The two spinners shown are each equally likely to land on any of their edges:

They are spun together to give a two digit number. The first spinner gives the tens digit and the second gives the units digit.

(i) List the twelve possible outcomes, starting with 11.
(ii) What is the probability that the outcome is (a) a multiple of 4,
     (b) a multiple of 5?

(b) A box contains six chocolates. The chocolates all look the same, but two have hard centres and four have soft centres.
   Alice takes a chocolate and eats it. Then Barbara takes a chocolate.
   (i) Copy and complete the tree diagram.

(ii) Find the probability that (a) both chose chocolates with hard centres,
     (b) one chose a chocolate with a hard centre and the other chose a chocolate with a soft centre,
     (c) Barbara chose a chocolate with a hard centre.
The diagram shows the face of a coin. It is a regular seven-sided polygon. $O$ is the centre of the face of the coin and $AO = 1.5$ centimetres.

(a) Show that, correct to two decimal places, angle $OAB = 64.29^\circ$.

(b) Giving your answers correct to three significant figures, calculate:
   (i) the length of the perpendicular, $OX$, from $O$ to $AB$,
   (ii) the length of $AB$,
   (iii) the area of triangle $AOB$,
   (iv) the area of the whole face.

(c) The coin is 3 millimetres thick. Calculate its volume.

5  (a) Construct triangle $ABC$ with $BC = 10$ cm, $AB = 9$ cm and $AC = 7$ cm.

(b) Using a straight edge and compasses only, construct the perpendicular bisectors of $BC$ and $AC$. Label their point of intersection $O$.

(c) Draw perpendicular lines from $A$ to $BC$ and from $B$ to $AC$. Label their point of intersection $H$.

(d) Draw the line $OH$ and label its mid-point $N$.

(e) $M$ is the mid-point of $BC$. Mark the point $M$ on your diagram. Draw the line through $M$ and $N$ to meet $AH$ at $U$.

(f) What do you notice about the lengths $AU$, $UH$ and $OM$?

(g) What can you say about triangle $OMN$ and triangle $HUN$?

(h) With $N$ as centre, draw a circle with radius $NM$. Measure and write down this radius.
6. Answer the whole of this question on a sheet of graph paper.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-6</td>
<td>-8</td>
<td>-6</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) Using a scale of 2 centimetres to represent 1 unit on the x-axis, and a scale of 2 centimetres to
represent 4 units on the y-axis, plot the points given in the table above.
Join the points with a smooth curve. [4]

(b) (i) On the same grid, draw the line with gradient 2 through the point (3, 0).
Label it L. [2]
(ii) Write down the equation of the line L. [2]
(iii) Write down the coordinates of the two points at which the line L meets the curve. [2]

(c) Draw the tangent to the curve at the point (3, -6) and use it to find an estimate of the gradient
of the curve at that point. [4]

(d) The equation of the curve is $y = ax^2 + bx$, where $a$ and $b$ are integers.
Using some of the values from the table above, calculate $a$ and $b$. [4]
The cumulative frequency diagram shows the weekly amount spent on food by 100 families.

(a) Use the diagram to find, to the nearest dollar,
   (i) the median,  
   (ii) the upper and lower quartiles,  
   (iii) the 60th percentile.  

(b) (i) Work out the inter-quartile range.
   (ii) The range is $50. Write the inter-quartile range as a percentage of the range. 

(c) (i) Copy and complete this table on your answer paper for the whole graph.

<table>
<thead>
<tr>
<th>Weekly amount ($x)</th>
<th>Frequency (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40 &lt; x \leq 50$</td>
<td>...............</td>
</tr>
<tr>
<td>$50 &lt; x \leq 60$</td>
<td>...............</td>
</tr>
<tr>
<td>...............</td>
<td>...............</td>
</tr>
</tbody>
</table>

(ii) Which is the modal class?

(iii) Calculate an estimate of the mean weekly amount spent on food. Show all your working.

(iv) Explain how to get a more accurate estimate of the mean, using only data displayed in the diagram.
8 Answer the whole of this question on a sheet of graph paper. Using a scale of 1 centimetre to represent 1 unit on each axis, draw $x$ and $y$ axes from 0 to 16.

(a) On your grid draw triangle $T$ whose vertices are $(2, 2), (2, 4)$ and $(6, 4)$. [2]

(b) Triangle $S$ is the image of triangle $T$ under the transformation represented by the matrix $M = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$.

(i) Draw and label triangle $S$ on your diagram. [4]

(ii) Calculate the area of triangle $S$. [2]

(iii) Describe fully the single transformation represented by the matrix $M$. [3]

(c) (i) Find $M^{-1}$, the inverse of the matrix $M$. [2]

(ii) What is the image of triangle $S$ under the transformation represented by $M^{-1}$? [2]

9 A "Pythagorean triple" is a set of three whole numbers that could be the lengths of the three sides of a right-angled triangle.

(a) Show that $\{5, 12, 13\}$ is a Pythagorean triple. [1]

(b) Two of the numbers in a Pythagorean triple are 24 and 25. Find the third number. [2]

(c) The largest number in a Pythagorean triple is $x$ and one of the other numbers is $x - 2$.

(i) If the third number is $y$, show that $y = \sqrt{4x - 4}$. [3]

(ii) If $x = 50$, find the other two numbers in the triple. [2]

(iii) If $x = 101$, find the other two numbers in the triple. [2]

(iv) Find two other Pythagorean triples in the form $\{y, x - 2, x\}$, where $x < 40$. Remember that all three numbers must be whole numbers. [4]
International General Certificate / Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICS
PAPER 4
Wednesday 3 JUNE 1998 Afternoon 2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (1 sheet)
Mathematical tables (optional)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.
Answer all questions.
Write your answers and working on the separate answer paper provided.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142.

This question paper consists of 8 printed pages.
1. The ratio of men : women : children living in Newtown is 6 : 7 : 3. There are 42 000 women.

   (a) (i) How many children live in Newtown?  
   (ii) How many people altogether live in Newtown?

   (b) The 42 000 women is an increase of 20% on the number of women 10 years ago. Calculate how many women lived in Newtown 10 years ago.

   (c) 12 000 of the children attend school. 48% of them are boys.
      (i) Calculate the number of boys and the number of girls at school.
      (ii) The average age of the 12 000 children is exactly 10.54 years. The average age of the boys is exactly 10.35 years. Calculate the average age of the girls, correct to 2 decimal places.

2. Draw the line $AB$, 10 cm long, in the centre of a new page.

   Construct a quadrilateral $ABCD$ such that $AD = 7.2$ cm, angle $DAB = 82^\circ$, angle $ADC = 68^\circ$ and angle $ABC = 112^\circ$.

   (b) Use a straight edge and compasses only to construct the perpendicular bisectors of $AB$ and $AD$. These meet at $E$. Leave all construction lines on your diagram.

   (c) (i) Measure and write down the length $AE$.
      (ii) Construct the locus of all points which are this distance from $E$.
      (iii) Write down the special name of quadrilateral $ABCD$.

   (d) Shade the region inside the quadrilateral which is nearer to $A$ than it is to $B$, and nearer to $A$ than it is to $D$.  

Triangle $ABC$ has $AB = 10$ cm, $BC = 9.2$ cm and $AC = 11.6$ cm.

(a) Calculate angle $ABC$.  

(b) Use the sine rule to calculate angle $BAC$.

(c) The triangle is cut from a square piece of card of side 12 cm.
    Calculate the area of card remaining.

---

4. Answer the whole of this question on a sheet of graph paper.

A table of values for $y = x(x + 2)(x - 3)$ is given below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$a$</td>
<td>0</td>
<td>$b$</td>
<td>0</td>
<td>-6</td>
<td>$c$</td>
<td>0</td>
<td>24</td>
</tr>
</tbody>
</table>

(a) Calculate the values of $a$, $b$ and $c$.  

(b) Using a scale of 2 cm to represent 1 unit, draw an $x$-axis for $-3 \leq x \leq 4$ and using a scale of 2 cm to represent 5 units, draw a $y$-axis for $-20 \leq y \leq 25$.

Draw the graph of $y = x(x + 2)(x - 3)$.

(c) Use your graph to solve

(i) $x(x + 2)(x - 3) = 10,$

(ii) $x(x + 2)(x - 3) + 15 = 0.$

(d) Draw the line $y = 2x - 6$ on your graph.

(e) The graphs meet when $x(x + 2)(x - 3) = 2x - 6$.

(i) Show that this equation can be written as $x^3 - x^2 - 8x + 6 = 0$.

(ii) Write down the solutions of this equation.
5 Mamoud tries to repair a broken toy. Each time he tries the probability that he succeeds is 0.8. Each time he fails he tries again.

(a) Copy and complete the tree diagram below.

\[
\begin{array}{ccc}
1st \text{ try} & 2nd \text{ try} & 3rd \text{ try} \\
& 0.8 \quad \text{succeeds} & \text{succeeds} \quad & \text{succeeds} \\
& \text{succeeds} & \text{succeeds} \quad & \text{succeeds} \\
& \text{fails} & \text{fails} \quad & \text{fails} \\
& \text{fails} & \text{fails} \quad & \text{fails} \\
\end{array}
\]

(b) Find the probability that, to succeed, it takes

(i) exactly two tries,

(ii) one, two or three tries,

(iii) exactly five tries.

(c) Write down a formula for the probability that he has not succeeded after \( n \) tries.

6 (a) Write the expression \( \frac{100}{x-2} - \frac{100}{x} \) as a single fraction and simplify your answer.

(b) Rice costs \( x \) francs for one kilogram. How many kilograms can I buy for 100 francs?

(c) When rice costs \( (x - 2) \) francs for a kilogram, I can buy 5 more kilograms for 100 francs. Write down an equation in \( x \).

Show that it simplifies to \( x^2 - 2x - 40 = 0 \).

(d) (i) Solve the equation \( x^2 - 2x - 40 = 0 \), giving your answers correct to 2 decimal places. Show all your working.

(ii) Write down the original price of one kilogram of rice.
Six cylindrical bales of hay, each with radius 0.8 m and length 1.5 m, are stacked as shown in the diagram. The centres of three of the bales are marked $A$, $B$ and $C$.

(a)  (i) What type of triangle is $ABC$?  [1]
(ii) What is the length of $AB$?  [1]
(iii) Calculate $h$, the vertical height of the stack in metres (see diagram above).  [4]

(b) Calculate the total volume of hay in the stack.  [3]

(c)  

Ten bales of hay are stacked as shown in the diagram. Calculate the vertical height of this stack.  [3]
(a) Describe fully a single transformation which maps
   (i) both $G$ onto $C$ and $H$ onto $B$, [2]
   (ii) both $G$ onto $D$ and $H$ onto $C$, [2]
   (iii) both $G$ onto $C$ and $H$ onto $D$. [2]

(b) Write down the new positions of the points $G$ and $H$ when they are
    (i) rotated $90^\circ$ clockwise about $O$, [2]
    (ii) reflected in the line $y = x$. [2]

(c) The matrix $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

   (i) Describe fully the single transformation represented by $M$. [2]
   (ii) Write down the new positions of $G$ and $H$ under $M$. [2]

(d) The matrix $N = \begin{pmatrix} 4 & -3 \\ -2 & 2 \end{pmatrix}$.

   (i) Find $N^{-1}$, the inverse of $N$. [2]
   (ii) Write down the positions of $G$ and $H$ after the transformation represented by $NN^{-1}$ [2]
9 (a) A large number of people attended a meeting. Simon asked 50 of them how much their journey had cost. His results are shown in the table below.

<table>
<thead>
<tr>
<th>Cost ($x$)</th>
<th>$0 &lt; x \leq 5$</th>
<th>$5 &lt; x \leq 10$</th>
<th>$10 &lt; x \leq 15$</th>
<th>$15 &lt; x \leq 25$</th>
<th>$25 &lt; x \leq 35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of people</td>
<td>13</td>
<td>12</td>
<td>10</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

(i) Find the modal class. 

(ii) Calculate an estimate of the mean cost, correct to the nearest cent. 

(b) Simon asked everyone attending the meeting how long their journey had taken. He put his results in a table and drew a histogram. He used a scale of 1 cm to represent 5 minutes on the horizontal axis and an area scale of 1 cm$^2$ to represent 5 people. Unfortunately, both his graph and table, shown below, were damaged.

![Histogram diagram]

<table>
<thead>
<tr>
<th>Time of journey in minutes</th>
<th>$4 &lt; t \leq 10$</th>
<th>$10 &lt; t \leq 15$</th>
<th>$15 &lt; t \leq 20$</th>
<th>$20 &lt; t \leq 30$</th>
<th>$30 &lt; t \leq T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of people</td>
<td>18</td>
<td>20</td>
<td>$e$</td>
<td>$f$</td>
<td>24</td>
</tr>
</tbody>
</table>

(i) Use what remains of the histogram, shown in the diagram, to find the missing values $e$, $f$ and $T$. 

(ii) Find the height, in centimetres, of the first bar representing the 18 people in the $4 < t \leq 10$ minute class.
The hands of a clock move at constant speeds.
The short hour hand completes one revolution every 12 hours.
The long minute hand completes one revolution every hour.

(a) Calculate the obtuse angle between the hands at

(i) 12:30,

(ii) 06:15.

(b) Calculate the angle turned in one minute by

(i) the minute hand,

(ii) the hour hand.

(c) Write down the number of degrees turned in $t$ minutes by

(i) the minute hand,

(ii) the hour hand.

(d)

At 03:00 the angle between the hands is 90°. At $t$ minutes past 3 o'clock, the minute hand and the hour hand are pointing in exactly the same direction.

(i) Write down an equation in $t$.

(ii) Show that $t = 16\frac{1}{4}$.

(iii) Write $16\frac{1}{4}$ minutes in minutes and seconds correct to the nearest second.
International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICS 0580/4, 0581/4
PAPER 4
Friday 6 NOVEMBER 1998 Morning 2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical Instruments
Graph paper (3 sheets)
Mathematical tables (optional)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.
Answer all questions.
Write your answers and working on the separate answer paper provided.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place.
For \( \pi \), use either your calculator value or 3.142.

This question paper consists of 7 printed pages and 1 blank page.
1  (a) A furniture salesman earned $24,600 last year.
   (i) He had to pay 28% of this amount as tax. How much was left after tax? [2]
   (ii) His earnings of $24,600 were made up of $15,000 basic salary and the rest as commission. The
   commission was 6% of the value of the furniture that he sold. Work out the value of the furniture he sold. [3]

   (b) He bought a table from the shop where he worked. Its marked price was $560, but because he
   worked there he only paid $392. What percentage discount on the marked price was this? [3]

2  (a) The Earth is 149 million kilometres from the Sun. Write this number in standard form. [1]

   Give the answers to the other parts of this question in standard form.

   (b) The planet Neptune is 30 times as far from the Sun as the Earth is. How far is Neptune from the Sun? [2]

   (c) The planet Venus travels around the Sun in a circular orbit of radius 108 million kilometres.
   Calculate the distance that Venus travels in one complete orbit of the Sun. [For \( \pi \), use either your calculator value or 3.142.] [3]

   (d) The planet Jupiter travels \( 4.89 \times 10^9 \) kilometres to complete one orbit of the Sun. This takes
   twelve years. Calculate the average speed of Jupiter, in kilometres per hour. [Take 1 year as 365\( \frac{1}{2} \) days.] [4]

3  \[ f(x) = x^2 - 16 \quad \text{and} \quad g(x) = 5x + 2 \quad \text{for all values of } x. \]

   (a) Find
      (i) \( f(10) \), [1]
      (ii) \( f(-2) \). [1]

   (b) Find \( g^{-1}(x) \), the inverse of \( g(x) \). [2]

   (c) Find \( fg(x) \), giving the answer in its simplest terms. [3]

   (d) Find the two values of \( x \) for which \( f(x) = g(x) \).
       Give your answers correct to two decimal places. [5]
The diagram shows a regular octahedron. All the edges are 3 centimetres long.

(a) For this solid, write down the number of
    (i) faces,
    (ii) vertices,
    (iii) edges. \[3\]

(b) The octahedron is split into two equal parts.

    One of the parts is shown in the diagram on the right.

    Calculate
    (i) the length of $AC$,
    (ii) the vertical height $OH$,
    (iii) the angle between $OA$ and the base $ABCD$. \[2\] \[2\] \[3\]

(c) The volume of a pyramid is $\frac{1}{3}$ base area $\times$ height.

    Calculate the volume of the octahedron. \[3\]

5 Answer the whole of this question on a sheet of graph paper.

(a) Using a scale of 2 cm to represent 1 unit on each axis, draw an $x$-axis for $-4 \leq x \leq 4$ and a $y$-axis for $-3 \leq y \leq 5$.

    Draw the triangle $T$ whose vertices are $(-1, 3)$, $(4, 3)$ and $(-3, -1)$. \[2\]

(b) Triangle $S$ is the image of triangle $T$ under the transformation represented by the matrix

    \[ M = \begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix} \]

    (i) Calculate the coordinates of the vertices of the triangle $S$. \[4\]

    (ii) Draw and label triangle $S$ on your diagram. \[1\]

(c) Describe fully the single transformation which maps triangle $T$ onto triangle $S$. \[3\]
6  Give each of your answers to this question as a fraction.

Peter has 10 geranium plants. He knows that 5 will flower red, 3 pink and 2 white.

(a) What is the probability that the first plant to flower is pink? [1]

(b) Copy the tree diagram below. Write the correct probability on each branch.

(c) What is the probability that, of the first two plants to flower,
   (i) both are red, [1]
   (ii) one is red and the other is pink, [2]
   (iii) at least one is pink? [3]

(d) What is the probability that the first three plants to flower are all white? [2]

7 \[ \mathbf{p} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}. \]

(a) Find \( |\mathbf{p}| \), the length of the vector \( \mathbf{p} \). [2]

(b) Find, as a single column vector,
   (i) \( \mathbf{p} + \mathbf{q} + \mathbf{r} \), [2]
   (ii) \( 10\mathbf{q} - 2\mathbf{r} \). [4]

(c) Without using an accurate diagram, explain why the vector \( 10\mathbf{q} - 2\mathbf{r} \) is parallel to the vector \( \mathbf{p} \). [2]

(d) If \( a\mathbf{p} + b\mathbf{r} = 5\mathbf{q} \), find the value of \( a \) and the value of \( b \). [5]
Answer the whole of this question on a sheet of graph paper.

A table of values for \( y = \frac{4}{x^2} + x \) is given below.

(The values of \( y \) are correct to one decimal place.)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.8</th>
<th>( 1 )</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>0.3</td>
<td>1.6</td>
<td>5.5</td>
<td>( m )</td>
<td>3.3</td>
<td>3</td>
<td>( n )</td>
<td>4.3</td>
</tr>
</tbody>
</table>

(a) Calculate the values of \( l, m \) and \( n \). [3]

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of \( y = \frac{4}{x^2} + x \) for \(-2 \leq x \leq -0.8 \) and \( 1 \leq x \leq 4 \). [5]

(c) Use your graph to solve

(i) \( \frac{4}{x^2} + x = 0 \),

(ii) \( \frac{4}{x^2} + x = 4 \). [4]

(d) By drawing a suitable tangent to the curve, estimate the gradient of the curve when \( x = 1.5 \). [4]
**9 (a)**

**Diagram A**

$ABCDE$ is a semicircle of diameter 10 centimetres. $AC = CE$ and angle $ACE = 90^\circ$.

[For $\pi$, use either your calculator value or 3.142.]

Calculate

(i) the area of the semicircle,

(ii) the area of triangle $ACE$,

(iii) the area of the segment $ABC$.  

**Diagram B**

$PQ$ and $QR$ are tangents to a semicircle with centre $O$ and diameter 10 centimetres. $POR$ is a straight line, $PQ = QR$ and angle $PQR = 90^\circ$.

Calculate the area of triangle $PQR$.  

---

[2][3][2][3]
10 Answer the whole of this question on a sheet of graph paper.

400 apples were weighed. Their masses are given in the table below.

<table>
<thead>
<tr>
<th>Mass (m grams)</th>
<th>$80 &lt; m \leq 100$</th>
<th>$100 &lt; m \leq 110$</th>
<th>$110 &lt; m \leq 120$</th>
<th>$120 &lt; m \leq 130$</th>
<th>$130 &lt; m \leq 160$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>50</td>
<td>70</td>
<td>113</td>
<td>92</td>
<td>75</td>
</tr>
</tbody>
</table>

(a) Using a scale of 2 centimetres to represent 10 grams on the horizontal axis, and an area scale of 1 square centimetre to represent 5 apples, draw a histogram to display this data. [6]

(b) Calculate an estimate of the mean mass of the apples. [4]

(c) A supermarket will only buy apples which have a mass greater than 110 grams. What percentage of the apples does the supermarket buy? [3]

11 Look at this table of numbers.

<table>
<thead>
<tr>
<th>Row</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 2</td>
<td>3  5</td>
</tr>
<tr>
<td>Row 3</td>
<td>7  9 11</td>
</tr>
<tr>
<td>Row 4</td>
<td>13 15 17 19</td>
</tr>
</tbody>
</table>

(a) (i) Write down the sum of each of the first four rows. [1]

(ii) What is the special name of the numbers that you have written down? [1]

(iii) What is the sum of the numbers in the hundredth row? [2]

(b) (i) Write down the sum of all the numbers in the first two rows, in the first three rows, in the first four rows. [2]

(ii) What is the special name of the numbers that you have written down? [1]

(iii) What is the sum of all the numbers in the first ten rows? [2]

(c) What is the last number in the fifteenth row? [3]
International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICs
PAPER 4

Wednesday 9 June 1999
Afternoon 2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables (optional)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
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For π, use either your calculator value or 3.1416.
1. A football club asks all its members to vote 'Yes' or 'No' for a new stadium. They receive 48,790 'Yes' votes. The ratio 'Yes' votes : 'No' votes is 7:5.

(a) How many members voted?  

(b) There were 14,760 members who did not vote. What percentage of members did not vote?  

(c) To build the new stadium, 50% of the total number of members have to vote 'Yes'. Will the new stadium be built? Show working to explain your answer.  

2. For a certain type of tree, \( C = 2.5y \) where \( C \) is the circumference in centimetres and \( y \) is the age of the tree in years.

[The cross-section of the tree trunk is a circle. For \( \pi \), use either your calculator value or 3.142.]

(a) Estimate the age of a tree with a circumference of 100 cm.  

(b) Find the radius of the trunk of a 20 year old tree.  

(c) The cross-sectional area of a tree trunk is 1200 cm\(^2\). Find

(i) the radius of the tree,  
(ii) the age of the tree, to the nearest year.  

(d) A three year old tree was planted in 1971. Calculate the year in which the diameter of its trunk will be one metre.
3

(a)

ACE is an isosceles triangle. CH is perpendicular to AE. Angle CAH = 70° and AH = 5 cm. Calculate the length of AC correct to 5 significant figures. Show that it rounds to 14.62 cm.

(b)

Pentagon ABCDE is formed from the isosceles triangle ACE together with congruent triangles ABC and EDC. BC = 7 cm, angle BCA = angle ECD = 20° and angle CAE = 70°.

(i) Use AC = 14.62 cm and the cosine rule to calculate the length of BA.

(ii) Find the area of triangle ABC.

(c) Triangles ABC and CDE are folded over onto triangle ACE, as shown on the diagram below.

Calculate the unshaded area.
In summer the probability of a wet day is 0.25.
On a wet day, the probability of wind is 0.2.
On a dry day, the probability of wind is 0.4.
You may assume that the weather each day is independent of the weather the day before.

(a) Copy and complete the tree diagram below.

(b) When it is wet and windy, concerts have to be postponed until the next day.
Find the probability that Monday's concert
(i) has to be postponed,
(ii) takes place on Tuesday.

(c) Sailing boats can only sail on a windy day.
Find the probability that they cannot sail on Monday.

(d) On a dry day with no wind, the probability that the temperature is more than 30°C is 0.9.
On a wet or windy day this will not happen.
Find
(i) the probability that the temperature is more than 30°C on Monday,
(ii) the probability that on Monday, Tuesday and Wednesday the temperature is more than 30°C.
On the grid above there are seven identical shapes, labelled A to G. Use these letters to answer the questions below.

(a) Which two shapes are a reflection of each other in the line $x = -1$?  
(b) Which two shapes are a reflection of each other in the line $x + y = 0$? 
(c) Which shape is a rotation of the shape D by $90^\circ$ clockwise? Write down the coordinates of the centre of rotation.
(d) Which two shapes are a translation of each other by a vector with magnitude exactly 6? Give the column vector of this translation.
(e) The transformation with matrix \[
\begin{pmatrix}
1 & 2 \\
0 & 1
\end{pmatrix}
\] maps the shape D onto another shape H.

(i) Find the coordinates of the 4 vertices of the shape H.
(ii) Describe fully this single transformation.
6 Answer the whole of this question on a sheet of graph paper.

(a) \( f(x) = x^3. \)

Draw the graph of \( y = f(x) \) for \(-3 \leq x \leq 3\). Use a scale of 2 cm to represent 1 unit on the \( x \)-axis and 2 cm to represent 10 units on the \( y \)-axis.

(b) Use your graph to solve

(i) \( f(x) = -20, \)

(ii) \( f^{-1}(x) = 1.7. \)

(c) The equation \( x^3 - 5x - 1 = 0 \) can be solved by drawing one straight line on your graph.

(i) Write down the equation of this straight line.

(ii) Draw the line and write down the three solutions of \( x^3 - 5x - 1 = 0. \)

7

In the circle, the chords \( AD \) and \( BC \) meet at \( M. \)

(a) (i) Show that triangles \( AMB \) and \( CD \) are similar.

(ii) \( AM = 10 \text{ cm} \) and \( MD = 4 \text{ cm}. \) If \( CM = MB = x \text{ cm}, \) calculate the value of \( x. \)

(b) \( CM = MB \) and \( AM = \frac{5}{2} MD. \)

\( \overrightarrow{MC} = p \) and \( \overrightarrow{MD} = q. \)

Write the following vectors in terms of \( p \) and/or \( q. \)

(i) \( \overrightarrow{BM}. \)

(ii) \( \overrightarrow{MA}. \)

(iii) \( \overrightarrow{BA}. \)

(iv) \( \overrightarrow{DC}. \)

(c) Use your answers to (b)(iii) and (b)(iv) to explain why \( BA \) is not parallel to \( DC. \)
Pedro and Anna measure the circumference (C) of 100 trees. Their results are shown in the table and the cumulative frequency diagram below.

<table>
<thead>
<tr>
<th>Circumference (C) in cm</th>
<th>$C \leq 20$</th>
<th>$20 &lt; C \leq 40$</th>
<th>$40 &lt; C \leq 70$</th>
<th>$70 &lt; C \leq 100$</th>
<th>$100 &lt; C \leq 120$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0</td>
<td>26</td>
<td>30</td>
<td>33</td>
<td>11</td>
</tr>
</tbody>
</table>

(a) (i) Estimate the number of trees whose circumferences are between 60 cm and 80 cm. [2]
(ii) Use the cumulative frequency graph to find the median, the quartiles and the interquartile range. [4]
(iii) Calculate an estimate of the mean circumference. [4]
(iv) Write down the modal class. [1]

(b) Anna wants to construct a histogram. She makes a table to show the heights of the bars she will draw, using a scale of 1 cm to represent 10 cm on the horizontal axis and 1 cm$^2$ to represent 1 tree.

<table>
<thead>
<tr>
<th>Circumference (C) in cm</th>
<th>$20 &lt; C \leq 40$</th>
<th>$40 &lt; C \leq 70$</th>
<th>$70 &lt; C \leq 100$</th>
<th>$100 &lt; C \leq 120$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of bar in cm</td>
<td>$x$</td>
<td>10</td>
<td>$y$</td>
<td>$z$</td>
</tr>
</tbody>
</table>

(i) Explain why the height of the bar for the $40 < C \leq 70$ class interval is 10 cm. [1]
(ii) Find the values of $x$, $y$ and $z$. Do NOT draw a histogram. [3]
(a) A rectangular tank with length 50 cm and width 30 cm contains 36 litres of water.
Show by calculation that the water is 2 cm deep. [2]

(b) A heavy rectangular block is 5 cm high and \(x\) cm wide.
Its length is 5 cm more than its width.
Write down an expression for the volume of the block in terms of \(x\). [2]

(c) The block is placed in the tank and the water level rises by 1 cm.
   (i) Write down an equation in \(x\) and show that it simplifies to
       \[ x^2 + 5x - 300 = 0. \] [4]
   (ii) Solve the equation \(x^2 + 5x - 300 = 0\). [4]
   (iii) Write down the width and length of the block. [1]

10 Answer the whole of this question on a sheet of graph paper.

Alberto, Bernard and Carlos sell houses.

Alberto charges $600 whatever the selling price.

Bernard charges 1% of the selling price.

Carlos charges $200 for selling prices up to $30 000.
For selling prices more than $30 000, he charges $200 and \(1\frac{1}{4}\)% of the value over $30 000.
For example, when the selling price is $50 000, Carlos charges

\[ $200 + 1\frac{1}{4}\% \text{ of } ($50 000 - $30 000) = $500. \]

(a) Use a scale of 2 cm to represent a selling price of $10 000 on the horizontal axis and 2 cm to
represent a charge of $100 on the vertical axis.
Draw on the same grid the three graphs to show the charges made by Alberto, Bernard and Carlos
for selling prices up to $80 000.
Label your graphs clearly. [7]

(b) (i) For which selling price is Alberto's charge the same as Bernard's? [1]
(ii) For what range of selling prices does Carlos charge the least? [2]
(iii) For which selling price, less than $50 000, does Bernard charge $50 less than Carlos? [2]
TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
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Answer all questions.
Write your answers on the separate answer paper provided.
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For \( \pi \), use either your calculator value or 3.142.
Anna, Bella and Carla enter competitions together. When they win a prize they divide it in the ratio 3 : 2 : 1 respectively.

(a) In one competition, Bella’s share of the prize was $30.

How much did Anna and Carla each receive? [2]

(b) In another competition, the prize which they won was $40 cash and 2 books worth $55 and $25.

(i) Anna, Bella and Carla managed to divide the prize so that the value of their shares was still in the ratio 3 : 2 : 1. What exactly did each girl receive? [3]

(ii) The total value of the prize ($120) was a 25% increase on its total value last year.

Calculate the value of the prize last year. [3]

The Venn diagram above shows two intersecting sets A and B, and the number of elements in each region. There are 70 elements in \( A \cup B \).

(a) (i) Write down an equation in \( x \). [2]

(ii) Show that your equation simplifies to \( x^2 - 5x - 50 = 0 \). [2]

(b) (i) Factorise \( x^2 - 5x - 50 \). [2]

(ii) Solve \( x^2 - 5x - 50 = 0 \). [1]

(c) When \( n(A \cap B') = x \),

(i) write down the value of \( x \), [1]

(ii) find \( n(B) \). [1]
3 On December 21st, the sun rises in Buenos Aires at 05 42 and sets at 20 13.

(a) Find the length of time between sunrise and sunset in hours and minutes. [2]

(b) A plane flies from Buenos Aires (B) to Cordoba (C). It continues to Mendoza (M) before returning to Buenos Aires. The flight distances are shown on the diagram above.

(i) Showing all your working, calculate angle $MCB$ to the nearest degree. [5]

(ii) The bearing of Buenos Aires from Cordoba is $124^\circ$. Write down the bearing of Mendoza from Cordoba. [2]

(c) The average speed of the plane was 500 kilometres per hour. The times spent at Cordoba and at Mendoza were 1 hour 30 minutes and 2 hours respectively.

(i) Calculate the total time from leaving Buenos Aires until landing there again.
   Give your answer in hours and minutes to the nearest minute. [3]

$VWXYZ$ is a straight line. A triangle has sides of length 3 cm, 4 cm and 5 cm. It starts in position $A$ and is rotated about its vertices through positions $B$ and $C$ to position $D$, as shown in the diagram above.

(a) Construct the diagram accurately. 

(b) Construct accurately the locus of vertex $P$ as it moves from $P$ to $P'$. 

(c) Describe fully the rotation which maps triangle $B$ onto triangle $C$. 

(d) Describe fully the single transformation which would map triangle $A$ onto triangle $D$ directly.

---

5 Answer the whole of this question on a sheet of graph paper.

The table below gives some values of $x$ and $y$ for the function $y = f(x)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-13</td>
<td>-3.6</td>
<td>2</td>
<td>4.6</td>
<td>5</td>
<td>3.9</td>
<td>2</td>
<td>0.1</td>
<td>-1</td>
<td>-0.6</td>
<td>2</td>
<td>7.6</td>
<td>17</td>
</tr>
</tbody>
</table>

(a) The graph of $y = f(x)$ is a smooth curve. Using 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 5 units on the $y$-axis, draw this graph for $-2 < x < 4$. 

(b) Use your graph to solve the equation $f(x) = 0$, giving your answers correct to 1 decimal place. 

(c) Write down a positive integer value of $k$ such that the equation $f(x) = k$ has 3 solutions. 

(d) Write down the order of rotational symmetry of this graph and the coordinates of the centre of rotation. 

(e) (i) By drawing a suitable tangent, estimate the gradient of the curve at $(-1, 2)$. 

(ii) Write down the coordinates of the other point where the curve has this gradient.
Doctors use a test to find who has a certain illness.

The probability of a positive test result is 0.20.

However 15% of those with positive test results do not have the illness;
5% of those with negative test results do have the illness.
(This means that these people are given an incorrect test result.)

(a) Copy and complete the tree diagram below.

```
0.70
Positive
 test result

0.15
has the illness

0.05
Negative
 test result

has the illness

does not have the illness

does not have the illness
```

(b) Find the probability that a person chosen at random
   (i) has a positive test result and has the illness,
   (ii) has the illness,
   (iii) is given an incorrect test result.

(c) In a certain town, 10 000 people, chosen at random, are given this test.
    Estimate how many of these people will
    (i) be told they have the illness,
    (ii) actually have the illness.
A large circular window is shown in the diagram. The unshaded part is glass and is made up of a small circle and 12 identical shapes. The shaded part is stone.

[For \(\pi\), use either your calculator value or 3.142.]

(a) The diagram shows one of the 12 identical shapes. 

\(ABC\) is an isosceles triangle and \(BCD\) is a semicircle. 

\(BC = 1.4\) m and angle \(BAC = 30^\circ\).

Calculate

(i) the area of the semicircle \(BCD\),

(ii) the length of \(AC\), showing that it rounds off to 2.705 m,

(iii) the area of triangle \(ABC\),

(iv) the area of the shape \(ABDC\).

(b) The radius of the small circle is 0.3 m.

Calculate the total area of glass, including the small circle.

(c) The radius of the large circular window is 4 m.

Calculate the percentage of the window's area which is stone.
Four lines are shown on the grid above.

(a) (i) Write down the equation of the line \( l_1 \). [1]
(ii) Write down the equation of the line \( l_2 \). [1]
(iii) Write down the value of \( m \) and the value of \( n \) for the line \( y = mx + n \). [2]
(iv) Write down the value of \( p \) and the value of \( q \) for the line \( y = px + q \). [3]

(b) Which of the letters \( A \) to \( K \) lie in the region where
(i) \( x > 2 \), \( y > mx + n \) and \( y > px + q \). [2]
(ii) \( y < 2 \), \( y < mx + n \) and \( y < px + q \).

(c) What is the maximum value of \( x + y \) in region \( G \)? [2]
Ricardo asks 200 people how much they spend each year on books. He puts his results in the table below.

<table>
<thead>
<tr>
<th>Amount ($x$)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; x &lt; 20$</td>
<td>50</td>
</tr>
<tr>
<td>$20 &lt; x &lt; 40$</td>
<td>40</td>
</tr>
<tr>
<td>$40 &lt; x &lt; 60$</td>
<td>48</td>
</tr>
<tr>
<td>$60 &lt; x &lt; 80$</td>
<td>30</td>
</tr>
<tr>
<td>$80 &lt; x &lt; 120$</td>
<td>32</td>
</tr>
</tbody>
</table>

(a) (i) Write down the modal class.  
(ii) Calculate an estimate of the mean.  
(iii) Explain briefly why, although you have done an exact calculation to find the mean, it is still an "estimate".

(b) Answer the rest of this question on a sheet of graph paper.

(i) Make a cumulative frequency table for the data above.
(ii) Using 2 cm to represent $20 on the horizontal axis and 2 cm to represent 20 people on the vertical axis, draw the cumulative frequency diagram.

(c) Use your graph to find

(i) the median,  
(ii) the upper and lower quartiles,  
(iii) the interquartile range.
The height of a cylinder is 10 cm and its radius is 2.5 cm.

[For \(\pi\), use either your calculator value or 3.142.]

(a) A piece of string is wound evenly once around the curved surface of the cylinder, starting at a point \(A\) on the circumference of the top circular face and finishing at \(B\), vertically below \(A\).

A sketch of the net of the curved surface of the cylinder, together with the string \(AB\), is shown above in the diagram on the right. Calculate the length of the string, \(AB\). [4]

(b) Another string, starting at \(A\), is wound evenly twice around the cylinder, finishing again at \(B\). Calculate the length of this string. [3]

(c) Sketch the net when a string, starting at \(A\), is wound evenly three times around the cylinder, finishing at \(B\). [2]

(d) A string is wound evenly \(n\) times around the cylinder, from \(A\) to \(B\). Find a formula, in terms of \(n\), for the length of the string. [3]
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If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142.
The line $y = x + 2$ cuts the $y$-axis at $P$. The line $3x + 4y = 22$ cuts the $x$-axis at $Q$. The two lines intersect at $R$.

(a) Find the coordinates of (i) $P$, [1]
    (ii) $Q$, [2]
    (iii) $R$. [3]

(b) $x > 0$ is one of the four inequalities which define the region $OPRQ$. Write down the other three inequalities. [3]

2 Winston and Anthony take a driving test. The probability that Winston will pass is $\frac{1}{4}$ and the probability that Anthony will pass is $\frac{3}{4}$.

(a) Which of them is more likely to pass? [1]

(b) Calculate the probability that they will both fail. [2]

(c) Calculate the probability that only one of them will pass. [3]

(d) If Winston fails he will take the test again. The probability that he will pass at any future attempt is $\frac{3}{4}$.
    (i) Draw a tree diagram to show the probabilities of Winston passing or failing on each of his first three attempts. [3]
    (ii) Calculate the probability that he will pass at his third attempt. [2]
    (iii) Calculate the probability that he will not need more than three attempts to pass. [3]
The line graph shows how many grams of bananas were eaten per person per week in Europe between 1984 and 1994.

(a) Between which two years did the largest yearly increase take place? [1]

(b) What was the percentage increase between 1984 and 1994? [2]

(c) Work out the fraction \[
\frac{\text{mass of bananas eaten per person per week in 1992}}{\text{mass of bananas eaten per person per week in 1994}}.
\]

Give your answer as a fraction in its lowest terms. [2]

(d) There were 497 million people in Europe in 1990.
Calculate the total mass of bananas eaten in Europe in 1990.
Give your answer in tonnes, correct to two significant figures. [3]
The points $P$, $Q$ and $R$ lie on the circumference of a circle, centre $O$.
$PQ = 5\text{ cm}$, $PR = 8\text{ cm}$ and angle $QPR = 70^\circ$.

(a) Calculate the area of triangle $PQR$.  [2]

(b) Calculate the length of the chord $QR$.  [4]

(c) Find the size of the obtuse angle $QOR$.  [1]

(d) Show that the radius of the circle is 4.18 cm, correct to three significant figures.  [4]

(e) Taking the radius of the circle as 4.18 cm, calculate the length of the minor arc $QR$.  [3]

(f) Find the size of the reflex angle $QOR$.  [1]

5 (a) (i) Write down the value of $\cos 295^\circ$.  [1]

(ii) If $\sin x$ and $\cos x$ are both negative, between what values must $x$ lie?  [2]

(b) The depth of water ($d$ metres) in a harbour is given by the formula

\[ d = 5 + 4 \sin 30^\circ t, \]

where $t$ is the time in hours after midnight.

(i) Find the depth of water at midnight.  [1]

(ii) Find the depth of water at 10 a.m.  [2]

(iii) What is the greatest depth of water in the harbour?  [2]

(iv) At what times of day is the depth of water greatest?  [2]

(v) What is the least depth of water in the harbour?  [1]
The points $A (-5, 1)$, $B (-1, -1)$ and $C (2, 5)$ are three vertices of a rectangle $ABCD$.

(a) (i) Write down the coordinates of $A$.

(ii) Calculate the length of the line segment $AB$.

(iii) Calculate the area of the rectangle $ABCD$.

(b) (i) Find the equation of the line in which $AD$ is the reflection of $BC$.

(ii) Describe fully a single transformation (not a reflection) which maps $A$ onto $D$ and $B$ onto $C$.

(c) The matrix $M = \begin{pmatrix} x^2 & 2x + 5 \\ 1 & 10 \end{pmatrix}$.

(i) The transformation represented by the matrix $M$ maps $A$ onto $C$. Show that $5x^2 - 2x - 3 = 0$.

(ii) Solve the equation $5x^2 - 2x - 3 = 0$.

(iii) Calculate the inverse matrix $M^{-1}$ when $x = 1$. [2]
7 The formula for the volume of a pyramid is \( \frac{1}{3} \) base area \( \times \) perpendicular height.

(a)

In the pyramid \( ABCD \), \( ABC \) is the base and \( D \) is the vertex.
Angle \( BCA = \text{angle} \ DAC = \text{angle} \ DAD = 90^\circ \).
\( AD = h \) cm, \( AC = b \) cm and \( BC = a \) cm.

(i) Write down a formula for the volume of the pyramid \( ABCD \) in terms of \( a \), \( b \) and \( h \). [2]
(ii) Calculate the volume of pyramid \( ABCD \) when \( a = 6 \), \( b = 5 \) and \( h = 8 \). [2]

(b)

The pyramid \( PQRST \) has a rectangular base with \( ST = x \) cm and \( RS = (x + 3) \) cm.
The height of the pyramid, \( OP \), is 12 cm, where \( O \) is the centre of the rectangle.

(i) Write down a formula for the volume of this pyramid in terms of \( x \). [2]
(ii) When the volume is numerically equal to the perimeter of the rectangular base, show that
\[ 2x^2 + 4x - 3 = 0. \] [2]
(iii) Solve the equation \( 2x^2 + 4x - 3 = 0 \), giving your answers correct to 2 decimal places.
Show all your working. [5]
(iv) Use your answer to part (iii) to write down the length of \( RS \). [1]
(v) \( M \) is the midpoint of \( ST \). Calculate angle \( PMO \). [3]

(c) (i) What is the name given to a pyramid with a circular base? [1]
(ii) Write down a formula for the volume of this pyramid if the radius of the circular base is \( r \) and the perpendicular height is \( h \). [1]
8 Answer the whole of this question on a sheet of graph paper.
500 eggs were sorted by mass into five different sizes.

<table>
<thead>
<tr>
<th>Mass (m grams)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>35 &lt; m &lt; 40</td>
</tr>
<tr>
<td>Medium</td>
<td>40 &lt; m &lt; 50</td>
</tr>
<tr>
<td>Standard</td>
<td>50 &lt; m &lt; 60</td>
</tr>
<tr>
<td>Large</td>
<td>60 &lt; m &lt; 75</td>
</tr>
<tr>
<td>Extra large</td>
<td>75 &lt; m &lt; 80</td>
</tr>
</tbody>
</table>

(a) Draw an accurate histogram to represent this information.
Use a scale of 2 cm to represent 5 grams on the horizontal axis, and an area scale of 1 square centimetre to represent 5 eggs.

(b) Calculate an estimate of the mean mass of these eggs.

(c) This cumulative frequency curve has been drawn using the information in the table above.

(i) Explain why the point (60, 280) is on the curve.
(ii) Estimate the median mass of the eggs.
(iii) Estimate the interquartile range of the masses of the eggs.
The diagram shows a window \( ABCDE \). \( ABDE \) is a rectangle. 
\( BCD \) is an arc of a circle with centre \( O \) and radius \( x \) cm. 
The total height of the window is 90 cm. 
\( AB = ED = 80 \) cm and \( AE = BD = 40 \) cm. 
The line \( OC \) is perpendicular to \( BD \), and \( BF = FD \).

(a) (i) Write down, in terms of \( x \), the length of \( OC \) and the length of \( OF \). [1]
(ii) Use Pythagoras' Theorem in triangle \( OFD \) to write down an equation in \( x \). [2]
(iii) By solving the equation, show that \( x = 25 \). [1]

(b) Using a scale of 1 cm to represent 10 cm, construct an accurate drawing of the window. [4]

(c) Find the area of the window. (For \( \pi \), use either your calculator value or 3.142.) [5]

(d) The window is made of glass 2 mm thick. 
The mass of 1 cm\(^3\) of the glass is 6.5 grams. 
Calculate the mass of glass in the window, giving your answer in kilograms. [3]

10 Throughout this question, remember that 1 is not a prime number.

(a) Find a prime number which can be written as the sum of two prime numbers. [1]

(b) Consider the statement

"All even numbers greater than 15 can be written as the sum of two different prime numbers in at least two different ways."

For example, \( 20 = 3 + 17 = 7 + 13 \).

(i) Show that the above statement is true for 16. [1]
(ii) Find a number between 30 and 50 which shows that the statement is false. [2]

(c) Show that 16 can be written as the sum of three different prime numbers. [1]

(d) Consider the statement

"All odd numbers greater than 3 can be written as the sum of two prime numbers".

Is it true or false? Justify your answer. [2]
International General Certificate of Secondary Education

UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

MATHEMATICS

PAPER 4

Wednesday 8 NOVEMBER 2000 Morning 2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (1 sheet)
Mathematical tables (optional)
Tracing paper (optional)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet

Answer all questions.

Write your answers on the separate answer paper provided.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 130.

Electronic calculators should be used.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place.

For π, use either your calculator value or 3.142.

This question paper consists of 8 printed pages.
A company's accounts for 1998 are shown in the pie chart above. The profit was $36 000 and the labour costs were $75 000.

(a) (i) The angle in the profit sector is \(x^\circ\). Show that \(x = 72\).  
(ii) Calculate the amount paid for materials.  
(iii) Find the ratio \(\text{tax : profit}\) in its lowest terms.

(b) In 1999 the labour costs were $78 000.

(i) Write 78 000 in standard form.  
(ii) Calculate the percentage increase in labour costs from 1998 to 1999.

(c) The labour costs of $78 000 in 1999 were 160% more than the labour costs in 1993. Calculate the labour costs for 1993.

\[
\begin{pmatrix}
3 & 0 & 0 \\
9 & 5 & 0 \\
4 & -3 & 2
\end{pmatrix}
\begin{pmatrix}
1 \\
q \\
r
\end{pmatrix}
=
\begin{pmatrix}
p \\
-26 \\
35
\end{pmatrix}.
\]

Find the values of \(p\), \(q\) and \(r\).

(b) \(M = \begin{pmatrix}
t & 6 \\
t & 5t
\end{pmatrix}\) and \(M^{-1} = \begin{pmatrix}
-5t & 6 \\
t & -t
\end{pmatrix}\) where \(M^{-1}\) is the inverse of \(M\) and \(t \neq 0\).

Write down an equation in \(t\) and solve it.

(c) \(A\begin{pmatrix} x \\
5
\end{pmatrix} = kx\) is a matrix equation.

(i) Find the value of \(k\) if \(x^2 + 8x + 10 = 0\).

(ii) Solve the equation \(x^2 + 8x + 10 = 0\). Show all your working and give your answers correct to 2 decimal places.
Diagram 1

The diagrams show a triangular prism $ABCDEF$ and a rough sketch of one of its possible nets. $AB = 4 \text{ cm}, BC = 5 \text{ cm}, AC = 6 \text{ cm}$ and $CF = 7 \text{ cm}$.

(a) Calculate, correct to one decimal place, the angle $ABC$ shown in Diagram 1. \[4\]

(b) Construct accurately the net of the prism shown in Diagram 2. Label every corner of your net with the appropriate letter $A, B, C, D, E$ or $F$. \[5\]

(c) Taking any measurements from your net which you may need and showing all your working, calculate

(i) the total surface area of the prism, \[3\]

(ii) the volume of the prism. \[2\]

4 Answer the whole of this question on a sheet of graph paper.

The equation $h = 20t - 5t^2 + 1$ gives the height $h$ metres above ground level of a stone $t$ seconds after it has been thrown vertically upwards. Some values of $h$ and $t$ are given in the following table.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>1</td>
<td>9.75</td>
<td>16</td>
<td>19.75</td>
<td>21</td>
<td>$a$</td>
<td>16</td>
<td>9.75</td>
<td>$b$</td>
<td>$c$</td>
</tr>
</tbody>
</table>

(a) Calculate the values of $a, b$ and $c$. \[3\]

(b) Using a scale of 2 cm to represent 0.5 seconds on the horizontal $t$-axis and 2 cm to represent 4 m on the vertical $h$-axis, draw the graph of $h = 20t - 5t^2 + 1$ for $0 \leq t \leq 4.5$. \[5\]

(c) Use your graph to answer these questions.

(i) What is the value of $t$ when the stone reaches ground level? \[1\]

(ii) For how long is the stone more than 12 m above the ground? Give your answer in seconds to 1 decimal place. \[2\]

(iii) How far does the stone travel altogether in the first 3 seconds? \[2\]

(d) (i) Draw a suitable tangent on your graph and use it to calculate an estimate of the gradient (slope) when $t = 1$. \[3\]

(ii) What quantity does the gradient measure and what are the units for this quantity? \[2\]
A belt $PQRSTU$ passes round two wheels, which turn when the belt moves. The larger wheel has radius 30 cm and centre $A$. The smaller wheel has radius 10 cm and centre $B$. Angle $TAP = 130^\circ$.

(a) Explain why the angles at $P$, $Q$, $S$ and $T$ are all $90^\circ$. [1]

(b) Find the size of angles $PUT$, $QBS$ and $QRS$. [4]

(c) The smaller wheel makes 12 revolutions in a clockwise direction. How many revolutions does the larger wheel make and in which direction? [2]

(d) NOT TO SCALE

This diagram represents part of the diagram at the top of the page. The perpendicular from $B$ meets $PA$ at $X$.

(i) Work out the length of $AX$ and the size of angle $PAB$. [1]

(ii) Find the distance between the centres $A$ and $B$. [3]
One teacher from Argentina, one from Brazil and three from Namibia attend an international conference. One of these five teachers is chosen at random to make a speech, and one of the remaining four is chosen at random to write a report.

(a) Copy and complete the probability tree diagram below, showing the countries from which the teachers were chosen.

(b) Calculate the probability that

(i) both the chosen teachers were from Namibia,

(ii) neither of the chosen teachers was from Namibia,

(iii) the teacher from Brazil was not chosen.

(c) One of the remaining three teachers is chosen at random to chair the conference. Calculate the probability that this is the teacher from Brazil.
(a) Triangle $T$ is mapped onto triangle $U$ by a single transformation. Describe this transformation fully. [2]

(b) Triangle $V$ is a rotation of triangle $U$ about the origin by $\theta^\circ$ anticlockwise. Calculate $\theta$ correct to 1 decimal place. [3]

(c) Triangle $W$ is an enlargement of triangle $V$, centre $(a, 0)$, with scale factor $k$.
   (i) Calculate the value of $k$. [2]
   (ii) Find the value of $a$. [2]

(d) (i) Find the area of triangle $U$. [2]
   (ii) Find the equation of the hypotenuse of triangle $U$. [2]

(e) A triangle similar to $T$ has an area 64 times larger than that of $T$. Calculate the length of the hypotenuse of this triangle. [3]
On television a weather forecaster uses a cloud symbol shown in the diagram.

Its perimeter consists of a straight line \( AE \), two semicircular arcs \( APB \) and \( DQE \) and the major arc \( BRD \) of a circle, centre \( C \).

\[ AE = 7.5 \text{ cm}, \ AB = DE = 3 \text{ cm} \text{ and } BC = CD = 2.8 \text{ cm}. \]

Angle \( BAE = \text{angle } DEA = 70^\circ \) and \( X \) is the midpoint of \( BD \).

[For \( \pi \), use either your calculator value or 3.142.]

\[ \begin{align*}
\text{(a) } & \quad \text{(i) Use the trapezium } ABDE \text{ to show that } BX = 2.724 \text{ cm.} \\
& \quad \text{(ii) Calculate angle } BCX. \\
\text{(b) } & \quad \text{Calculate} \\
& \quad \text{(i) the area of triangle } BCD, \\
& \quad \text{(ii) the area of the trapezium } ABDE, \\
& \quad \text{(iii) the area of the major sector } BCD, \\
& \quad \text{(iv) the total area of the cloud symbol.}
\end{align*} \]
A teacher asks four students to write down an expression using each of the integers 1, 2, 3 and \( n \) exactly once.

Ahmed's expression was \((3n + 1)^2\).

Bumni's expression was \((2n + 1)^3\).

Cesar's expression was \((2n)^{3+1}\).

Dan's expression was \((3 + 1)^{2n}\).

The value of each expression has been worked out for \( n = 1 \) and put in the table below.

(a) Copy and complete this table, giving the values for each student's expression for \( n = 2, 0, -1 \) and \(-2\).

<table>
<thead>
<tr>
<th></th>
<th>( n = 2 )</th>
<th>( n = 1 )</th>
<th>( n = 0 )</th>
<th>( n = -1 )</th>
<th>( n = -2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahmed</td>
<td></td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bumni</td>
<td></td>
<td>27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cesar</td>
<td></td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dan</td>
<td></td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Whose expression will always give the greatest value?

(i) if \( n < -2 \),

(ii) if \( n > 2 \)?

(c) Cesar's expression \((2n)^{3+1}\) can be written as \( an^b \) and Dan's expression \((3 + 1)^{2n}\) can be written as \( c^n \). Find the values of \( a, b \) and \( c \).

(d) Find any expression, using 1, 2, 3 and \( n \) exactly once, which will always be greater than 1 for any value of \( n \).
International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICS
PAPER 4
MAY/JUNE SESSION 2001

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables (optional)
Tracing paper (optional)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.
Answer all questions.
Write your answers and working on the separate answer paper provided.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer.
Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.
1 (a) 12 friends take a holiday together in Jordan and Saudi Arabia.
9 have already been to Jordan and 4 have already been to Saudi Arabia.
The probability that one of the 12 friends, chosen at random, has already been to both countries is \( \frac{1}{4} \).

(i) Write down the number of friends who have already been to both countries.  \( \text{[1]} \)

(ii) Copy the Venn diagram below.

\[
\begin{array}{c}
\text{J} \\
\text{S} \\
\end{array}
\]

\( J = \{ \text{those who have already been to Jordan} \} \) and
\( S = \{ \text{those who have already been to Saudi Arabia} \} \).
Write the number of friends in each part of your Venn diagram.  \( \text{[4]} \)

(iii) Write down the value of \( n(J \cup S) \).  \( \text{[1]} \)

(b) Ahmed changed 10000 Riyals into Dinars. The exchange rate was 1 Dinar = 5.28 Riyals.
He then divided the Dinars between himself, Yousef and Ibrahim in the ratio 2 : 3 : 1.
How many Dinars did Ahmed keep for himself? Give your answer to the nearest Dinar.  \( \text{[3]} \)

2

\[
\begin{array}{c}
\text{North} \\
\text{North} \\
\end{array}
\]

A boat \( B \) is 1200 metres from a lighthouse \( L \) and 750 metres from a rock \( R \). Angle \( LBR = 110^\circ \).

(a) Calculate

(i) the length \( LR \), correct to the nearest metre,

(ii) angle \( BLR \), correct to the nearest degree.

(b) The bearing of \( B \) from \( L \) is 053\(^\circ\).
Calculate

(i) the bearing of \( L \) from \( B \).

(ii) the bearing of \( B \) from \( R \).

(c) The boat is sailing due south.
Calculate, to the nearest metre, its closest distance to the lighthouse.
Answer the whole of this question on one sheet of graph paper.

(a) Using a scale of 1 centimetre to represent 1 unit on each axis, draw an $x$-axis for $-6 \leq x \leq 10$ and a $y$-axis for $-6 \leq y \leq 8$.
Copy the letters IGCSE accurately from the diagram above onto the same position on your graph paper. Each letter is 2 cm high and 1 cm wide.
[For example, the letter I lies in the rectangle $1 \leq x \leq 2$ and $4 \leq y \leq 6$.] [2]

(b) Draw accurately the image of your letters under the following transformations.

(i) Rotate your letter I by $90^\circ$ clockwise about the origin. [2]
(ii) Reflect your letter G in the $y$-axis. [2]
(iii) Enlarge your letter C, scale factor 4, centre (7, 7). [2]
(iv) Translate your letter S by the vector $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$. [2]
(v) Stretch your letter E parallel to the $y$-axis, stretch factor 0.5, with the $x$-axis invariant. [2]

(c) (i) Find the transformation matrix $M$ which represents rotation by $90^\circ$ clockwise about the origin. [2]

(ii) Find the inverse matrix $M^{-1}$ and describe in words the transformation which it represents. [3]
Answer this question without using graph paper.

\( A \) is the point \((2, 5)\) and \(B\) is the point \((8, 2)\).

(a) Find the equation of the line \(AB\). \[2\]

(b) Calculate the length of the line \(AB\), giving your answer correct to 2 decimal places. \[2\]

(c) Find the coordinates of the point \(C\) such that \(A\) is the midpoint of \(BC\). \[3\]

(d) Point \(D\) lies on the line \(y = 2\) and has coordinates \((x, 2)\).

Find two possible values of \(x\) if the area of triangle \(ABD\) is 15 cm². \[3\]

5

Answer the whole of this question on a sheet of graph paper.

(a) Using a scale of 1 centimetre to represent 1 unit on each axis, draw an \(x\)-axis for \(-6 \leq x \leq 10\), and a \(y\)-axis for \(-2 \leq y \leq 12\).

Mark the points \(A\) \((-6, 1)\), \(B\) \((-3, 10)\) and \(C\) \((9, 6)\).
Draw the triangle \(ABC\). \[2\]

(b) Construct the locus of points

(i) 7 cm from \(A\) and inside triangle \(ABC\), \[2\]
(ii) equidistant from \(B\) and from \(C\), \[2\]
(iii) equidistant from \(BC\) and from \(AC\). \[2\]

(c) Shade the region inside triangle \(ABC\) which is less than 7 cm from \(A\) and nearer to \(BC\) than to \(AC\).
Label this region \(R\). \[2\]

(d) Shade the region inside triangle \(ABC\) which is nearer to \(C\) than to \(B\) and nearer to \(BC\) than to \(AC\).
Label this region \(S\). \[2\]
6 Monica received $x$ marks in a test. Sandra received 4 marks more than Monica.

(a) Write down Sandra's mark in terms of $x$.

(b) When Monica subtracts 7 from her mark and squares the result, her answer is 1 more than Sandra's mark.

(i) Write down an equation in $x$ and show that it simplifies to $x^2 - 15x + 44 = 0$.

(ii) Solve the equation $x^2 - 15x + 44 = 0$.

(c) The test was marked out of 10. Write down the mark received by each girl.

7

Diagram 1

The wheel in Diagram 1 has one of the numbers 0 to 4 in each of its five identical sectors. The wheel is spun and when it stops the arrow points to a number. All numbers are equally likely. Diagram 2 shows the possible outcomes when the wheel is spun twice.

(a) Find the probability that

(i) both numbers are 3,

(ii) both numbers are the same,

(iii) the sum of the two numbers is 6,

(iv) the product of the two numbers is 8 or more,

(v) the product of the two numbers is less than the sum.

(b) $S = \{\text{all possible values of the sum of the two numbers}\}$, $P = \{\text{all possible values of the product of the two numbers}\}$.

(i) List the elements of $S$.

(ii) List the elements of $P$.

(iii) Find $S \cap P'$.
The heights, \( h \) centimetres, of 60 children are shown in the pie chart above.

(i) What is the angle in the \( 100 < h \leq 125 \) sector? [1]

(ii) How many children have a height less than or equal to 100 centimetres? [1]

(iii) How many children have a height less than or equal to 150 centimetres? [1]

(iv) Write down the width of each class interval.

Which of the four groups in the pie chart would give the highest bar in a histogram?

Explain your answer, but do not draw an accurate histogram. [3]

(b) The 60 children are asked how many pets they have. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Number of pets</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>16</td>
<td>14</td>
<td>3</td>
<td>9</td>
<td>7</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Find

(i) the mode, [1]

(ii) the median, [2]

(iii) the mean number of pets per child. [2]
A cone, \(ATB\), and a section of a sphere, \(ASB\), share the same circular base, centre \(C\), radius \(r\). The height, \(TC\), of the cone is \(h\) and \(STOC\) is a straight line. The radius, \(OB\), of the sphere is \(R\) and the height, \(CS\), of the section of the sphere is \(H\).

(a) \(r = 6\) cm, \(h = 14\) cm and \(R = 10\) cm.

(i) Calculate the volume of the cone \(ABT\). \([2]\)

[The volume of a cone with base \(\pi r^2\) and height \(h\) is \(\frac{1}{3}\pi r^2 h\).]

(ii) Show that the height, \(SC\), of the section of the sphere is 18 cm. \([2]\)

(iii) Calculate the volume of the section of the sphere \(ASB\). \([2]\)

[The volume of a section of a sphere, radius \(R\), height \(H\) is \(\frac{1}{3}\pi H^2 (3R - H)\).]

(iv) Find the percentage of the volume in the section of the sphere not occupied by the cone. \([2]\)

(b) In a different sphere section, \(R = 3\) cm, \(h = 2r\) cm and \(TS = 1\) cm.

(i) Write down the height, \(SC\), in terms of \(r\) and show that \(OC = (2r - 2)\) cm. \([2]\)

(ii) Use Pythagoras' theorem in triangle \(OCB\) to find \(OC^2\) in terms of \(r\). \([1]\)

(iii) Use your answers to parts (b)(i) and (b)(ii) to show that \(5r^2 - 8r - 5 = 0\). \([3]\)

(iv) Solve the equation \(5r^2 - 8r - 5 = 0\). Show all your working, and give your answers correct to 2 decimal places. \([5]\)

(v) Write down the height of the cone. \([1]\)
(a) Write down which one of the sketch graphs above labelled A to H shows each of the following:

(i) a speed – time graph for a car which starts from rest and has constant acceleration; [2]
(ii) \( y = x^3 + 1 \); [2]
(iii) \( y \) is inversely proportional to \( x^2 \); [2]
(iv) the sum of \( x \) and \( y \) is constant; [2]
(v) \( y = \cos x \) for \( 0^\circ \leq x \leq 90^\circ \); [2]
(vi) a distance – time graph when the speed is decreasing. [2]

(b) Write down an equation for sketch graph D if it passes through the points (1, 1) and (2, 4) and, when extended to the left, has line symmetry about the vertical axis. [2]
International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
MATHEMATICS
PAPER 4
OCTOBER/NOVEMBER SESSION 2001
2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables (optional)
Tracing paper (optional)

TIME
2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.
Answer all questions.
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For π, use either your calculator value or 3.142.
1 In an election in Anyville, the Blue party got 40% of the votes. The Orange party got 11 424 votes which was seven eighths of the Blue party vote. Some people voted for other parties and so...e did not vote at all.

(a) Calculate

(i) how many people in Anyville voted for the Blue party.

(ii) how many people in Anyville voted.

(b) There were 42 320 people in Anyville.

Calculate the percentage of people in Anyville who did not vote.

(c) There were 572 senators in the new National Assembly.

The numbers of senators in the Blue, Orange and other parties were in the ratio


Calculate

(i) the number of senators in the Orange party,

(ii) the difference between the number of senators in the Blue party and the number who were not in the Blue party.

2

The diagram shows the circular cross-section of a horizontal pipe.

The shaded area shows water lying in the pipe.

The circle, centre \( O \), has a radius 63.7 cm and angle \( AOB = 46^\circ \).

(a) Calculate the arc length \( AWB \).

(b) Calculate the length of the straight line \( AB \).

(c) Calculate the length of the perpendicular from \( O \) to \( AB \).

(d) Write down the greatest depth of water in the pipe.

Give your answer correct to the nearest millimetre.
In the diagram, \(MN\) and \(PQ\) are parallel and \(MQ\) and \(NP\) meet at \(O\).

(i) Show that triangles \(MNO\) and \(QPO\) are similar.

(ii) \(OM = 5\) cm and \(ON = 4\) cm. \(OP = (y + 1)\) cm and \(OQ = (2y - 2)\) cm.

Explain why \(\frac{2y - 2}{5} = \frac{y + 1}{4}\).

(iii) Solve the equation in part (a)(ii).

(iv) Find the length of \(NP\).

(b) (i) Write down the value of \(\sin 30^\circ\) as a fraction.

(ii)

In triangle \(ABC\), angle \(ACB = 90^\circ\) and angle \(ABC = 30^\circ\).

\(AC = (x - 3)^2\) and \(AB = (30 - 4x)\).

Use your answer to part (b)(i) to write down an equation in \(x\).

Show that it simplifies to \(x^2 - 4x - 6 = 0\).

(iii) Solve the equation \(x^2 - 4x - 6 = 0\).

Show all your working and give your answers correct to 2 decimal places.

(iv) Find the length of \(AB\) when \(AC\) is greater than 10.
You do not need to copy this diagram.

The coordinates of $P$, $Q$ and $R$ are $(4, 2)$, $(6, 2)$ and $(8, 4)$ respectively.

The points $Q$ and $R$ lie on the line $l$.

(a) Find the new coordinates for

(i) $P$, after reflection in the line $l$.

(ii) $Q$, after translation by the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

(iii) $R$, after a rotation of $90^\circ$ anticlockwise about centre $P$.

(b) The coordinates of $S$, $T$ and $U$ are $(2, 2)$, $(3, 2)$ and $(4, 3)$ respectively.

(i) Describe fully the single transformation which maps triangle $PQR$ onto the shaded triangle $STU$.

(ii) Find, in the form $1:n$, the ratio $\text{area of triangle } STU : \text{area of triangle } PQR$.

(c) Find the new area of triangle $PQR$ when it is stretched parallel to the $y$-axis with scale factor 3 and the $x$-axis invariant.

(d) (i) Find the inverse of the matrix $\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$.

(ii) A point $W$ has coordinates $(x, y)$ such that $\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

Find the coordinates of $W$. 


5 Answer the whole of this question on a sheet of graph paper.

(a) 100 seeds were treated with fertilizer and the heights (H cm) of the plants which grew are given in the cumulative frequency table below.

<table>
<thead>
<tr>
<th>Height (H cm)</th>
<th>( H &lt; 5 )</th>
<th>( H &lt; 10 )</th>
<th>( H &lt; 15 )</th>
<th>( H &lt; 20 )</th>
<th>( H &lt; 30 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative frequency</td>
<td>0</td>
<td>5</td>
<td>25</td>
<td>85</td>
<td>100</td>
</tr>
</tbody>
</table>

(i) Using a scale of 2 cm to represent 5 cm on the horizontal \( H \)-axis and 2 cm to represent 20 plants on the vertical axis, draw a cumulative frequency diagram for these plants. [5]

(ii) Find the median, the lower quartile and the interquartile range. [3]

(iii) Write down the number of plants in the \( 15 < H < 20 \) group. [1]

(b) Another 100 seeds were not treated with fertilizer and the heights (h cm) of the plants which grew from these seeds are given in the grouped frequency table below.

<table>
<thead>
<tr>
<th>Height (h cm)</th>
<th>( 0 &lt; h &lt; 5 )</th>
<th>( 5 &lt; h &lt; 10 )</th>
<th>( 10 &lt; h &lt; 15 )</th>
<th>( 15 &lt; h &lt; 20 )</th>
<th>( 20 &lt; h &lt; 30 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>20</td>
<td>45</td>
<td>23</td>
<td>2</td>
</tr>
</tbody>
</table>

You do not need to draw another graph.

(i) Calculate an estimate of the mean value of \( h \). [4]

(ii) Write down the class which contains the lower quartile. [1]

(iii) Write down the class which contains the 90th percentile. [1]

(c) The two groups of 100 plants are combined. One of these 200 plants is chosen at random.

(i) Find the probability that its height is less than or equal to 10 cm. [2]

(ii) Given that the plant height is less than or equal to 10 cm, what is the probability that it was treated with fertilizer? [2]
The graphs of \( y = f(x) \) and \( y = g(x) \) are drawn on the grid above.

The line \( y = g(x) \) is the tangent to the curve \( y = f(x) \) at \( x = 0.75 \).

(a) Find

(i) \( f(1.5) \). [1]

(ii) \( g(0) \). [1]

(iii) \( g^{-1}(-0.5) \). [1]

(b) (i) Find the range of values of \( x \) such that \( f(x) > g(x) \). [2]

(ii) Solve \( f(x) = 0 \). [2]

(iii) Find the range of values for \( k \) when \( f(x) = k \) has 3 different solutions. [2]

(c) Calculate the gradient of \( y = f(x) \) where \( x = 0.75 \). [3]

(d) Solve \( 1 - f(x) = 0 \). [3]
Grumpy Guy and Happy Hal are children's toys. Each is made from a solid hemisphere, a cylinder and a cone.

[The volume of a sphere, radius r, is \( \frac{4}{3}\pi r^3 \) and the surface area of a sphere is \( 4\pi r^2 \).
The volume of a cone, base radius r and perpendicular height h is \( \frac{1}{3}\pi r^2 h \) and the surface area of a cone is \( \pi rl \) where l is the slant height.]

(a) Grumpy Guy has a radius of 3 cm. The height of his cylinder is 7 cm and the perpendicular height, h, of his cone is 4 cm.

Calculate for Guy

(i) his volume.
(ii) his surface area.

(b) Happy Hal has a radius of x cm. The height of his cylinder is x cm and the perpendicular height of his cone is also x cm.

Find for Hal

(i) his volume in terms of \( \pi \) and x.
(ii) his volume when \( x = 5 \).

(c) Happy Hal is made from two materials. The hemisphere is made from a heavy material and the rest from a lighter material. The mass of the hemisphere is half the mass of the whole toy.

Find, in its simplest form, the ratio

mass of hemisphere : mass of cylinder : mass of cone.
8 Answer the whole of this question on a sheet of graph paper.

There are \( x \) girls and \( y \) boys in a school choir.

(a)  
(i) The number of girls is more than 1.5 times the number of boys in the choir.

Show that \( y < \frac{2x}{3} \).

(ii) There are more than 12 girls in the choir.

There are more than 5 boys in the choir.

The maximum number of children in the choir is 35.

Write down three more inequalities.

(b)  
(i) Using a scale of 2 cm to represent 5 children on each axis, draw an \( x \)-axis for \( 0 < x < 40 \) and a \( y \)-axis for \( 0 < y < 40 \).

(ii) Draw 4 lines on your graph to represent the inequalities in part (a).

Shade the unwanted parts of the grid.

(c) The school buys a uniform for each choir member.

A girl's uniform costs $25. A boy's uniform costs $20.

Find the maximum possible cost for the choir uniforms. Mark clearly the point \( P \) on your graph which you use to calculate this cost.

9  
(a) Write down the next two terms in the sequence

\[ 7, \ 10, \ 15, \ 22, \ \ldots \]

(b) Write down the next term and the 50th term in the sequence

\[ \frac{2}{5}, \ \frac{3}{7}, \ \frac{4}{9}, \ \frac{5}{11}, \ \ldots \]

(c) Write down the next two terms and find an expression for the \( n \)th term in the sequence

\[ 17, \ 13, \ 9, \ 5, \ \ldots \]
INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer all questions.

Write your answers and working on the separate answer paper provided.

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For π, use either your calculator value or 3.142.
1 (a) One day Amit works from 0800 until 1700.
The time he spends on filing, computing, writing and having lunch is in the ratio

Calculate the time he spends
(i) writing, [1]
(ii) having lunch, giving this answer in minutes. [1]

(b) The amount earned by Amit, Bernard and Chris is in the ratio 2:5:3.
Bernard earns $855 per week.
Calculate how much
(i) Amit earns each week, [1]
(ii) Chris earns each week. [1]

(c) After 52 weeks Bernard has saved $2964.
What fraction of his earnings has he saved?
Give your answer in its lowest terms. [2]

(d) Chris saves $3500 this year. This is 40% more than he saved last year.
Calculate how much he saved last year. [3]

2

\[ \begin{array}{c}
\text{North} \\
A \\
\text{88 m} \\
O \\
B \\
C \\
\text{NOT TO SCALE} \\
\end{array} \]

\( \triangle OABC \) is a field.
A is 88 metres due North of \( O \).
\( B \) is 146 metres from \( O \) on a bearing of 040°.
\( C \) is equidistant from \( A \) and from \( B \). The bearing of \( C \) from \( O \) is 098°.

(a) Using a scale of 1 centimetre to represent 10 metres, make an accurate scale drawing of the field \( \triangle OABC \), by
(i) constructing the triangle \( OAB \), [3]
(ii) drawing the locus of points equidistant from \( A \) and from \( B \), [2]
(iii) completing the scale diagram of \( OABC \). [2]
(b) Use your scale drawing to write down

(i) the distance $OC$ correct to the nearest metre,

(ii) the size of angle $OAB$ correct to the nearest degree.

(c) Find the bearing of $A$ from $B$.

(d) A donkey in the field is not more than 40 metres from $C$ and is closer to $B$ than to $A$. Shade the area where the donkey could be and label it $D$.

(e) A horse in the field is not more than 20 metres from the side $AB$ and is closer to $A$ than to $B$. Shade the area where the horse could be and label it $II$.

3 Paula and Tarek take part in a quiz.
The probability that Paula thinks she knows the answer to any question is 0.6.
If Paula thinks she knows, the probability that she is correct is 0.9.
Otherwise she guesses and the probability that she is correct is 0.2.

(a) Copy and complete the tree diagram.

(b) Find the probability that Paula

(i) thinks she knows the answer and is correct.

(ii) gets the correct answer.

(c) The probability that Tarek thinks he knows the answer to any question is 0.55.
If Tarek thinks he knows, he is always correct.
Otherwise he guesses and the probability that he is correct is 0.2.

(i) Draw a tree diagram for Tarek. Write all the probabilities on your diagram.

(ii) Find the probability that Tarek gets the correct answer.

(d) There are 100 questions in the quiz.
Estimate the number of correct answers given by

(i) Paula,

(ii) Tarek.
A sphere, centre C, rests on horizontal ground at A and touches a vertical wall at D. A straight plank of wood, GBW, touches the sphere at B, rests on the ground at G and against the wall at W. The wall and the ground meet at X.
Angle \( \angle WGX = 42^\circ \).

(a) Find the values of \( a \), \( b \), \( c \), \( d \) and \( e \) marked on the diagram. \([5]\)

(b) Write down one word which completes the following sentence.

'Angle CGA is 21° because triangle GBC and triangle GAC are .................'. \([1]\)

(c) The radius of the sphere is 54 cm.
   (i) Calculate the distance GA. Show all your working. \([3]\)
   (ii) Show that \( GX = 195 \) cm correct to the nearest centimetre. \([1]\)
   (iii) Calculate the length of the plank GW. \([3]\)
   (iv) Find the distance BW. \([1]\)

---

5 Answer the whole of this question on a sheet of graph paper.

Dimitra stands by a river and watches a fish. The distance (\( d \) metres) of the fish from Dimitra after \( t \) minutes is given by

\[
d = (t + 1)^2 + \frac{48}{(t + 1)} - 20.
\]

Some values for \( d \) and \( t \) are given in the table below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d  )</td>
<td>14.3</td>
<td>8</td>
<td>5.5</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>10.9</td>
<td>14.6</td>
<td>( q )</td>
<td>35.9</td>
<td>( r )</td>
<td></td>
</tr>
</tbody>
</table>

(a) Find the values of \( p \), \( q \) and \( r \). \([3]\)
(b) Using a scale of 2 cm to represent 1 minute on the horizontal \( t \)-axis and 2 cm to represent 10 metres on the vertical \( d \)-axis, draw the graph of \[ d = (t + 1)^2 + \frac{48}{(t + 1)} - 20 \] for \( 0 \leq t \leq 7 \). [6]

(c) Mark and label \( F \) the point on your graph when the fish is 12 metres from Dimitra and swimming away from her. Write down the value of \( t \) at this point, correct to one decimal place. [2]

(d) For how many minutes is the fish less than 10 metres from Dimitra? [2]

(e) By drawing a suitable line on your grid, calculate the speed of the fish when \( t = 2.5 \). [4]

---

**An equilateral 16-sided figure \( APAB \) is formed when the square \( ABCD \) is rotated 45° clockwise about its centre to position \( A'B'C'D' \). \( AB = 12 \text{ cm} \) and \( AP = x \text{ cm} \).**

(a) (i) Use triangle \( PA'Q \) to explain why \( 2x^2 = (12 - 2x)^2 \). [3]

(ii) Show that this simplifies to \( x^2 - 24x + 72 = 0 \). [3]

(iii) Solve \( x^2 - 24x + 72 = 0 \). Give your answers correct to 2 decimal places. [4]

(b) (i) Calculate the perimeter of the 16-sided figure. [2]

(ii) Calculate the area of the 16-sided figure. [3]
(a) Describe fully a single transformation which maps both

(i) A onto C and B onto D. [2]
(ii) A onto D and B onto C. [2]
(iii) A onto P and B onto Q. [3]

(b) Describe fully a single transformation which maps triangle OAB onto triangle JFE. [2]

(c) The matrix M is

\[
\begin{pmatrix}
0 & -1 \\
-1 & 0
\end{pmatrix}
\]

(i) Describe the transformation which M represents. [2]
(ii) Write down the co-ordinates of P after transformation by matrix M. [2]

(d) (i) Write down the matrix R which represents a rotation by 90° anticlockwise about O. [2]
(ii) Write down the letter representing the new position of P after the transformation RM(F). [2]
(a) A sector of a circle, radius 6 cm, has an angle of 20°.

Calculate

(i) the area of the sector,
(ii) the arc length of the sector. [2] [2]

(b) A whole cheese is a cylinder, radius 6 cm and height 5 cm.
The diagram shows a slice of this cheese with sector angle 20°.

Calculate

(i) the volume of the slice of cheese,
(ii) the total surface area of the slice of cheese. [2] [4]

(c) The radius, \( r \), and height, \( h \), of cylindrical cheeses vary but the volume remains constant.

(i) Which one of the following statements \( A, B, C \) or \( D \) is true?

- \( A \): \( h \) is proportional to \( r \).
- \( B \): \( h \) is proportional to \( r^2 \).
- \( C \): \( h \) is inversely proportional to \( r \).
- \( D \): \( h \) is inversely proportional to \( r^2 \). [2]

(ii) What happens to the height \( h \) of the cylindrical cheese when the volume remains constant but the radius is doubled? [2]
(a) The number of people living in six houses is 3, 8, 4, $x$, $y$ and $z$.

The median is 7½.
The mode is 8.
The mean is 7.

Find a value for each of $x$, $y$ and $z$.

(b) The grouped frequency table below shows the amount ($\$A$) spent on travel by a number of students.

<table>
<thead>
<tr>
<th>Cost of travel ($$A$)</th>
<th>$0 &lt; A \leq 10$</th>
<th>$10 &lt; A \leq 20$</th>
<th>$20 &lt; A \leq 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>15</td>
<td>$m$</td>
<td>$n$</td>
</tr>
</tbody>
</table>

(i) Write down an estimate for the total amount in terms of $m$ and $n$. [2]

(ii) The calculated estimate of the mean amount is $\$13$ exactly.

Write down an equation containing $m$ and $n$.

Show that it simplifies to $2m + 17n = 120$. [3]

(iii) A student drew a histogram to represent this data.

The area of the rectangle representing the $0 < A \leq 10$ group was equal to the sum of the areas of the other two rectangles.

Explain why $m + n = 15$. [1]

(iv) Find the values of $m$ and $n$ by solving the simultaneous equations

$$2m + 17n = 120,$$

$$m + n = 15.$$ [3]
CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

MATHEMATICS
PAPER 4

OCTOBER/NOVEMBER SESSION 2002

2 hours 30 minutes

Additional materials:
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Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables (optional)
Tracing paper (optional)

TIME 2 hours 30 minutes

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For π, use either your calculator value or 3.142.
1. (a) At an athletics meeting, Ben’s time for the 10,000 metres race was 33 minutes exactly and he finished at 15:17.

(i) At what time did the race start? [1]

(ii) What was Ben’s average speed for the race? Give your answer in kilometres per hour. [2]

(iii) The winner finished 51.2 seconds ahead of Ben. How long did the winner take to run the 10,000 metres? [1]

(b) The winning distance in the javelin competition was 80 metres. Otto’s throw was 95% of the winning distance. Calculate the distance of Otto’s throw. [2]

(c) Pamela won the long jump competition with a jump of 6.16 metres. This was 10% further than Mona's jump. How far did Mona jump? [2]

2. The diagram shows a sketch of the net of a solid tetrahedron (triangular prism). The right-angled triangle $ABC$ is its base. $AC = 8$ cm, $BC = 6$ cm and $AB = 10$ cm. $FC = CE = 5$ cm.

(a) (i) Show that $BE = \sqrt{61}$ cm. [1]

(ii) Write down the length of $DB$. [1]

(iii) Explain why $DA = \sqrt{89}$ cm. [2]

(b) Calculate the size of angle $DBA$. [4]

(c) Calculate the area of triangle $DBA$. [3]

(d) Find the total surface area of the solid. [3]

(e) Calculate the volume of the solid. [The volume of a tetrahedron is $\frac{1}{3}$ (area of the base) $\times$ perpendicular height.] [3]
Answer the whole of this question on a sheet of graph paper.

(a) Using a scale of 1 cm to represent 1 unit on each axis, draw an $x$-axis for $-6 \leq x \leq 10$ and a $y$-axis for $-8 \leq y \leq 8$.

Copy the word EXAM onto your grid so that it is exactly as it is in the diagram above.

Mark the point $P(6,6)$. [2]

(b) Draw accurately the following transformations.

(i) Reflect the letter E in the line $x = 0$. [2]

(ii) Enlarge the letter X by scale factor 3 about centre $P(6,6)$. [2]

(iii) Rotate the letter A $90^\circ$ anticlockwise about the origin. [2]

(iv) Stretch the letter M vertically with scale factor 2 and $x$-axis invariant. [2]

(c) (i) Mark and label the point $Q$ so that $PQ = \left(-\frac{3}{2}\right)$. [1]

(ii) Calculate $|PQ|$ correct to two decimal places. [2]

(iii) Mark and label the point $S$ so that $PS = \left(-\frac{4}{1}\right)$. [1]

(iv) Mark and label the point $R$ so that $PQRS$ is a parallelogram. [1]
A wheel is divided into 10 sectors numbered 1 to 10 as shown in the diagram. The sectors 1, 2, 3 and 4 are shaded. The wheel is spun and when it stops the fixed arrow points to one of the sectors. (Each sector is equally likely.)

(a) The wheel is spun once so that one sector is selected. Find the probability that

(i) the number in the sector is even,
(ii) the sector is shaded,
(iii) the number is even or the sector is shaded,
(iv) the number is odd and the sector is shaded.

(b) The wheel is spun twice so that each time a sector is selected. Find the probability that

(i) both sectors are shaded,
(ii) one sector is shaded and one is not,
(iii) the sum of the numbers in the two sectors is greater than 20,
(iv) the sum of the numbers in the two sectors is less than 4,
(v) the product of the numbers in the two sectors is a square number.
5 Answer the whole of this question on a sheet of graph paper.

(a) The table gives values of \( f(x) = \frac{24}{x^2} + x^2 \) for \( 0.8 \leq x \leq 6 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.8</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
<th>5.5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>38.1</td>
<td>25</td>
<td>12.9</td>
<td>10</td>
<td>10.1</td>
<td>11.7</td>
<td>( l )</td>
<td>( m )</td>
<td>( n )</td>
<td>26</td>
<td>31</td>
<td>36.7</td>
</tr>
</tbody>
</table>

Calculate, correct to 1 decimal place, the values of \( l, m \) and \( n \). \[3\]

(b) Using a scale of 2 cm to represent 1 unit on the \( x \)-axis and 2 cm to represent 5 units on the \( y \)-axis, draw an \( x \)-axis for \( 0 \leq x \leq 6 \) and a \( y \)-axis for \( 0 \leq y \leq 40 \).

Draw the graph of \( y = f(x) \) for \( 0.8 \leq x \leq 6 \). \[6\]

(c) Draw the tangent to your graph at \( x = 1.5 \) and use it to calculate an estimate of the gradient of the curve at this point. \[4\]

(d) (i) Draw a straight line joining the points \((0, 20)\) and \((6, 32)\).
(ii) Write down the equation of this line in the form \( y = mx + c \). \[1\]
(iii) Use your graph to write down the \( x \)-values of the points of intersection of this line and the curve \( y = f(x) \). \[2\]
(iv) Draw the tangent to the curve which has the same gradient as your line in part d(i). \[1\]
(v) Write down the equation for the tangent in part d(iv). \[2\]

6 (a) On 1st January 2000, Ashraf was \( x \) years old.
Bukki was 5 years older than Ashraf and Claude was twice as old as Ashraf.

(i) Write down in terms of \( x \), the ages of Bukki and Claude on 1st January 2000. \[2\]
(ii) Write down in terms of \( x \), the ages of Ashraf, Bukki and Claude on 1st January 2002. \[1\]
(iii) The product of Claude’s age and Ashraf’s age on 1st January 2002 is the same as the square of Bukki’s age on 1st January 2000.
Write down an equation in \( x \) and show that it simplifies to \( x^2 - 4x - 21 = 0 \). \[4\]
(iv) Solve the equation \( x^2 - 4x - 21 = 0 \). \[2\]
(v) How old was Claude on 1st January 2002? \[1\]

(b) Claude’s height, \( h \) metres, is one of the solutions of \( h^2 + 8h - 17 = 0 \).

(i) Solve the equation \( h^2 + 8h - 17 = 0 \).

Show all your working and give your answers correct to 2 decimal places. \[4\]

(ii) Write down Claude’s height, to the nearest centimetre. \[1\]
7 (a) A group of students sat an examination. Each student got one of the grades A, B, C or D. The pie chart shows these results.

36 students got grade A, shown by an angle of $108^\circ$.

(i) Calculate the total number of students who sat the examination.

(ii) How many students did not get grade A?

(iii) The ratio of the number of students getting grades B, C or D is 4 : 5 : 3. Find the number of students getting each grade.

(iv) Work out the angles in the pie chart for grades B, C and D.

(v) Find the ratio, in its lowest terms, the number of students with grade A : the number of students with grade B.

(b) A group of children were asked how much money they had saved. The histogram and table show the results.

<table>
<thead>
<tr>
<th>Money saved ($/m)</th>
<th>$0 &lt; m \leq 20$</th>
<th>$20 &lt; m \leq 30$</th>
<th>$30 &lt; m \leq 40$</th>
<th>$40 &lt; m \leq 70$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>25</td>
<td>$p$</td>
<td>$q$</td>
<td>$r$</td>
</tr>
</tbody>
</table>

Use the histogram to calculate the values of $p$, $q$ and $r$. [4]
NOT TO SCALE

Sarah investigates cylindrical plant pots.
The standard pot has base radius $r$ cm and height $h$ cm.
Pot $A$ has radius $3r$ and height $h$. Pot $B$ has radius $r$ and height $3h$. Pot $C$ has radius $3r$ and height $3h$.

(a) (i) Write down the volumes of pots $A$, $B$ and $C$ in terms of $\pi$, $r$ and $h$.  
(ii) Find in its lowest terms the ratio of the volumes of $A : B : C$.  
(iii) Which one of the pots $A$, $B$ or $C$ is mathematically similar to the standard pot? Explain your answer.  
(iv) The surface area of the standard pot is $Scm^2$. Write down in terms of $S$ the surface area of the similar pot.

(b) Sarah buys a cylindrical plant pot with radius $15$ cm and height $20$ cm. She wants to paint its outside surface (base and curved surface area).
(i) Calculate the area she wants to paint.  
(ii) Sarah buys a tin of paint which will cover $30m^2$. How many plant pots of this size could be completely painted on their outside surfaces using this tin of paint?

9 (a) Write down the 10th term and the $n$th term of the following sequences.
(i) $1, 2, 3, 4, 5 \ldots, \ldots, \ldots, \ldots$  
(ii) $7, 8, 9, 10, 11 \ldots, \ldots, \ldots$  
(iii) $8, 10, 12, 14, 16 \ldots, \ldots, \ldots$  

(b) Consider the sequence
$1(8 - 7), 2(10 - 8), 3(12 - 9), 4(14 - 10), \ldots, \ldots, \ldots, \ldots$
(i) Write down the next term and the 10th term of this sequence in the form $a(b - c)$ where $a$, $b$ and $c$ are integers.  
(ii) Write down the $n$th term in the form $a(b - c)$ and then simplify your answer.
CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

MATHEMATICS

Paper 4

0580/04
0581/04

May/June 2003

2 hours 30 minutes

Additional Materials: Answer Booklet/Paper
Electronic calculator
Geometric instruments
Graph paper (2 sheets)
Mathematical tables (optional)
Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer.
Marks will be given for working which shows that you know how to solve the problem even if you get the
answer wrong.
The total of the marks for this paper is 130.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to
three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.
1 Tickets for the theatre cost either $10 or $16.

(a) Calculate the total cost of 197 tickets at $10 each and 95 tickets at $16 each. [1]

(b) On Monday, 157 tickets at $10 and $ tickets at $16 were sold. The total cost was $4018. Calculate the value of $n$. [2]

(c) On Tuesday, 319 tickets were sold altogether. The total cost was $3784. Using x for the number of $10 tickets sold and y for the number of $16 tickets sold, write down two equations in x and y.

Solve your equations to find the number of $10 tickets and the number of $16 tickets sold. [5]

(d) On Wednesday, the cost of a $16 ticket was reduced by 15%. Calculate this new reduced cost. [2]

(e) The $10 ticket costs 25% more than it did last year. Calculate the cost last year. [2]

2

\[ \text{NOT TO SCALE} \]

In quadrilateral \(ABCD\), \(AB = 77\) m, \(BC = 120\) m, \(CD = 60\) m and diagonal \(AC = 55\) m. Angle \(CAD = 45^\circ\), angle \(BAC = x^\circ\) and angle \(ADC = y^\circ\).

(a) Calculate the value of \(x\). [4]

(b) Calculate the value of \(y\). [4]

(c) The bearing of \(D\) from \(A\) is 090°. Find the bearing of 
   (i) \(A\) from \(C\). [2]
   (ii) \(B\) from \(A\). [2]
There are 2 sets of road signals on the direct 12 kilometre route from Acity to Beeton.
The signals say either “GO” or “STOP”.
The probabilities that the signals are “GO” when a car arrives are shown in the tree diagram.

(a) Copy and complete the tree diagram for a car driver travelling along this route.

\[
\begin{array}{c}
\text{1st signal} & \text{2nd signal} \\
0.4 & 0.65 \\
\text{GO} & \text{GO} \\
\text{STOP} & \text{STOP} \\
0.45 & 0.45 \\
& \text{STOP} \\
& \text{GO} \\
\end{array}
\]

(b) Find the probability that a car driver
   (i) finds both signals are “GO”, [2]
   (ii) finds exactly one of the two signals is “GO”, [3]
   (iii) does not find two “STOP” signals. [2]

(c) With no stops, Damon completes the 12 kilometre journey at an average speed of 40 kilometres per hour.
   (i) Find the time taken in minutes for this journey. [1]
   (ii) When Damon has to stop at a signal it adds 3 minutes to this journey time.

   Calculate his average speed, in kilometres per hour, if he stops at both road signals. [2]

(d) Elsa takes a different route from Acity to Beeton.
   This route is 15 kilometres and there are no road signals.
   Elsa’s average speed for this journey is 40 kilometres per hour.
   Find
   (i) the time taken in minutes for this journey. [1]
   (ii) the probability that Damon takes more time than this on his 12 kilometre journey. [2]
4 Answer the whole of this question on a sheet of graph paper.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-8</td>
<td>4.5</td>
<td>8</td>
<td>5.5</td>
<td>0</td>
<td>-5.5</td>
<td>-8</td>
<td>-4.5</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 4 units on the $y$-axis, draw axes for $-4 < x < 4$ and $-8 < y < 8$.
   Draw the curve $y = f(x)$ using the table of values given above.  
   5

(b) Use your graph to solve the equation $f(x) = 0$.  
   2

(c) On the same grid, draw $y = g(x)$ for $-4 < x < 4$, where $g(x) = x + 1$.  
   2

(d) Write down the value of
   (i) $g(1)$,
   (ii) $f_{g(1)}$,
   (iii) $g^{-1}(4)$,
   (iv) the positive solution of $f(x) = g(x)$.  
   4

(e) Draw the tangent to $y = f(x)$ at $x = 3$. Use it to calculate an estimate of the gradient of the curve at this point.  
   3

5 (a) Calculate the area of an equilateral triangle with sides 10 cm.  
   2

(b) Calculate the radius of a circle with circumference 10 cm.  
   2

(c)

The diagrams represent the nets of 3 solids. Each straight line is 10 cm long. Each circle has circumference 10 cm. The arc length in Diagram 3 is 10 cm.

(i) Name the solid whose net is Diagram 1. Calculate its surface area.  
   3

(ii) Name the solid whose net is Diagram 2. Calculate its volume.  
   4

(iii) Name the solid whose net is Diagram 3. Calculate its perpendicular height.  
   4
A rectangular-based open box has external dimensions of $2x$ cm, $(x + 4)$ cm and $(x + 1)$ cm.

(a) (i) Write down the volume of a cuboid with these dimensions. 
(ii) Expand and simplify your answer. 

(b) The box is made from wood 1 cm thick.
   (i) Write down the internal dimensions of the box in terms of $x$.
   (ii) Find the volume of the inside of the box and show that the volume of the wood is $8x^3 + 12x$ cubic centimetres.

(c) The volume of the wood is 1980 cm$^3$.
   (i) Show that $2x^2 + 3x - 495 = 0$ and solve this equation.
   (ii) Write down the external dimensions of the box.
A star is made up of a regular hexagon, centre \(X\), surrounded by 6 equilateral triangles. \(\overline{OA} = a\) and \(\overline{OB} = b\).

(a) Write the following vectors in terms of \(a\) and/or \(b\), giving your answers in their simplest form.

(i) \(\overline{OS}\),
(ii) \(\overline{AB}\),
(iii) \(\overline{CD}\),
(iv) \(\overline{OR}\),
(v) \(\overline{CF}\).

(b) When \(|a| = 5\), write down the value of

(i) \(|b|\),
(ii) \(|a - b|\).

(c) Describe fully a single transformation which maps

(i) triangle \(OBA\) onto triangle \(OQS\),
(ii) triangle \(OBA\) onto triangle \(RDE\), with \(O\) mapped onto \(R\) and \(B\) mapped onto \(D\).

(d) (i) How many lines of symmetry does the star have?
(ii) When triangle \(OQS\) is rotated clockwise about \(X\), it lies on triangle \(PRT\), with \(O\) on \(P\). Write down the angle of rotation.
8 Answer the whole of this question on a sheet of graph paper.

In a survey, 200 shoppers were asked how much they had just spent in a supermarket. The results are shown in the table.

<table>
<thead>
<tr>
<th>Amount ($x$)</th>
<th>$0 &lt; x &lt; 20$</th>
<th>$20 &lt; x &lt; 40$</th>
<th>$40 &lt; x &lt; 60$</th>
<th>$60 &lt; x &lt; 80$</th>
<th>$80 &lt; x &lt; 100$</th>
<th>$100 &lt; x &lt; 140$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of shoppers</td>
<td>10</td>
<td>32</td>
<td>48</td>
<td>54</td>
<td>36</td>
<td>20</td>
</tr>
</tbody>
</table>

(a) (i) Write down the modal class. [1]

(ii) Calculate an estimate of the mean amount, giving your answer correct to 2 decimal places. [4]

(b) (i) Make a cumulative frequency table for these 200 shoppers. [2]

(ii) Using a scale of 2 cm to represent $20 on the horizontal axis and 2 cm to represent 20 shoppers on the vertical axis, draw a cumulative frequency diagram for this data. [4]

(c) Use your cumulative frequency diagram to find

(i) the median amount, [1]

(ii) the upper quartile, [1]

(iii) the interquartile range, [1]

(iv) how many shoppers spent at least $75. [2]

Question 9 is on the next page
Diagram 1 shows a triangle with its base divided in the ratio 1:3.

Diagram 2 shows a parallelogram with its base divided in the ratio 1:3.

Diagram 3 shows a kite with a diagonal divided in the ratio 1:3.

Diagram 4 shows two congruent triangles and a trapezium each of height 1 unit.

For each of the four diagrams, write down the percentage of the total area which is shaded. [7]

Diagram 5 shows a semicircle, centre $O$.

Diagram 6 shows two circles with radii 1 unit and 5 units.

Diagram 7 shows two sectors, centre $O$, with radii 2 units and 3 units.

For each of diagrams 5, 6 and 7, write down the fraction of the total area which is shaded. [6]
READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer.

Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

The total of the marks for this paper is 130.

Electronic calculators should be used.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Answers in degrees should be given to one decimal place.

For \( \pi \), use either your calculator value or 3.142.
1 A train starts its journey with 240 passengers. 144 of the passengers are adults and the rest are children.

(a) Write the ratio Adults : Children in its lowest terms. [2]

(b) At the first stop, 37\% of the adults and \frac{1}{3} of the children get off the train. 20 adults and \( x \) children get onto the train. The total number of passengers on the train is now 200.

(i) How many children got off the train? [1]
(ii) How many adults got off the train? [1]
(iii) How many adult passengers are on the train as it sets off again? [1]
(iv) What is the value of \( x \)? [1]

(c) After a second stop, there are 300 passengers on the train and the ratio Men : Women : Children is 6 : 5 : 4. Calculate the number of children now on the train. [2]

(d) On Tuesday the train journey took 7 hours and 20 minutes and began at 13 53.

(i) At what time did the train journey end? [1]
(ii) Tuesday’s time of 7 hours 20 minutes was 10\% more than Monday’s journey time. How many minutes longer was Tuesday’s journey? [2]

2 (a) The surface area of a person’s body, \( A \) square metres, is given by the formula

\[ A = \sqrt{\frac{hm}{3600}} \]

where \( h \) is the height in centimetres and \( m \) is the mass in kilograms.

(i) Dolores is 167 cm high and has a mass of 70 kg. Calculate the surface area of her body. [1]

(ii) Erik has a mass of 80 kg. Find his height if \( A = 1.99 \). [2]

(iii) Make \( h \) the subject of the formula. [3]

(b) Factorise

(i) \( x^2 - 16 \), [1]
(ii) \( x^2 - 16x \), [1]
(iii) \( x^2 - 9x + 8 \). [2]
(c) Erik runs a race at an average speed of \(x\) m/s. His time is \((3x - 9)\) seconds and the race distance is \((2x^2 - 8)\) metres.

(i) Write down an equation in \(x\) and show that it simplifies to
\[x^2 - 9x + 8 = 0.\] [2]

(ii) Solve \(x^2 - 9x + 8 = 0\). [2]

(iii) Write down Erik's time and the race distance. [2]

---

Felipe \(F\) stands 17 metres from a bridge \(B\) and 32 metres from a tree \(T\). The points \(F\), \(B\) and \(T\) are on level ground and angle \(BFT = 40^\circ\).

(a) Calculate

(i) the distance \(BT\), [4]

(ii) the angle \(BTF\). [3]

(b) The bearing of \(B\) from \(F\) is 085°. Find the bearing of

(i) \(T\) from \(F\), [1]

(ii) \(F\) from \(T\), [1]

(iii) \(B\) from \(T\). [1]

(c) The top of the tree is 30 metres vertically above \(T\). Calculate the angle of elevation of the top of the tree from \(F\). [2]
4 Answer the whole of this question on a sheet of graph paper.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(t)$</td>
<td>0</td>
<td>25</td>
<td>37.5</td>
<td>43.8</td>
<td>46.9</td>
<td>48.4</td>
<td>49.2</td>
<td>49.6</td>
</tr>
</tbody>
</table>

(a) Using a scale of 2 cm to represent 1 unit on the horizontal $t$-axis and 2 cm to represent 10 units on the $y$-axis, draw axes for $0 < t < 7$ and $0 < y < 60$. Draw the graph of the curve $y = f(t)$ using the table of values above. [5]

(b) $f(t) = 50\left(1 - 2^{-t}\right)$.
   (i) Calculate the value of $f(8)$ and the value of $f(9)$. [2]
   (ii) Estimate the value of $f(t)$ when $t$ is large. [1]

(c) (i) Draw the tangent to $y = f(t)$ at $t = 2$ and use it to calculate an estimate of the gradient of the curve at this point. [3]
   (ii) The function $f(t)$ represents the speed of a particle at time $t$. Write down what quantity the gradient gives. [1]

(d) (i) On the same grid, draw $y = g(t)$ where $g(t) = 6t + 10$, for $0 < t < 7$. [2]
   (ii) Write down the range of values for $t$ where $f(t) > g(t)$. [2]
   (iii) The function $g(t)$ represents the speed of a second particle at time $t$. State whether the first or second particle travels the greater distance for $0 < t < 7$. You must give a reason for your answer. [2]

5 Adam writes his name on four red cards and Daniel writes his name on six white cards.

(a) One of the ten cards is chosen at random. Find the probability that
   (i) the letter on the card is D, [1]
   (ii) the card is red, [1]
   (iii) the card is red or the letter on the card is D, [1]
   (iv) the card is red and the letter on the card is D, [1]
   (v) the card is red and the letter on the card is N. [1]
(b) Adam chooses a card at random and then Daniel chooses one of the remaining 9 cards at random.
Giving your answers as fractions, find the probability that the letters on the two cards are
(i) both D, [2]
(ii) both A, [2]
(iii) the same, [2]
(iv) different. [2]

6  (a) Calculate the volume of a cylinder with radius 30 cm and height 50 cm. [2]

(b) A cylindrical tank, radius 30 cm and length 50 cm, lies on its side.
It is partially filled with water.
The shaded segment \( AXBY \) in the diagram shows the cross-section of the water.
The greatest depth, \( XY \), is 12 cm.
\( OA = OB = 30 \) cm.

(i) Write down the length of \( OX \). [1]

(ii) Calculate the angle \( AOB \) correct to two decimal places, showing all your working. [3]

(c) Using angle \( AOB = 106.3^\circ \), find
(i) the area of the sector \( AOBY \), [3]
(ii) the area of triangle \( AOB \), [2]
(iii) the area of the shaded segment \( AXBY \). [1]

(d) Calculate the volume of water in the cylinder, giving your answer
(i) in cubic centimetres, [2]
(ii) in litres. [1]

(e) How many more litres must be added to make the tank half full? [2]
Use one of the letters $A, B, C, D, E$ or $F$ to answer the following questions.

(i) Which triangle is $T$ mapped onto by a translation? Write down the translation vector. [2]

(ii) Which triangle is $T$ mapped onto by a reflection? Write down the equation of the mirror line. [2]

(iii) Which triangle is $T$ mapped onto by a rotation? Write down the coordinates of the centre of rotation. [2]

(iv) Which triangle is $T$ mapped onto by a stretch with the $x$-axis invariant? Write down the scale factor of the stretch. [2]

(v) $M = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$. Which triangle is $T$ mapped onto by $M$?

Write down the name of this transformation. [2]

(b) $P = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}$, $Q = (-1, -2)$, $R = (1, 2, 3)$, $S = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$

Only some of the following matrix operations are possible with matrices $P, Q, R$ and $S$ above: $PQ, \quad QP, \quad P + Q, \quad PR, \quad RS$

Write down and calculate each matrix operation that is possible. [6]
8 Answer the whole of this question on a sheet of graph paper.  
120 passengers on an aircraft had their baggage weighed. The results are shown in the table.

<table>
<thead>
<tr>
<th>Mass of baggage (M kg)</th>
<th>$0 &lt; M &lt; 10$</th>
<th>$10 &lt; M &lt; 15$</th>
<th>$15 &lt; M &lt; 20$</th>
<th>$20 &lt; M &lt; 25$</th>
<th>$25 &lt; M &lt; 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of passengers</td>
<td>12</td>
<td>32</td>
<td>28</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>

(a) (i) Write down the modal class. [1]

(ii) Calculate an estimate of the mean mass of baggage for the 120 passengers. Show all your working. [4]

(iii) Sophia draws a pie chart to show the data. What angle should she have in the $0 < M < 10$ sector? [1]

(b) Using a scale of 2 cm to represent 5 kg, draw a horizontal axis for $0 < M < 40$. Using an area scale of 1 cm² to represent 1 passenger, draw a histogram for this data. [7]

9 In each of the diagrams below, triangle $ABC$ is an isosceles right-angled triangle. $AB = AC = 6$ cm.

A straight line or a circular arc divides the triangle into two parts, one of which is shaded.

(a) Which diagram has a shaded region showing all the points in the triangle which are

(i) closer to $BC$ than to $BA$, [1]

(ii) more than 3 cm from $A$, [1]

(iii) closer to $C$ than to $A$? [1]

(b) For each of the five diagrams, calculate the shaded area. [7:1]