① Standard Form

$A \times 10^n$

$1 \leq A < 10$ & n can be +ve or -ve

Example

Express in standard form:

a) $321000 = 3.21 \times 10^5$

b) $0.000678 = 6.78 \times 10^{-4}$

② Prime number

Memorise all prime numbers from 2 to 71.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71

③ Upper & Lower Bound

Example

Each of the length is measured correct to the nearest centimetre

Find:

(a) the upper bound for the perimeter &

(b) the lower bound for the perimeter

Answer

(a) Upper bound ⇒ round all reading up by 0.5 cm.

10cm ⇒ 10.5cm

5cm ⇒ 5.5cm

Perimeter = 10.5 + 10.5 + 5.5 + 5.5

= 32cm

b) Lower bound ⇒ round all reading down by 0.5 cm.

10cm ⇒ 9.5cm

5cm ⇒ 4.5cm

Perimeter = 9.5 + 9.5 + 4.5 + 4.5

= 28cm

④ Direct & Inverse Proportion

Example 1

x is directly proportional to y.

When $y = 10$, $x = 5$

Find x when $y = 20$.

Answer

$x = ky$

$5 = k(10)$

$k = \frac{5}{10} = \frac{1}{2}$

$x = \frac{1}{2}y$

When $y = 20$,

$x = \frac{1}{2}(20)$

$= 10$

Example 2

x is inversely proportional to y.

When $y = 10$, $x = 2$

Find x when $y = 30$.

Answer

$x = \frac{k}{y}$

$2 = \frac{k}{10}$

$k = 2 \times 10$

$= 20$

$x = \frac{20}{y}$

When $y = 30$

$x = \frac{20}{30}$

$= \frac{2}{3}$

⑤ Percentage

Example

Express 64 as a percentage of 80.

Answer

$\frac{64}{80} \times 100\% = 80\%$

⑦ Simple & Compound Interest

Simple interest $(I) = \frac{PRT}{100}$

P = principal, R = rate, T = time

Example 1

Calculate the interest owed if a man borrows $300 from a bank charging 2% simple interest per month for 3 months?

$P = 300$, $R = 2\%$ $T = 3$

$I = \frac{300 \times 2 \times 3}{100}$

$= \$18$

Compound Interest

$A = P(1 + \frac{R}{100})^n$

A = total amount after time, n

P = principal

R = rate

n = time

Example 2

Caculate the total amount owed if a man borrows $300 from a bank charging 2% compound interest per month for 3 months?

$P = 300$. $R = 2$, $n = 3$

Total amount owed $= P(1 + \frac{R}{100})^n$

$= 300(1 + \frac{2}{100})^3$

$= \$318.36$

⑧ Gradient of a straight line

$\text{Gradient} = \frac{y_1 - y_2}{x_1 - x_2}$

Example

Calculate the gradient of a line that passes through point A(-2, -1) & B(4, 2)

Answer

A(-2, -1) B(4, 2)

x_1 y_1 x_2 y_2

$\text{Gradient} = \frac{-1 - 2}{-2 - 4}$

$= \frac{-3}{-6}$

$= \frac{1}{2}$

⑨ Equation of a line

$y = mx + c$

① find the gradient, m.

② find the y-intercept, c.

Example

Find the equation of a line that passes through A(-2, -1) & B(4, 2)

Answer

Eqn of a line $\Rightarrow y = mx + c$

$m = \frac{1}{2}$ (found above in ⑧)

$y = \frac{1}{2}x + c$

To find c, sub in point A.

$-1 = \frac{1}{2}(-2) + c$

$-1 = -1 + c$

$c = 0$

Sub in the value of m & c to $y = mx + c$

$y = \frac{1}{2}x + 0$

$y = \frac{1}{2}x$

(10) Midpoint of 2 given points

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$$

Example

Find the midpoint of $P(-2, 8)$ & $Q(4, -4)$

Answer

$P(-2, 8)$ $Q(4, -4)$
x_1 y_1 x_2 y_2

$$\text{Midpoint} = \left(\frac{-2+4}{2}, \frac{8+(-4)}{2}\right)$$
$$= (1, 2)$$

(11) Length between 2 points

$$\text{Length} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Example

Find the distance between $P(-2, 8)$ & $Q(4, -4)$

Answer

$P(-2, 8)$ $Q(4, -4)$
x_1 y_1 x_2 y_2

$$\text{Distance} = \sqrt{(-2-4)^2 + (8-(-4))^2}$$
$$= \sqrt{(-6)^2 + 12^2}$$
$$= \sqrt{180}$$
$$= 13.4 \text{ units.}$$

(12) Function

Example

$f(x) = 4x + 1$ $g(x) = x^3 + 1$

$f(2) \Rightarrow$ sub $x = 2$ & solve for $f(x)$

$$f(2) = 4(2) + 1$$
$$= 8 + 1$$
$$= 9$$

$fg(x) \Rightarrow$ sub $x = g(x)$

$$fg(x) = 4(x^3 + 1) + 1$$
$$= 4x^3 + 4 + 1$$
$$= 4x^3 + 5$$

$f^{-1}(x) \Rightarrow$ Let $y = f(x)$ & make x the subj.

Let $y = f(x) = 4x + 1$

$$y = 4x + 1$$
$$4x = y - 1$$
$$x = \frac{y-1}{4}$$
$$f^{-1}(x) = \frac{x-1}{4}$$

(13) Indices

$$a^m \times a^n = a^{m+n}$$

Example $3x^5 \times 4x^3 = 12x^{5+3}$
$= 12x^8$

$$a^m \div a^n = a^{m-n}$$

Example: $24x^7 \div 6x^3 = 4x^{7-3}$
$= 4x^4$

$$a^0 = 1$$

Example: $24x^7 \div 3x^7 = 8x^{7-7}$
$= 8x^0$
$= 8(1)$
$= 8$

$$(a^m)^n = a^{m \times n} = a^{mn}$$

Example: $(3x^2)^4 = 3^4 x^{2\times4}$

$= 81x^8$

$$(a \times b)^n = a^n \times b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n}$$

Example: $x^2y^5 \div x^7y^3 = x^{2-7}y^{5-3}$

$= x^{-3}y^2$

$= \frac{1}{x^3}y^2$

$= \frac{y^2}{x^3}$

$$a^{\frac{1}{n}} = \sqrt[n]{a}, \quad n \neq 0$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}, \quad n \neq 0$$

Solving Eqn involving Indices

$3^x \times 3^2 = 81$
$3^{x+2} = 81$
$3^{x+2} = 3^4$
} Make the base the same

$x+2 = 4$
$x = 4-2$
$= 2$
} Compare the powers

(14) Solving Quadratic Eqn

- By factorisation

Example: $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x-3=0 \quad x+2=0$

$x=3 \quad x=-2$

	x	-3	$-3x$
x	x	2	$2x$
x^2	-6		$-x$

- By formula

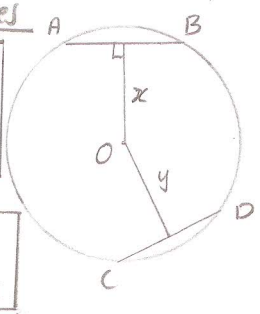
* When question says "give your answers correct to 2 decimal places.", USE FORMULA *

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(15) Symmetry Properties of Circles

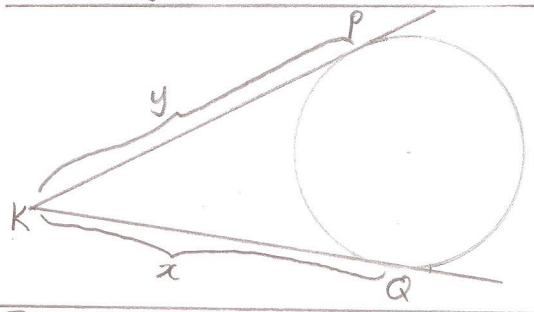
a) Equal chords are equidistant from centre

If AB = CD $\Rightarrow x = y$



b) Perpendicular bisector of chord passes through centre

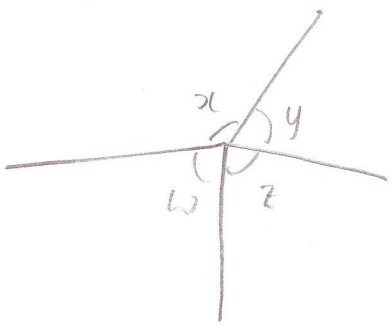
c) Tangents from an external point are equal in length $\Rightarrow$ KP = KQ $\Rightarrow x = y$



Paper 1 Qn

(16) Angle Properties

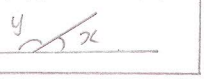
a) Angles at a point = 360°



$\angle w + \angle x + \angle y + \angle z = 360°$

b) Angles on a straight line = 180°

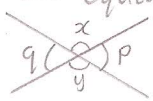
$\angle x + \angle y = 180°$



c) Vertically opp. angles are equal.

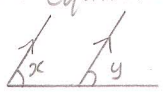
$\angle x = \angle y$

$\angle p = \angle q$



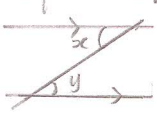
d) Corresponding angles are equal. (F)

$\angle x = \angle y$



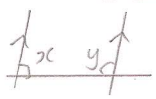
e) Alternate angles are equal (Z)

$\angle x = \angle y$



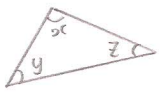
f) Interior angles = 180° (U)

$\angle x + \angle y = 180°$



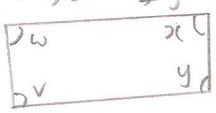
g) Angles in a Δ = 180°

$\angle x + \angle y + \angle z = 180°$



Angles in a quadrilateral = 360°

$\angle w + \angle v + \angle x + \angle y = 360°$

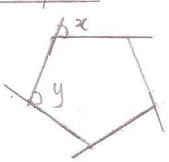


h) Polygons & their angles

For regular polygon with n sides, ext. $\angle = \frac{360°}{n}$

For regular polygon with n sides, int. $\angle = 180° - \frac{360°}{n}$

Example



For a 5-sided polygon, n = 5

ext. $\angle = \angle x = \frac{360}{5} = 72°$

int. $\angle = \angle y = 180° - 72°$

$= 108°$

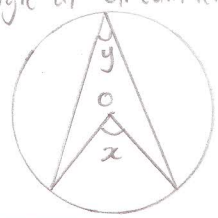
i) Irregular Polygon & their angles

Total ext. $\angle$s = 360°

Total interior $\angle$s = $(n-2) \times 180°$

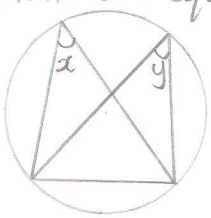
j) Angle at centre = 2 × angle at circumference

$\angle x = 2 \times \angle y$



k) Angles in the same segments are equal

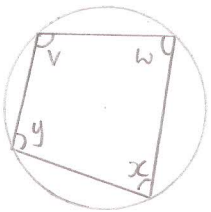
$\angle x = \angle y$



l) Opp. $\angle$s in a cyclic quadrilateral = 180°

$\angle v + \angle x = 180°$

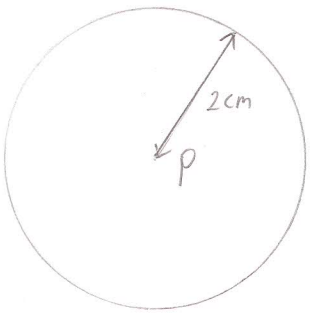
$\angle y + \angle w = 180°$



(17) Locus

a) Given distance from a given point

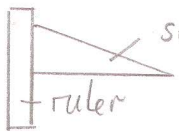
Example: Construct a locus 2cm from P.



b) Given distance from a given line

Example : construct a locus 2cm from PQ

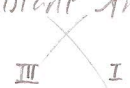
① Place a set square on PQ & a ruler perpendicular to the set square as shown

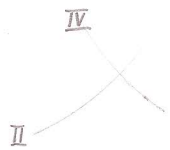


② Slide the set square 2cm up & 2cm down to draw the 2 locus as shown

c) Equidistant from 2 given point

Example : Construct a locust that is equidistant from point P & Q





① Open the compass to more than half the length of PQ.

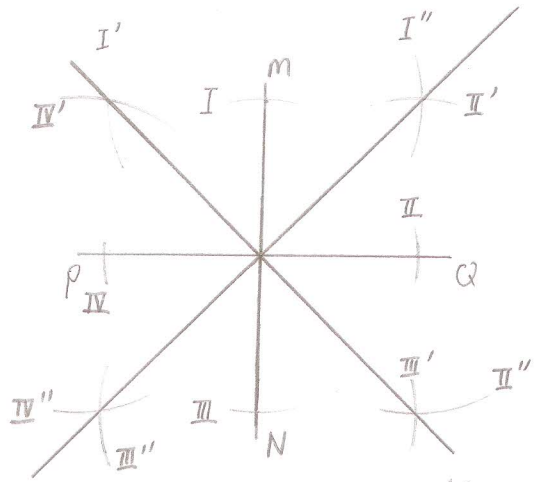
② Without changing the width of the compass, draw arc I & II from P & arc III & IV from Q

③ 2 crosses will be formed.

④ Connect the 2 crosses together.

d) Equidistant from 2 given intersecting line

Example : Construct a locus equidistant from PQ & MN.



① Open the compass to a reasonable width (recommended 5cm). * DO NOT CHANGE this WIDTH THROUGHOUT.

② From the intersecting point of PQ & MN, draw arc I, II, III, IV

③ From I, draw I' & II''
From II, draw II' & II''
From III, draw III' & III''
From IV, draw IV' & IV''

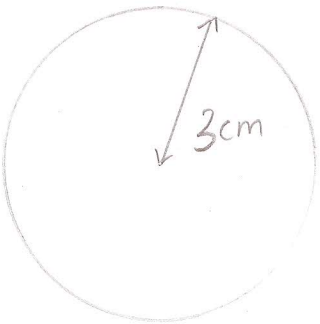
④ 4 crosses will be formed

⑤ Connect the 4 crosses as shown.

(18) Mensuration

a) Circumference of circle = $2\pi r$

b) Area of circle = πr^2

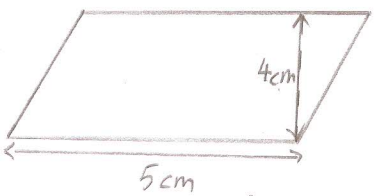


Example: Find the circumference & area of the circle shown above.

Circumference = $2\pi r$
= $2 \times \pi \times 3$
= 18.8cm (3s.f.)

Area = πr^2
= $\pi \times 3^2$
= $28.3 cm^2$ (3s.f.)

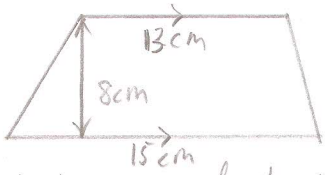
c) Area of parallelogram = Length × ⊥ height



Example: Find the area of the parallelogram

Area = length × ⊥ height
= 5 × 4
= $20 cm^2$

d) Area of trapezium = $\frac{1}{2}(l_1 + l_2)h$

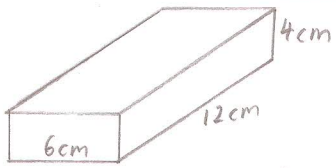


Example: Find the area of the trapezium

Area = $\frac{1}{2}(l_1 + l_2)h$
= $\frac{1}{2}(13 + 15) \times 8$
= $112 cm^2$

e) Volume of cuboid = $l \times b \times h$

Surface Area of cuboid = $2(lb) + 2(bh) + 2(lh)$



Example: Find the volume & surface area

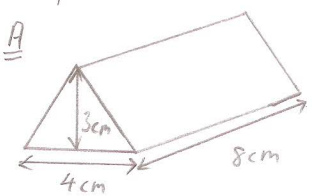
Volume = $l \times b \times h$
= $12 \times 6 \times 4$
= $288 cm^3$

Surface Area = $2(lb) + 2(bh) + 2(lh)$
= $2(12 \times 6) + 2(6 \times 4) + 2(12 \times 4)$
= $288 cm^2$

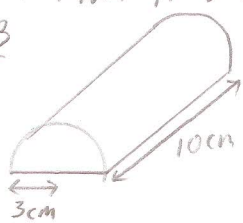
f) Volume of Prism = base area × height

Example: Find the volume of Prism A & B.

A



B

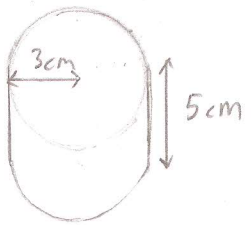


Volume of Prism A = base area × height
= $\frac{1}{2} \times 4 \times 3 \times 8$
= $48 cm^3$

Volume of Prism B = base area × height
= $\frac{1}{2} \times \pi \times 3^2 \times 10$
= $141 cm^3$ (3s.f.)

g) **Volume of Cylinder = $\pi r^2 h$**

Surface Area of Cylinder = $2\pi r^2 + 2\pi rh$



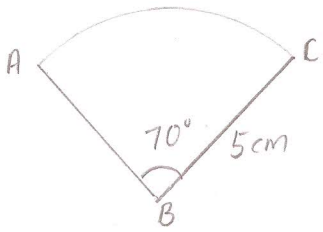
Example: Find the volume & surface area.

Volume = $\pi r^2 h$
$= \pi \times 3^2 \times 5$
$= 141\ cm^3$ (3s.f.)

Surface area = $2\pi r^2 + 2\pi rh$
$= 2\times\pi\times 3^2 + 2\times\pi\times 3\times 5$
$= 151\ cm^2$ (3s.f.)

h) **Arc Length = $\frac{\theta}{360} \times 2\pi r$**

Area of Sector = $\frac{\theta}{360} \times \pi r^2$



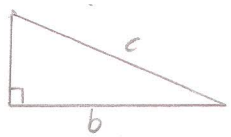
Arc Length AC = $\frac{\theta}{360} \times 2\pi r$
$= \frac{70}{360} \times 2\pi \times 5$
$= 6.11\ cm$ (3s.f.)

Area of minor sector ABC = $\frac{\theta}{360} \times \pi r^2$
$= \frac{70}{360} \times \pi \times 5^2$
$= 15.3\ cm^2$ (3s.f.)

(19) Trigonometry

For Right-Angled Triangle

a) **$a^2 + b^2 = c^2$**



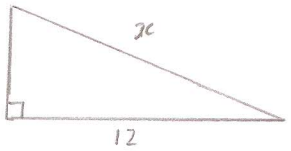
* Use to find 3rd side when given 1st 2 sides.

Example: Find x

$5^2 + 12^2 = x^2$

$x = \sqrt{5^2 + 12^2}$

$= 13\ cm$

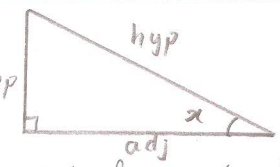


b) TOA CAH SOH

$\tan x = \frac{opp}{adj}$

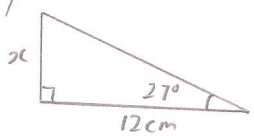
$\cos x = \frac{adj}{hyp}$

$\sin x = \frac{opp}{hyp}$



* Use to find angle when given 2 sides or
- Use to find 1 side when given 1 side & 1A

Example: ① Find side x

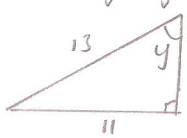


$\tan 27 = \frac{x}{12}$

$x = 12 \tan 27$

$= 6.11\ cm$

② Find angle y



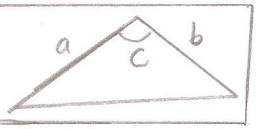
$\sin y = \frac{11}{13}$

$y = \sin^{-1}\left(\frac{11}{13}\right)$

$= 58.0°$

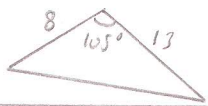
Not a right-angled Triangle

a) **Area of $\Delta = \frac{1}{2}ab\sin C$**



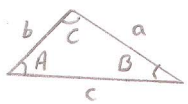
Example: Area $= \frac{1}{2} \times 8 \times 13 \times \sin 105$

$= 50.2\ cm^2$



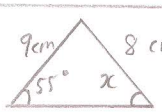
b) **Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$**

Use to: ① find 1 side when given 1 side & 2 angles.
② find 1 angle when given 1 angle & 2 sides.



Example: Find x.

$\frac{9}{\sin x} = \frac{8}{\sin 55}$



$\sin x = \frac{9\sin 55}{8} \Rightarrow x = \sin^{-1}\frac{9\sin 55}{8}$

$= 67.2°$

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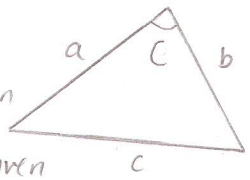
c) Cosine Rule

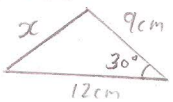
$c^2 = a^2 + b^2 - 2ab\cos C$



* Use to: ① find 1 side when given 2 sides & 1 angle
② find 1 angle when given 3 sides.

Example:

① Find side x



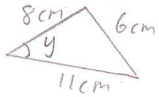
$x^2 = 9^2 + 12^2 - 2(9)(12)\cos 30°$

$= 37.94$

$x = \sqrt{37.94}$

$= 6.16\text{cm}$ (3 s.f.)

② Find angle y.



$6^2 = 8^2 + 11^2 - 2(8)(11)\cos y$

$36 = 185 - 176\cos y$

$\cos y = \frac{185 - 36}{176}$

$y = \cos^{-1}\left(\frac{149}{176}\right)$

$= 32.2°$

b) For histogram,

Frequency density = $\frac{\text{frequency}}{\text{width}}$

(21) Probability

If we call a particular event 'A' then the probability of 'A' happening is

$$P(A) = \frac{\text{Number of different way A can happen}}{\text{Total number of outcomes}}$$

The 'and' rule

$P(A \text{ and } B) = P(A) \times P(B)$

The 'or' rule

$P(A \text{ or } B) = P(A) + P(B)$

(20) Statistics

a) Mode, median & mean

Example: normal dice, numbered 1 to 6, rolled 50 times

Score	1	2	3	4	5	6
Frequency	15	10	7	5	6	7

Mode = 15 => score with highest frequency.

Median => score in the middle position.

$\frac{50+1}{2} = 25.5$ => 25th (↓ 2) & 26th (↓ 3) position

median $= \frac{2+3}{2}$

$= 2.5$

Mean $= \frac{\text{total score}}{\text{frequency}}$

$= \frac{1\times15 + 2\times10 + 3\times7 + 4\times5 + 5\times6 + 6\times7}{50}$

$= 2.96$

(22) Matrice

For a matrix, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

determinant $A = ad - bc$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example: Find A^{-1} of $A = \begin{bmatrix} -6 & 7 \\ -4 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{(-6)(3) - (7)(-4)}\begin{bmatrix} 3 & -7 \\ 4 & -6 \end{bmatrix}$

$= \frac{1}{-18+28}\begin{bmatrix} 3 & -7 \\ 4 & -6 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 3 & -7 \\ 4 & -6 \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{10} & -\frac{7}{10} \\ \frac{2}{5} & -\frac{3}{5} \end{bmatrix}$

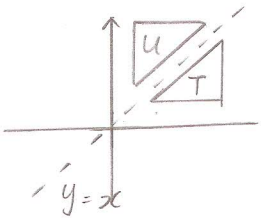
(23) Transformation

a) Reflection

Example: Describe transformation T to U

Reflection [1 mark]

$y = x$ [1 mark]



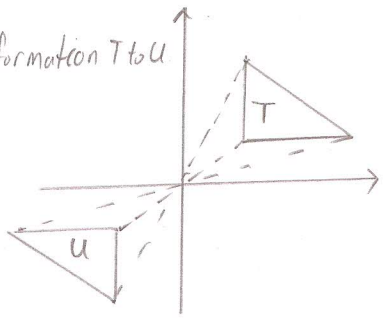
b) Rotation

Example: Describe transformation T to U

Rotation [1 mark]

Centre (0,0) [1 mark]

180° [1 mark]

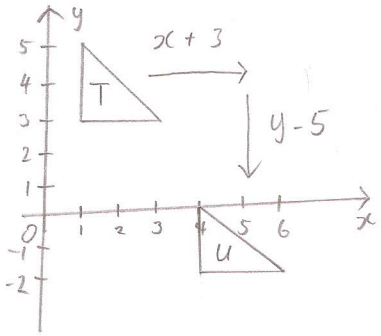


c) Translation

Example: describe the transformation T to U

Translation [1 mark]

$\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ [1 mark]



d) Enlargement

Example: describe the transformation T to U

Enlargement [1 mark]

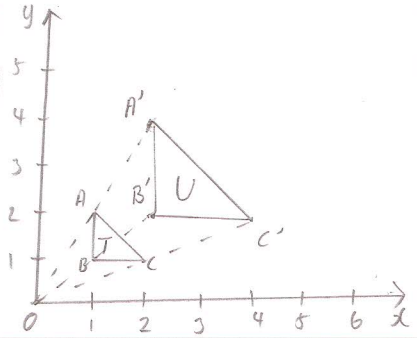
Centre (0,0) [1 mark]

⇒ Draw 2 lines AA' & BB'
intersection is the centre

scale factor $= \frac{OA'}{OA}$

$= \frac{4}{2}$

$= 2$



e) Shearing

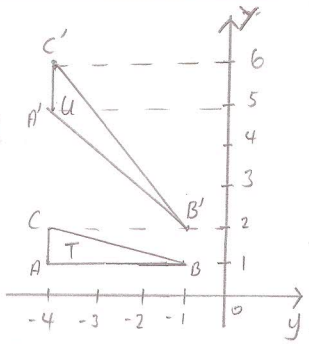
Example:

Describe the transformation T to U.

Shearing [1 mark]

Invariant line, y-axis [1 mark]

Shear factor = −1



How to find invariant line?

① Draw 2 lines AB & A'B' & find interception point 1

② Draw 2 lines CB & C'B' & find interception point 2.

③ Connect interception point 1 & 2 to get invariant line.

How to find shear factor?

$$\text{shear factor} = \frac{\text{distance from invariant to old pt}}{\text{distance old point to new pt.}}$$

Note: dist to the left ⇒ −ve
" " " right ⇒ +ve
dist upward ⇒ +ve
dist downward ⇒ −ve

f) Stretching

Example: Describe transformation T to U.

Stretching [1 mark]

Invariant [1 mark]

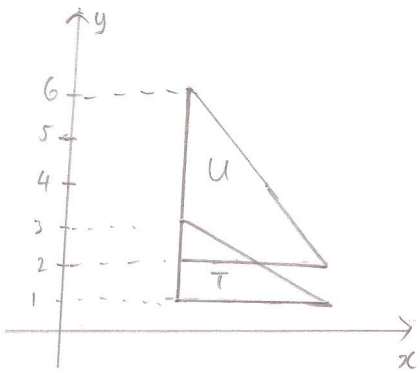
x-axis

stretch factor

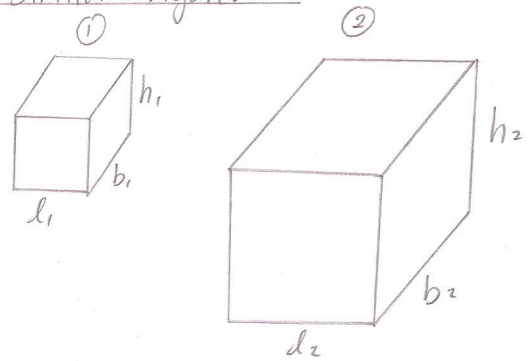
$= \frac{\text{invariant to new pt}}{\text{invariant to old pt}}$

$= \frac{6}{3}$

$= 2$



(24) Similar Figures.



a) $\left(\frac{l_1}{l_2}\right) = \left(\frac{b_1}{b_2}\right) = \left(\frac{h_1}{h_2}\right)$

b) $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

c) $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$