GCE 'O' LEVEL MATHEMATICS

Dengan kerjasama Yayasan MENDAKI In Quest of Excellence

Scheme of Assessment

Paper	Duration	Descriptions	Marks	Weightage
4016/1	2 h	There will be about 25 short answer questions testing more on the fundamental skills and concepts. ALL questions must be answered. The use of calculator is allowed.	80	50%
4016/2	2.5 h	There will be 10 to 11 questions of varying marks and lengths, testing on candidates' higher order thinking skills. ALL questions must be answered. The use of calculator is allowed.	100	50%

Assessment Notes

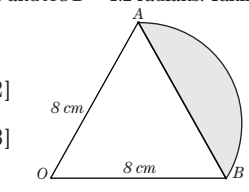
1. Omission of essential working will result in loss of marks.
2. For Paper 1, candidates are expected to show all their working and answers in the spaces provided below that question. For Paper 2, writing and graph papers will be provided.
3. Candidates are expected to cover the whole syllabus. Each paper may contain questions on any part of the syllabus and will not necessarily be restricted to a single topic.
4. Relevant mathematical formulae will be provided for both Paper 1 and Paper 2. Candidates should be familiar with formulae that are not provided in the formulae list.
5. The use of approved scientific calculators is allowed for both Paper 1 and Paper 2. Candidates should be familiar with the use of their calculators. Prior to the examination, they should also ensure that their calculators are in working condition.
6. Candidates are expected to provide their own geometrical instruments and other necessary stationary for both Paper 1 and Paper 2.
7. For non-exact answers and unless otherwise stated within a question, the final answer should be rounded off to three significant figures. Throughout the working, including in cases where answers are used in subsequent parts of the question, four significant figures (minimum) should be used. Premature approximation will be penalized. Angles in degrees should be rounded off to one decimal place.
8. Unless the question requires the answers in terms of π, either the calculator value for π or $\pi = 3.142$ should be used.

Sample questions and solutions

1. OAB is a sector of a circle, centre O, radius 8 cm. ΔAOB is an equilateral triangle and $A\hat{O}B = 1.2$ radians. Taking π to be 3.142, calculate

(a) the perimeter of the shaded region, [2]

(b) the area of the shaded region. [3]



Observe that the angle is given in radians. Therefore, set your calculator to the RADIAN mode. Use the RADIAN mode for the entire question.

(a) Perimeter of the shaded region
= arc length AB + chord length AB
= $r\theta + 8$
= $8(1.2) + 8$
= 17.6 cm

(b) Area of shaded region = Area of sector OAB – Area of ΔAOB
$= \frac{1}{2} r^2\theta - \frac{1}{2} r^2 \sin\theta$
$= \frac{1}{2} r^2 (\theta - \sin\theta)$
$= \frac{1}{2} (8)^2 (1.2 - \sin 1.2)$
$= 8.57 \text{ cm}^2$

Note that the formulae Area of sector *OAB and* Area of ΔAOB *will be provided in the formulae list.*

2. The diagram below shows the line segment AB = 7cm. Construct a triangle ABC with sides BC = 5 cm and AC = 8.5 cm. [2]

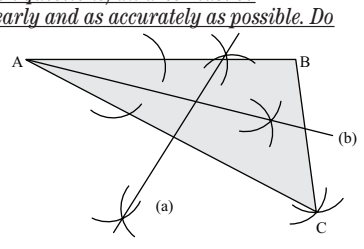
In the same diagram, construct,

(a) the perpendicular bisector of line AC, [1]

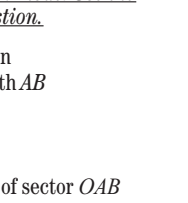
(b) the angle bisector of $C\hat{A}B$. [1]

A_____B

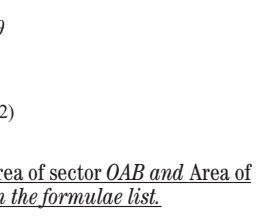
For construction questions, all arcs must be constructed clearly and as accurately as possible. Do not erase the arcs drawn! Candidates are expected to use a pair of compass to construct all arcs. Label all points clearly.



3. (a) In the following figure, PQ = PS and QR = ST. State, with reasons, the pair of congruent triangles. [2]



(b) (i) In the figure below, $B\hat{A}C = D\hat{E}C$. State which pair of triangles is similar, stating your reasons clearly. [2]



(c) (ii) Calculate the length of BE. [1]

To show or prove that any two triangles are congruent, either one of these 4 tests may be used - (1) SSS (2) SAS (3)RHS or (4) AAS. Write 3 relevant statements, with reasons, to support the proof.

(a) PQ = PS (given)
PR = PT (since QR =ST)
$R\hat{P}S = T\hat{P}Q$ (common angle)
$\therefore \Delta RPS \equiv \Delta TPQ$ *(SAS)*

The easiest way to show or prove that two triangles are similar is to show that their 2 corresponding angles are equal. Write 2 relevant statements, with reasons, to support the proof. Ensure that the correct vertices of the two triangles are written in the final statement.

(b)(i)$\angle$BAC = $\angle$DEC (given)
$\angle$BCA = $\angle$DCE (common angle)
Hence, ΔBCA is similar to ΔDCE.

From the statement" ΔBCA is similar to ΔDCE", the ratio of the corresponding sides of the two triangles can be obtained. To solve the problem, you need 2 pairs of corresponding sides. Form an equation in them and use this equation to solve for the unknown side. Study the diagram carefully and decide which 2 pairs can be used to solve the problem.

(ii) $\frac{BC}{DC} = \frac{CA}{CE}$
$\frac{16}{8} = \frac{12}{CE}$
$EC = 6$ cm.
$BE = 16 - 6 = 10$ cm

4. Ali and Mazlan have been receiving the Edusave Scholarships awards since they were in Secondary One (Express). The cash awards which they received every year were given at the end of each academic year.

(a) Ali deposited all $500 in his account annually. This account paid simple interest at the rate of 1.55% per year. Calculate the total amount in his account at the end of JC2. [2]

(b) Mazlan also deposited all $500 in his account annually. This account paid compound interest at the rate of 1.5% per year. Calculate how much more or less money there was in his account after JC2 than there was in Ali's account. [3]

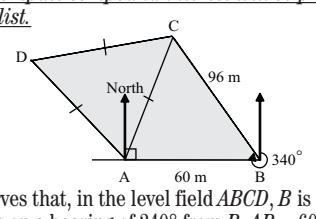
The formula to compute simple interest is I=(PRT)/100, where P is the principal amount (or initial amount) deposited, R is the rate in percentage and T is the term in years. This formula will not be provided in the formulae list.

(a) Interest earned at the end of JC2 $= \frac{500 \times 1.55 \times 6}{100}$
= $ 46.50
Total amount in account at the end of JC2 = 500 + 46.50
= $ 546.50

(b) Interest earned at the end of JC2 $= 500\left(1+\frac{1.5}{100}\right)^6$
= $ 546.72
Difference in interest earned = 546.72 - 546.50
= $ 0.22
Mazlan will have $ 0.22 more than Ali.

The formula to compute compound interest will be provided in the formulae list.

5.



A man observes that, in the level field $ABCD$, B is due east of A, C is on a bearing of 340° from B, AB = 60 m and BC = 96 m. He also notes that ΔACD is equilateral. Calculate

(a) the length AC, [3]

(b) the angle CAB, [3]

(c) the bearing of D from A. [2]

A bird is hovering at a point vertically above C at a height of 40m above the ground.

(d) Calculate the angle of elevation of the bird from point B. [2]

The whole question must be answered with calculator set to DEGREE mode as the angles and bearings are given in degrees.

(a) $C\hat{B}A = 340° - 270°$
$= 70°$
$AC^2 = 96^2 + 60^2 - 2(96)(60)\cos 70°$
AC = 94.212
= 94.2 m

It is encouraged to write the answer to 5 significant figures before rounding off the final answer to 3 significant figures. Use this value (5 significant value) in the subsequent parts if required.

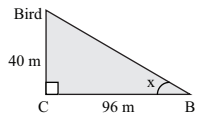
(b) $\frac{\sin C\hat{A}B}{96} = \frac{\sin 70°}{94.212}$
$C\hat{A}B = 73.24$
$= 73.2°$(1 dec place)

Bearing is measured in aclockwise direction from the specified North line. It is written as a three-digit number.

(c) $N\hat{A}D = 60° - (90° - 73.24°)$
$= 43.24°$
Bearing of D from A = 360° – 43.24°
= 316.8°

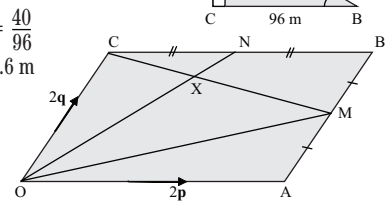
For questions involving angle of elevation or depression, draw a right angle triangle with all given information. Apply trigonometric ratio to solve for unknowns.

d) Let the angle of elevation be x



$\tan x = \frac{40}{96}$
$x = 22.6$ m

6.



$OABC$ is a parallelogram. M is a midpoint of AB and N is a midpoint of CB. $\vec{OA} = 2\underset{\sim}{p}$ and $\vec{OC} = 2\underset{\sim}{q}$.

(a) Express in terms of $\underset{\sim}{p}$ and/or $\underset{\sim}{q}$

(i) $\vec{CN}$, [1]

(ii) $\vec{OM}$, [1]

(iii) $\vec{CM}$. [1]

(b) The point X divides CM in the ratio 2 : 3. Express in terms of $\underset{\sim}{p}$ and/or $\underset{\sim}{q}$

(i) $\vec{CX}$, [1]

(ii) $\vec{OX}$, [2]

(iii) $\vec{XN}$. [2]

(c) Write down the ratio $OX:XN$. [2]

(d) Find the numerical value of

(i) $\frac{\text{Area of OCM}}{\text{Area of OAM}}$, [1]

(ii) $\frac{\text{Area of OCX}}{\text{Area of CNX}}$. [1]

Use geometrical properties of parallelograms to determine parallel vectors and equal vectors in the diagram given.

(a) (i) $\vec{CN} = \underset{\sim}{p}$
(ii) $\vec{AM} = \underset{\sim}{q}$
$\vec{OM} = 2\underset{\sim}{p} + \underset{\sim}{q}$
(iii) $\vec{CM} = 2\underset{\sim}{p} - \underset{\sim}{q}$

(b) (i) $\vec{CX} = \frac{2}{5}\left(2\underset{\sim}{p} - \underset{\sim}{q}\right)$

(ii) $\vec{OX} = 2\underset{\sim}{q} + \frac{4}{5}\underset{\sim}{p} - \frac{2}{5}\underset{\sim}{q}$
$= \frac{4}{5}\underset{\sim}{p} + \frac{8}{5}\underset{\sim}{q}$
$= \frac{4}{5}\left(\underset{\sim}{p} + 2\underset{\sim}{q}\right)$

(iii) $\vec{ON} = \left(\underset{\sim}{p} + 2\underset{\sim}{q}\right)$
$\vec{XN} = \frac{1}{5}\left(\underset{\sim}{p} + 2\underset{\sim}{q}\right)$

(c) OX : XN = 4 : 1

(d)(i) *Both triangles share the same base, OM. Compare ratio of OC : AM*
$\frac{\text{Area of OCM}}{\text{Area of OAM}} = \frac{2}{1}$

(ii) Both triangles share the same height, CX. Compare OX : XN
$\frac{\text{Area of OCX}}{\text{Area of CNX}} = \frac{4}{1}$

7. (a) Balls are drawn at random from a bag containing 6 green balls and 4 blue balls. Find the probability that

(i) the first ball drawn will be blue, [1]

(ii) the first two balls drawn, without replacement, will be different in colours. [2]

(b) Another bag contains w green and b blue balls. The probability of drawing a blue ball at random is $\frac{3}{7}$.

(i) Find a relation between w and b. [1]

(ii) It is also given that when two balls are drawn, without replacement, the probability that both balls are blue is $\frac{6}{35}$. Hence show that
$5(b-1) = 2(w+b-1)$ [2]

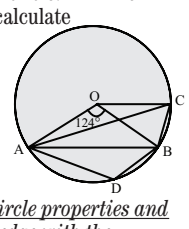
(a) (i) P(blue ball drawn) $= \frac{4}{10}$
$= \frac{2}{5}$

(ii) P(different colours drawn) = P(blue and green or green and blue)
$= \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{6}{9}$
$= \frac{8}{15}$

(b) (i) $\frac{3}{7} = \frac{b}{b+w}$
$3b + 3w = 7b$
$3w = 4b$
$w = \frac{4}{3} b$

(ii) Without replacement, $\frac{b}{(b+w)} \times \frac{(b-1)}{(b+w-1)} = \frac{6}{35}$
$\frac{3}{7} \times \frac{(b-1)}{(b+w-1)} = \frac{6}{35}$
$\frac{(b-1)}{(b+w-1)} = \frac{2}{5}$
$5(b-1) = 2(w+b-1)$
(shown)

8. In the figure, O is the centre of the circle. If AB is parallel to OC and $\angle AOB = 124°$, calculate



(a) $\angle$ACB, [1]

(b) $\angle$ADB, [1]

(c) $\angle$CAB [1]

Candidates need to be familiar with circle properties and apply them accordingly. Relate knowledge with the diagram given. Wherever possible, state the properties used beside the statement you write. Use the amount of marks to gauge your solutions. A 1 mark solution should not exceed more than 1 to 3 lines.

(a) $\angle ACB = 62°$ ($\angle$ *at centre* = 2$\angle$ *at circumference*)

(b) $\angle ADB = 180° - 62°$ (*opp*$\angle$ *cyclic quad*)
$= 118°$

(c) $\angle OBA = \frac{180° - 124°}{2}$ (*isos* Δ)
$= 28°$
$\angle COB = 28°$ (*alt*$\angle$*s*)
$\angle CAB = 28° \div 2$ ($\angle$ *at centre* = 2$\angle$ *at circumference*)
$= 14°$

9. A manufacturing plant assembles printers and normally produces 500 printers in x days.

(a) Write down an expression, in terms of x, for the number of printers produced per day. [1]

(b) The rate of production is increased so that 500 printers are now produced in one day less. Write down an expression, in terms of x, for the number of printers produced in this case. [1]

(c) If the difference between these daily rates of production is 60, form an equation in x and show it reduces to $3x^2 - 3x - 25 = 0$. [3]

(d) Solve the equation $3x^2 - 3x - 25 = 0$. Explain why you would reject one of the two solutions. [3]

(e) Hence, find the number of printers originally produced per day. [1]

(a) $\frac{500}{x}$

(b) $\frac{500}{(x-1)}$

(c) $\frac{500}{(x-1)} - \frac{500}{x} = 60$ *Multiply throughout by the LCM x (x-1)*
$500x - 500(x-1) = 60x(x-1)$
$500 = 60x^2 - 60x$
$3x^2 - 3x - 25 = 0$ (shown)

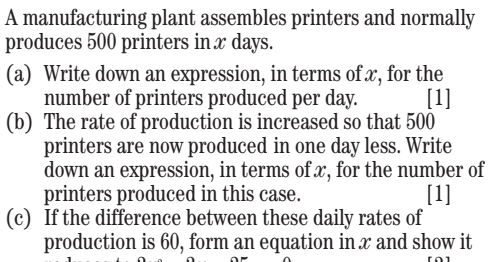
If you are unable to show the equation in part (c), proceed to solve the remaining part of the question. Do not leave the remaining parts blank.

(d) $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-25)}}{2(3)}$
= 3.4297 or - 2.4297
= 3.43 or - 2.43 (rejected)
One of the solutions has to be rejected as the number of days cannot be a negative value.

The equation above will NOT be provided in the formula list. Ensure correct substitution of values.

(e) Number of printers originally produced $= \frac{500}{3.4297}$
$= 145.78$
$= 145$

10.



The diagram represents the speed-time graph of a moving object.

(a) Calculate the speed of the object when $t = 4$. [2]

(b) Calculate the distance traveled in the first 15 seconds. [2]

(c) Given that the rate at which the object slows down after $t = 15$ is equal to half the rate at which it accelerates during the first 6 seconds, calculate the time at which it stops. [2]

Apply concept of similar triangles to answer part (a)

(a) Let the speed when t = 4 be v
$\frac{40-16}{v-16} = \frac{6}{4}$
$v = 32$ m/s

Distance traveled is equal to the area under the speed-time graph.

(b) Distance traveled in the first 15s $= \frac{1}{2}(16+40)\times 6 + (40 \times 9)$
$= 528$ m

Acceleration is obtained from the gradient of the speed time graph. Negative acceleration represents retardation.

(c) Acceleration from 0 to 6 s $= \frac{40-16}{6}$
$= 4 \text{ m/s}^2$
Acceleration after 15 s = -2 m/s^2
Let the time it stops be t.
$\frac{40-0}{15-t} = -2$
$t = 35$ s

11. **Diagram 1** shows a solid consisting of a hemisphere of radius 5cm, a cylinder of radius 5 cm and height x cm, and a right circular cone of radius 5 cm and height 12cm, joined together.

(a) Find the volume of the cone, leaving your answer in terms of π. [2]

(b) The total volume of the solid is $300\pi \text{ cm}^3$. Show that the height, x, of the cylinder is $4\frac{2}{3}$ cm. [3]

Diagram 1

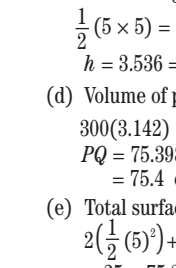
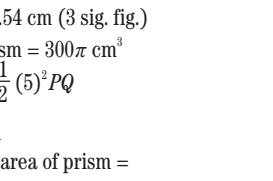


Diagram 2



The solid is melted completely and recast to form a prism as shown in **Diagram 2**.
The cross-section of the prism is a right-angled triangle with sides $PR = PS = 5$ cm. Taking π to be 3.142, calculate

(c) the vertical height, h, [2]

(d) the length PQ of the prism, [2]

(e) the total surface area of the prism. [3]

Formula for the volume of cone and sphere is provided in the formulae list. For parts (a) and (b), leave final answers in terms of π.

(a) Volume of cone $= \frac{1}{3}\pi(5)^2 12$
$= 100\pi \text{ cm}^3$

(b) Volume of cylinder and hemisphere = $200\pi \text{ cm}^3$
$\pi(5)^2 x + \frac{2}{3}\pi(5)^3 = 200\pi$
$x = \frac{200\pi - \frac{2}{3}\pi(5)^3}{\pi(5)^2}$
$x = 4\frac{2}{3}$ cm

(c) $RS = \sqrt{5^2 + 5^2}$
= 7.071 cm
Area of a right-angled triangle $= \frac{1}{2}$ *base* $\times$ *height*
$\frac{1}{2}(5 \times 5) = \frac{1}{2}(7.071 \times h)$
$h = 3.536 = 3.54$ cm (3 sig. fig.)

(d) Volume of prism = $300\pi \text{ cm}^3$
$300(3.142) = \frac{1}{2}(5)^2 PQ$
$PQ = 75.398$
= 75.4 cm

(e) Total surface area of prism =
$2\left(\frac{1}{2}(5)^2\right) + 2(5 \times 75.398) + (7.071 \times 75.398)$
= 25 + 75.398 + 533.139
= 1312 cm^2

12. (a) The Latiff family went to Beijing to watch the 2008 Olympics.

(i) Mr Latiff bought 5000 CNY (¥) at a rate of ¥1= 0.1885 SGD (S$) on 6 August. How much did this cost in SGD? [1]

(ii) The family returned home with ¥298 and changed this back into SGD. They received S$56.30. Calculate how many SGD they received for each CNY, giving your answer correct to 4 significant figures. [1]

(iii) Ms Latiff bought a bag for ¥99. The same bag costs S$37.50 in Singapore. Calculate, as a percentage of the cost in Singapore, how much less does it cost in Beijing. [2]

(iv) Mrs Latiff bought a few dresses that cost ¥1000. She paid ¥100 in cash and used her credit card to pay for the remaining amount. When she returned home, her credit card bill for this item was S$184.74. Calculate the rate of exchange, in CNY to SGD, giving your answer correct to 4 significant figures. [1]

(v) The plane took 5 hours 15 minutes to return to Singapore. The return journey was 3801 km. Calculate the average speed of the plane in metres per second. [2]

(a) (i) It cost $5000 \times 0.1885 =$ S$ 942.50
(ii) They received $(56.30 \div 298) = 0.18892$
$\approx$S$0.1889 for each CNY
(iii) ¥ 99 = $99 \times 0.1885 =$ S$18.6615
$\frac{37.50 - 18.6615}{37.50} \times 100\% = 50.236\% \approx 50.2\%$
(iv) ¥1 = 184.74 ÷ 900 = 0.20526 × S$ 0.2053
(v) Average speed $= \frac{3801 \times 10^3}{315 \times 60}$
$= 201.1 \approx 201 \text{ ms}^{-1}$

13. Answer the whole of this question on a single sheet of graph paper.

The variables x and y are connected by the equation $y = x + \frac{15}{x} - 6$. Some corresponding values of x and y, correct to 1 decimal place, are given in the table below.

x	1	1.5	2	3	4	5	6	7	8
y	10	a	3.5	2	1.8	2	b	3.1	3.9

(a) Calculate the value of a and of b. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal axis for. $0 \le x \le 8$. Using a scale of 2 cm to represent 1 unit, draw a vertical axis for $0 \le x \le 10$. On these axes, draw the graph of $y = x + \frac{15}{x} - 6$. [3]

(c) Using your graph, find
(i) value(s) of x when $y = 8$, [1]
(ii) range of values of x when $x + \frac{15}{x} - 6 > 4$ [2]

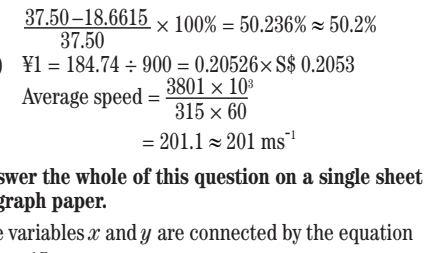
(d) Use your graph to find the value of x when the gradient of the curve $y = x + \frac{15}{x} - 6$ is equal to zero. [1]

(e) By drawing a tangent, find the gradient of the graph at $x = 2$. [2]

As specified in the instruction above, the whole of this question must be answered on a SINGLE sheet of graph paper. All necessary working should be done on either side of the graph paper. Follow instructions closely and plan where to draw the axes. Ensure axes are labeled and scale is consistent. The graph must appear smooth. Label the curve and lines clearly.

(a) $a = 5.5, b = 2.5$

(b)



(c) (i) $x = 1.25$ (1.2 ~1.3)
(ii) $x + \frac{15}{x} - 6 > 4$ *refers to values of x when the graph of* $y = x + \frac{15}{x} - 6$ *is greater than or diagrammatically, above the line y = 4.*
x < 1.8 or x > 8.25 [(1.75 ~ 1.85) , (8.2 ~ 8.3) respectively.]

(d) Gradient is zero at the minimum or the lowest point of the graph. $x = 3.75$

(e) Tangent must be sufficiently long and should touch the graph, as best as possible, at one point only. Obtain coordinates of the endpoints and use the gradient formula to calculate the gradient at this point, in this case, x = 2.
Gradient at $x = 2$ $= \frac{5.9 - 0.5}{0.7 - 3.1}$
$= -2.25$