



ADDITIONAL MATHEMATICS

2002 – 2011

CLASSIFIED LOGSsurdsINDICES

**Compiled & Edited
By**

Dr. Eltayeb Abdul Rhman

www.drtayeb.tk

**First Edition
2011**

4 You must not use a calculator in Question 4.For
Examiner's
Use

In the triangle ABC , angle $B = 90^\circ$, $AB = 4 + 2\sqrt{2}$ and $BC = 1 + \sqrt{2}$.

(i) Find $\tan C$, giving your answer in the form $k\sqrt{2}$. [2]

(ii) Find the area of the triangle ABC , giving your answer in the form $p + q\sqrt{2}$, where p and q are integers. [2]

(iii) Find the area of the square whose side is of length AC , giving your answer in the form $s + t\sqrt{2}$, where s and t are integers. [2]

1 Without using a calculator, express $\frac{(5 + 2\sqrt{3})^2}{2 + \sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers. [4]

(b) Given that $\frac{\sqrt{a^{\frac{4}{3}}b^{-\frac{2}{5}}}}{a^{-\frac{1}{3}}b^{\frac{3}{5}}} = a^p b^q$, find the value of p and of q . [2]

- 8 The temperature, T° Celsius, of an object, t minutes after it is removed from a heat source, is given by

$$T = 55e^{-0.1t} + 15.$$

For
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Use

- (i) Find the temperature of the object at the instant it is removed from the heat source. [1]
- (ii) Find the temperature of the object when $t = 8$. [1]
- (iii) Find the value of t when $T = 25$. [3]
- (iv) Find the rate of change of T when $t = 16$. [3]

- 8 (a) (i) Solve $3^x = 200$, giving your answer to 2 decimal places.

[2]

For
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- (ii) Solve $\log_5 (5y + 40) - \log_5 (y + 2) = 2$.

[4]

- (b) Given that $\frac{(24z^3)^2}{27 \times 12z} = 2^a 3^b z^c$, evaluate a , b and c .

[3]

- 3 (i) Express $\log_x 2$ in terms of a logarithm to base 2.

[1]

*For
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Use*

- (ii) Using the result of part (i), and the substitution $u = \log_2 x$, find the values of x which satisfy the equation $\log_2 x = 3 - 2 \log_x 2$. [4]

- 7 (i) Show that $\frac{(4 - \sqrt{x})^2}{\sqrt{x}}$ can be written in the form $px^{-\frac{1}{2}} + q + rx^{\frac{1}{2}}$, where p , q and r are integers to be found. [3]

- (ii) A curve is such that $\frac{dy}{dx} = \frac{(4 - \sqrt{x})^2}{\sqrt{x}}$ for $x > 0$. Given that the curve passes through the point $(9, 30)$, find the equation of the curve. [5]

- 1 Given that $\frac{\left(6x^{\frac{3}{2}}y^{\frac{4}{5}}\right)^4}{2x^{\frac{1}{2}}y^{-1}} = ax^py^q$, find the values of the constants a , p and q .

[3]

*For
Examiner's
Use*

-
- 2 Express $\sqrt{\frac{1 - \cos^2 \theta}{4 \sec^2 \theta - 4}}$ in the form $k \cos \theta$, where k is a constant to be found.

[4]

5 It is given that $\lg p^3 q = 10a$ and $\lg \left(\frac{p}{q^2} \right) = a$.

For
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Use

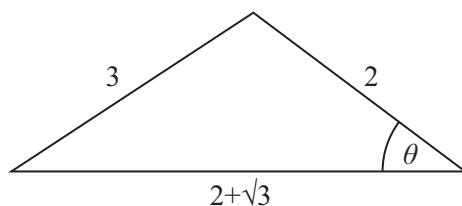
(i) Find, in terms of a , expressions for $\lg p$ and $\lg q$.

[5]

(ii) Find the value of $\log_p q$.

[1]

3

For
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Use

Without using a calculator, find the value of $\cos\theta$, giving your answer in the form $\frac{a+b\sqrt{3}}{c}$, where a , b and c are integers. [5]

5 (i) Express $\frac{1}{\sqrt{32}}$ as a power of 2. [1]

(ii) Express $(64)^{\frac{1}{x}}$ as a power of 2. [1]

(iii) Hence solve the equation $\frac{(64)^{\frac{1}{x}}}{2^x} = \frac{1}{\sqrt{32}}$. [3]

8 (i) Given that $\log_9 x = a \log_3 x$, find a . [1]

(ii) Given that $\log_{27} y = b \log_3 y$, find b . [1]

(iii) Hence solve, for x and y , the simultaneous equations

$$6\log_9 x + 3\log_{27} y = 8,$$

$$\log_3 x + 2\log_9 y = 2.$$

[4]

8 Solve the equation

(i) $2^{2x+1} = 20$, [3]

(ii) $\frac{5^{4y-1}}{25^y} = \frac{125^{y+3}}{25^{2-y}}$. [4]

11 (a) (i) Solve $\frac{5^{2x+3}}{25^{2x}} = \frac{25^{2-x}}{125^x}$. [3]

(ii) Solve $\lg y + \lg(y - 15) = 2$. [4]

Solve the equation $x^3 + 3x^2 = 2$, giving your answers to 2 decimal places where necessary. [5]

5 Solve the equation $3x(x^2 + 6) = 8 - 17x^2$. [6]

9 (a) Given that $u = \log_4 x$, find, in simplest form in terms of u ,

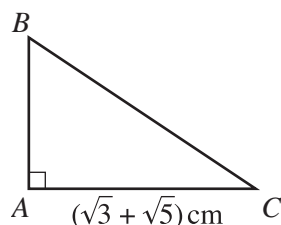
(i) x ,

(ii) $\log_4\left(\frac{16}{x}\right)$,

(iii) $\log_x 8$. [5]

(b) Solve the equation $(\log_3 y)^2 + \log_3(y^2) = 8$. [4]

5



The diagram shows a right-angled triangle ABC in which the length of AC is $(\sqrt{3} + \sqrt{5})$ cm. The area of triangle ABC is $(1 + \sqrt{15})$ cm².

(i) Find the length of AB in the form $(a\sqrt{3} + b\sqrt{5})$ cm, where a and b are integers. [3]

(ii) Express $(BC)^2$ in the form $(c + d\sqrt{15})$ cm², where c and d are integers. [3]

3 The roots of the equation $x^2 - \sqrt{28}x + 2 = 0$ are p and q , where $p > q$. Without using a calculator, express $\frac{p}{q}$ in the form $m + \sqrt{n}$, where m and n are integers. [5]

6 Solve the equation $x^2(2x + 3) = 17x - 12$. [6]

(b) Without using a calculator, and showing each stage of your working, find the value of

$$2 \log_{12} 4 - \frac{1}{2} \log_{12} 81 + 4 \log_{12} 3. \quad [3]$$

9 Solve

(i) $\log_4 2 + \log_9 (2x + 5) = \log_8 64,$ [4]

(ii) $9^y + 5(3^y - 10) = 0.$ [4]

4 Without using a calculator, find the positive root of the equation

$$(5 - 2\sqrt{2})x^2 - (4 + 2\sqrt{2})x - 2 = 0,$$

giving your answer in the form $a + b\sqrt{2}$, where a and b are integers. [6]

4 Simplify $\frac{16^{x+1} + 20(4^{2x})}{2^{x-3} 8^{x+2}}.$ [4]

3 A rectangle has sides of length $(2 + \sqrt{18})$ metres and $(5 - \frac{4}{\sqrt{2}})$ metres. Express, in the form $a + b\sqrt{2}$, where a and b are integers,

(i) the value of A , where A square metres is the area of the rectangle,

(ii) the value of D^2 , where D metres is the length of the diagonal of the rectangle. [5]

1 Express $\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are integers. [3]

10 Solve the equation

(i) $\lg(2x) - \lg(x - 3) = 1,$ [3]

(ii) $\log_3 y + 4\log_y 3 = 4.$ [4]

7 Solve, for x and y , the simultaneous equations

$$125^x = 25(5^y),$$

$$7^x \div 49^y = 1.$$

[6]

2 The area of a rectangle is $(1 + \sqrt{6})\text{m}^2$. The length of one side is $(\sqrt{3} + \sqrt{2})\text{m}$. Find, without using a calculator, the length of the other side in the form $\sqrt{a} - \sqrt{b}$, where a and b are integers. [4]

8 Solve

(i) $\log_3(2x + 1) = 2 + \log_3(3x - 11),$ [4]

(ii) $\log_4 y + \log_2 y = 9.$ [4]

6 Given that $\log_8 p = x$ and $\log_8 q = y$, express in terms of x and/or y

(i) $\log_8 \sqrt{p} + \log_8 q^2$, [2]

(ii) $\log_8 \left(\frac{q}{8} \right)$, [2]

(iii) $\log_2 (64p)$. [3]

8 (a) Solve the equation $(2^{3-4x}) (4^{x+4}) = 2$. [3]

(b) (i) Simplify $\sqrt{108} - \frac{12}{\sqrt{3}}$, giving your answer in the form $k\sqrt{3}$, where k is an integer. [2]

(ii) Simplify $\frac{\sqrt{5}+3}{\sqrt{5}-2}$, giving your answer in the form $a\sqrt{5} + b$, where a and b are integers. [3]

8 (i) Show that $(5 + 3\sqrt{2})^2 = 43 + 30\sqrt{2}$. [1]

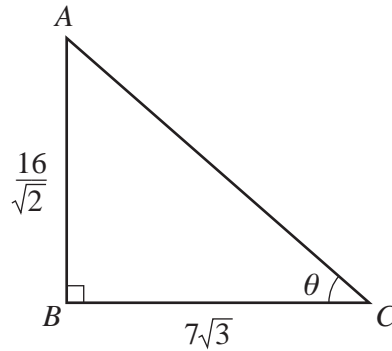
Hence find, **without using a calculator**, the positive square root of

(ii) $86 + 60\sqrt{2}$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers, [2]

(iii) $43 - 30\sqrt{2}$, giving your answer in the form $c + d\sqrt{2}$, where c and d are integers, [1]

(iv) $\frac{1}{43 + 30\sqrt{2}}$, giving your answer in the form $\frac{f + g\sqrt{2}}{h}$, where f , g and h are integers. [3]

4



The diagram shows a right-angled triangle ABC in which the length of AB is $\frac{16}{\sqrt{2}}$, the length of BC is $7\sqrt{3}$ and angle BCA is θ .

- (i) Find $\tan \theta$ in the form $\frac{a\sqrt{b}}{b}$, where a and b are integers. [2]
- (ii) Calculate the length of AC , giving your answer in the form $c\sqrt{d}$, where c and d are integers and d is as small as possible. [3]

5 Solve the equation $2x^3 - 3x^2 - 11x + 6 = 0$. [6]

10 (a) Given that $\log_p X = 6$ and $\log_p Y = 4$, find the value of

(i) $\log_p \left(\frac{X^2}{Y} \right)$. [2]

(ii) $\log_Y X$. [2]

(b) Find the value of 2^z , where $z = 5 + \log_2 3$. [3]

(c) Express $\sqrt{512}$ as a power of 4. [2]

4 (a) Solve the equation $16^{3x-2} = 8^{2x}$. [3]

8 Solve the equation

(i) $\lg(5x + 10) + 2\lg 3 = 1 + \lg(4x + 12)$, [4]

(ii) $\frac{9^{2y}}{3^{7-y}} = \frac{3^{4y+3}}{27^{y-2}}$. [3]

4 You must not use a calculator in Question 4.

In the triangle ABC , angle $B = 90^\circ$, $AB = 4 + 2\sqrt{2}$ and $BC = 1 + \sqrt{2}$.

(i) Find $\tan C$, giving your answer in the form $k\sqrt{2}$. [2]

(ii) Find the area of the triangle ABC , giving your answer in the form $p + q\sqrt{2}$, where p and q are integers. [2]

(iii) Find the area of the square whose side is of length AC , giving your answer in the form $s + t\sqrt{2}$, where s and t are integers. [2]

5 Solve the equation

(i) $\frac{4^x}{2^{5-x}} = \frac{2^{4x}}{8^{x-3}}$, [3]

(ii) $\lg(2y + 10) + \lg y = 2$. [3]

1 Without using a calculator, express $\frac{(5 + 2\sqrt{3})^2}{2 + \sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers. [4]

4 A cuboid has a square base of side $(2 - \sqrt{3})\text{ m}$ and a volume of $(2\sqrt{3} - 3)\text{ m}^3$. Find the height of the cuboid in the form $(a + b\sqrt{3})\text{ m}$, where a and b are integers. [4]

- 8 The temperature, T° Celsius, of an object, t minutes after it is removed from a heat source, is given by

$$T = 55e^{-0.1t} + 15.$$

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Use

- (i) Find the temperature of the object at the instant it is removed from the heat source. [1]

- (ii) Find the temperature of the object when $t = 8$. [1]

- (iii) Find the value of t when $T = 25$. [3]

- (iv) Find the rate of change of T when $t = 16$. [3]

- 5 (a) Solve the equation $9^{2x-1} = 27^x$. [3]

- (b) Given that $\frac{a^{-\frac{1}{2}}b^{\frac{2}{3}}}{\sqrt{a^3b^{-\frac{2}{3}}}} = a^pb^q$, find the value of p and of q . [2]

- (a) At the beginning of 1960, the number of animals of a certain species was estimated at 20 000. This number decreased so that, after a period of n years, the population was

$$20\,000e^{-0.05n}.$$

Estimate

- (i) the population at the beginning of 1970, [1]

- (ii) the year in which the population would be expected to have first decreased to 2000. [3]

- (b) Solve the equation $3^{x+1} - 2 = 8 \times 3^{x-1}$. [6]

8 (a) (i) Solve $3^x = 200$, giving your answer to 2 decimal places. [2]

(ii) Solve $\log_5(5y + 40) - \log_5(y + 2) = 2$. [4]

7 (a) Solve $\log_7(17y + 15) = 2 + \log_7(2y - 3)$. [4]

(b) Evaluate $\log_p 8 \times \log_{16} p$. [3]

2 Given that $\sqrt{a + b\sqrt{3}} = \frac{13}{4 + \sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and of b . [4]

9 (a) Find, in its simplest form, the product of $a^{\frac{1}{3}} + b^{\frac{2}{3}}$ and $a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}$. [3]

(b) Given that $2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$, evaluate 10^x . [4]

3 Without using a calculator, solve, for x and y , the simultaneous equations

$$\begin{aligned} 8^x \div 2^y &= 64, \\ 3^{4x} \times \left(\frac{1}{9}\right)^{y-1} &= 81. \end{aligned} \quad [5]$$

(b) Given that $\frac{(24z^3)^2}{27 \times 12z} = 2^a 3^b z^c$, evaluate a , b and c . [3]

2 Without using a calculator, solve the equation $\frac{2^{x-3}}{8^{-x}} = \frac{32}{4^{\frac{1}{2}x}}$. [4]

4 A rectangular block has a square base. The length of each side of the base is $(\sqrt{3} - \sqrt{2})$ m and the volume of the block is $(4\sqrt{2} - 3\sqrt{3})$ m³. Find, without using a calculator, the height of the block in the form $(a\sqrt{2} + b\sqrt{3})$ m, where a and b are integers. [5]

6 (i) Solve the equation $2t = 9 + \frac{5}{t}$. [3]

(ii) Hence, or otherwise, solve the equation $2x^{\frac{1}{2}} = 9 + 5x^{-\frac{1}{2}}$. [3]

- 1 Variables V and t are related by the equation

$$V = 1000e^{-kt},$$

where k is a constant. Given that $V = 500$ when $t = 21$, find

(i) the value of k , [2]

(ii) the value of V when $t = 30$. [2]

8 (a) Solve the equation $\lg(x + 12) = 1 + \lg(2 - x)$. [3]

(b) Given that $\log_2 p = a$, $\log_8 q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b . [4]

9 (a) Express $(2 - \sqrt{5})^2 - \frac{8}{3 - \sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers. [4]

(b) Given that $\frac{a^x}{b^{3-x}} \times \frac{b^y}{(a^{y+1})^2} = ab^6$, find the value of x and of y . [4]

3 Given that $p = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$, express in its simplest surd form,

(i) p , [3]

(ii) $p - \frac{1}{p}$. [2]

7 (i) Use the substitution $u = 2^x$ to solve the equation $2^{2x} = 2^{x+2} + 5$. [5]

(ii) Solve the equation $2\log_9 3 + \log_5(7y - 3) = \log_2 8$. [4]

3 (i) Express 9^{x+1} as a power of 3. [1]

(ii) Express $\sqrt[3]{27^{2x}}$ as a power of 3. [1]

(iii) Express $\frac{54 \times \sqrt[3]{27^{2x}}}{9^{x+1} + 216(3^{2x-1})}$ as a fraction in its simplest form. [3]

7 Given that $\log_p X = 9$ and $\log_p Y = 6$, find

(i) $\log_p \sqrt{X}$, [1]

(ii) $\log_p \left(\frac{1}{X} \right)$, [1]

(iii) $\log_p (XY)$, [2]

(iv) $\log_Y X$. [2]

5 Solve the simultaneous equations

$$\log_3 a = 2 \log_3 b,$$

$$\log_3 (2a - b) = 1. \quad [5]$$

3 A number, N_0 , of fish of a particular species are introduced to a lake. The number, N , of these fish in the lake, t weeks after their introduction, is given by $N = N_0 e^{-kt}$, where k is a constant. Calculate

(i) the value of k if, after 34 weeks, the number of these fish has fallen to $\frac{1}{2}$ of the number introduced, [2]

(ii) the number of weeks it takes for the number of these fish to have fallen to $\frac{1}{5}$ of the number introduced. [3]

8 Solve the equation

(i) $\lg(5x + 10) + 2\lg 3 = 1 + \lg(4x + 12)$, [4]

(ii) $\frac{9^{2y}}{3^{7-y}} = \frac{3^{4y+3}}{27^{y-2}}$. [3]



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