Math 0580

June 2000

Paper 2

1- Least possible Length = $4810 - \frac{10}{2} = 4805\ m$

2- Next two prime numbers after 29 are 31 and 37

Mean = $\frac{31 + 37}{2} = 34$

3- Length of a "Life time" = $\frac{650 \times 10^6}{1000}$

= 650000 hours

= $\frac{650000}{24 \times 365} = 74.2$ year

= 74 to the nearest year.

4- $X = 3.4 \times 10^{-3} = 0.0034$

$y = 1.2 \times 10^{-1} = 0.12$

$z = 4.6 \times 10^{-4} = 0.00046$

(a) X < Y

(b) x + y > z

5- (a) Bearing of Z from C = 024°

Therefore bearing of C from Z = 180 + 24 = 204°

(b) Angle AZ makes with the south direction = 90 + 24 = 114

Bearing of Z from A = 114° (alternate angles)

6- 1 min 31.649 sec = 1 x 60 + 31.649 = 91.649 sec

208.303 Km / h = $\frac{208.303 \times 1000}{3600}$ m/s = 57.862 m/s

Length of Lap = 57.862 x 91.649

= 5302.99 = 5303 m

7- (a) $(8x^4y)^2 = 64x^8y^2$

(b) $(8x^4y)^2 \div x^2y^{-1} = \frac{64x^8y^2}{x^2y^{-1}} = 64x^6y^3$

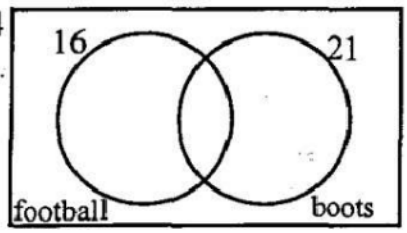
8- (a)(i) $\overrightarrow{QM} = -\frac{1}{2}r$

(ii) $\overrightarrow{RM} = P - \frac{1}{2}r$

(b) $\overrightarrow{OS} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

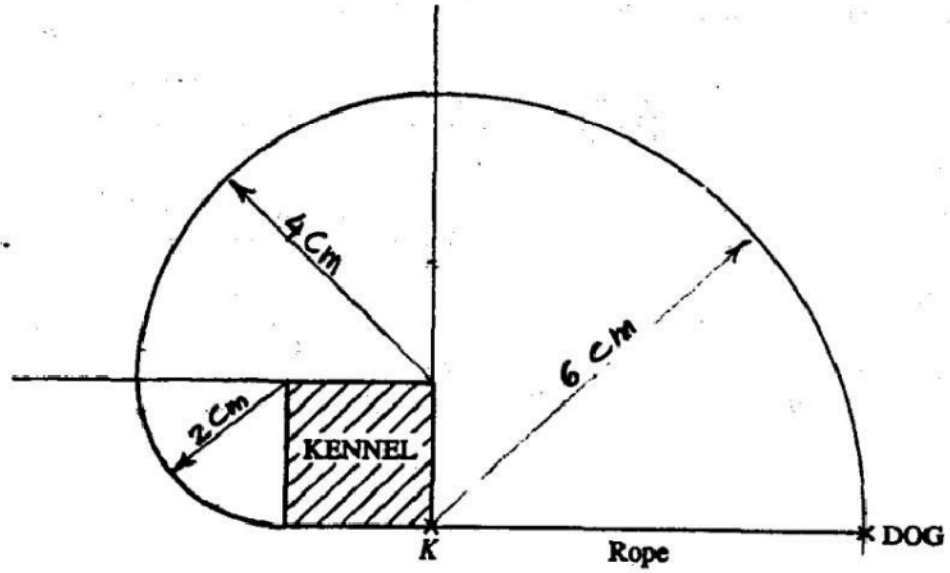
9- (a) The number of students should be exactly divisible by 3 and 8 , Therefore the number of students = 24

(b)



16 + 21 - 24 = 13

10-



11- Monday and Tuesday temperatures are - 2.4°
Wednesday (the median) is -1.3°
Friday is 4.5°, the maximum
Let Thursday temperature be x

$$\therefore \frac{(-2.4)+(-2.4)+(-1.3)+x+4.5}{5} = 0$$

x - 1.6 = 0

x = 1.6

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Temperature (°C)	-2.4	-2.4	-1.3	1.6	4.5

12- (a) x = 0

(b) $10^y = 0.01 = \frac{1}{100} = 10^{-2}$ y = -2

(c) $16^z = 2$ $(2^4)^z = 2$ $2^{4z} = 2^1$

$4^z = 1$ $z = \frac{1}{4}$

13- (a)

Cost	Loss	Selling
100	22.5	77.5
8400		?

Selling price = $\frac{8400 \times 77.5}{100} = \$6510$

(b)

Cost	Profit	Selling
100	40	140
?		8400

Amount paid for the car = $\frac{8400 \times 100}{140} = \$6000$

14- $y = 3 + \sqrt{2x}$

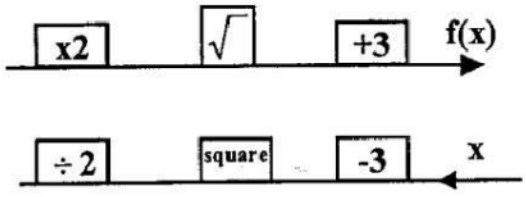
$y - 3 = \sqrt{2x}$

$2x = (y-3)^2$

$x = \frac{1}{2}(y-3)^2$

$f^{-1}(x) = \frac{1}{2}(x-3)^2$

OR



$f^{-1}(x) = \frac{(x-3)^2}{2}$

15- (a) $B = \frac{K}{d^2}$

$2 = \frac{K}{(12)^2}$

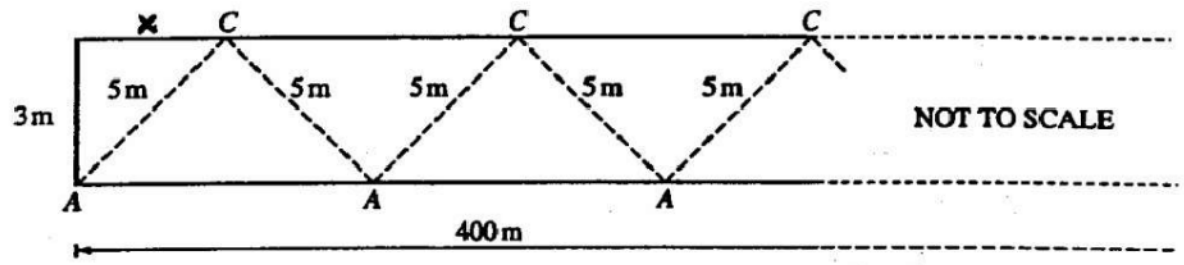
$\therefore$ K = 2 x 144 = 288

$B = \frac{288}{d^2}$

(b) d = 3

$B = \frac{288}{3^2} = 32$

16-



(a) distance x $= \sqrt{5^2 - 3^2} = 4$ m

distance between two apple trees = 8 m

number of apple trees $= \frac{400}{8} + 1 = 51$

(b) number of cherry trees is one less than apple trees i.e 50

17- (a) $x = \frac{1}{2} \times 100 = 50°$

y = angle OBA = 90 - 50 = 40°

z = angle ABT = 40°

(b) No, angle z = 40° and angle OAT = 50° alternate angles are not equal.

18- (a)

Alex		Bukki		chris
2	:		:	1
3	:	4	:	
6		$\frac{6 \times 4}{3} = 8$		$\frac{6 \times 1}{2} = 3$
Ratio		6 : 8 : 3		

(b) Sum of shares = 6 + 8 + 3 = 17

Cost of present for Bukki $= \frac{8}{17} \times 21.25 = \$10$

19- (a) Similar triangles $\frac{h}{1.59} = \frac{8}{3}$

$h = \frac{8 \times 1.59}{3} = 4.24m$

(b) $\frac{x}{4.24} = \frac{2x}{2x+5}$

$\frac{x}{2x} = \frac{4.24}{2x+5}$

$\frac{1}{2} = \frac{4.24}{2x+5}$

2x + 5 = 2 x 4.24 = 8.48

2x = 3.48 x = 1.74

20- (a) (i) Acceleration $= \frac{27}{3} = 9 m/s^2$

(ii) distance = area $= \frac{1}{2} \times 3 \times 27 = 40.5 m$

(b) 1000 - 112 = 888 m

$\frac{888}{2} = 444$ sec

total time = 444 + 12 = 456 sec

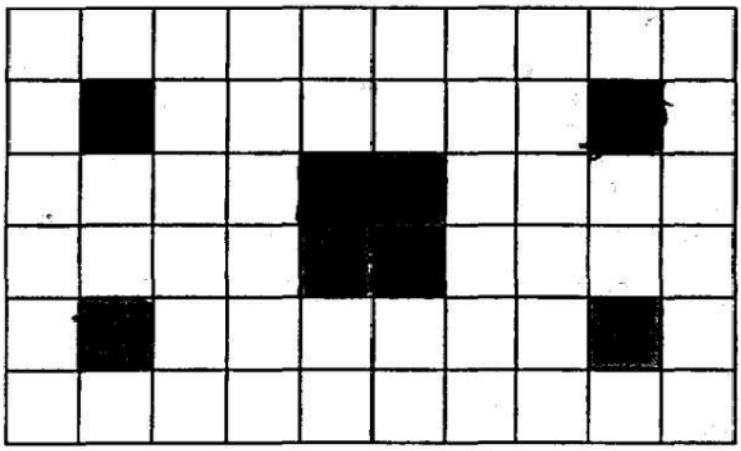
= 7.6 min = 7 min 36 sec

21- (a) $\frac{1}{x-3} - \frac{1}{x} = \frac{x-(x-3)}{x(x-3)} = \frac{3}{x(x-3)}$

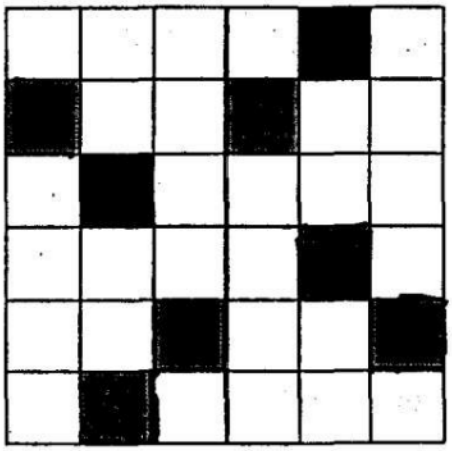
(b) $\frac{1}{y} = \frac{1}{x-3} - \frac{1}{x} = \frac{3}{x(x-3)}$

$y = \frac{x(x-3)}{3} = \frac{x^2-3x}{3} = \frac{1}{3}x^2 - x$

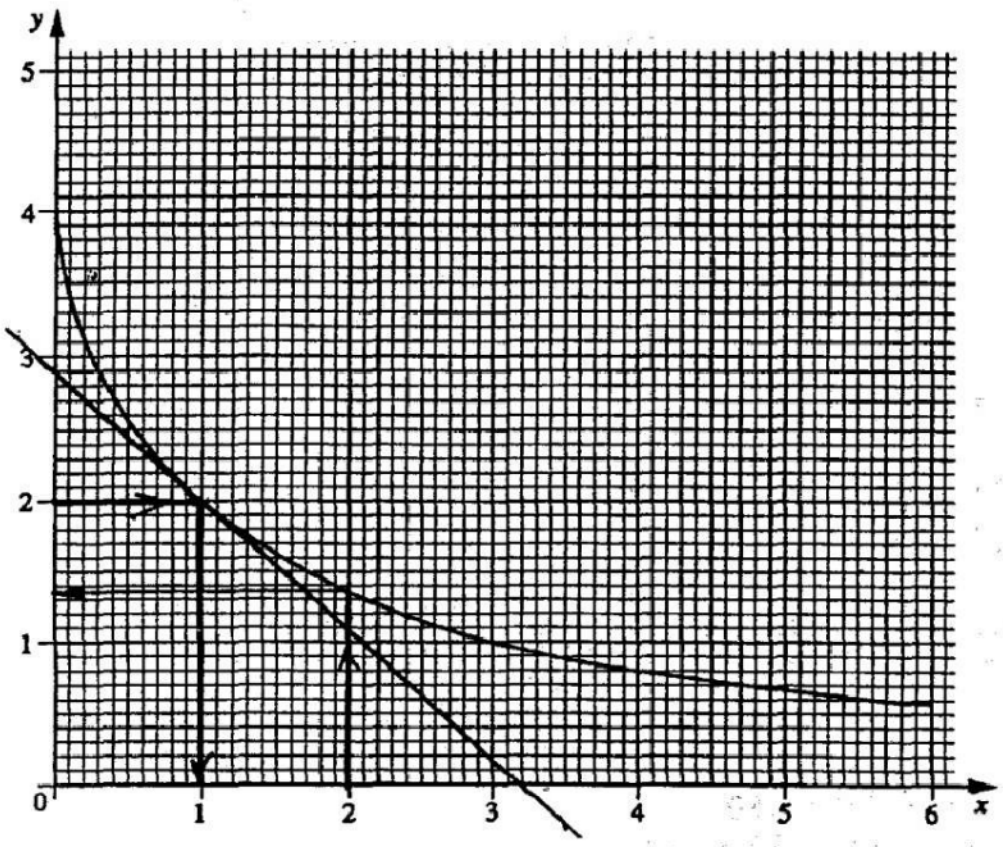
22- (a)



(b)



23-



(a)(i) f(2) means to find y when x = 2

from graph $y = 1.35$ $f(2) = 1.35$

(ii) $y = f(x)$ $x = f^{-1}(y)$ so $y = 2$

from graph $x = 1$ $f^{-1}(2) = 1$

(b) Tangent is drawn on graph

Two points on the tangent are (0, 2.9) (3.2, 0)

Gradient $= \dfrac{2.9-0}{0-3.2} = \dfrac{2.9}{-3.2} = -0.906 = -0.91$

Mathematics 0580

November 2000

Paper 2

1- $7 - 5(6-1) = 7 - 5(5) = 7 - 25 = -18$

2- $g = \sqrt{h+i}$ $\qquad g^2 = h+i$ $\qquad h = g^2 - i$

3- $\left(\frac{9}{4}\right)^{-\frac{3}{2}} = \left(\frac{4}{9}\right)^{\frac{3}{2}} = \frac{8}{27}$

4- $1\frac{1}{4} \div \frac{2}{3} - 1\frac{1}{3} = \frac{5}{4} \div \frac{2}{3} - \frac{4}{3}$

$= \frac{5}{4} \times \frac{3}{2} - \frac{4}{3} = \frac{15}{8} - \frac{4}{3} = \frac{45-32}{24} = \frac{13}{24}$

5- $36 - x + 7x = 180$

$6x = 180 - 36 = 144$

$x = \frac{144}{6} = 24$

6- $21.65 \leq$ Perimeter < 21.75

$7.65 \leq$ One side < 7.75

Smallest possible third side

$= 21.65 - 2(7.75)$

$= 6.15$

7- $2x - y = 81$ (1)

$x + 2y = 23$ (2)

(1) x 2 $\qquad 4x - 2y = 162$

(2) $\qquad x + 2y = 23$

$5x = 185$

$x = 37$

from (2) $\qquad 37 + 2y = 23$

$2y = -14$

$y = -7$

$x = 37 \qquad y = -7$

8- (a) time $= \frac{43.4}{2.8} = 15.5$ h

(b) Using calculator

20 [,,,] 40 [,,,] + 15.5 - 24 = shift [,,,] 1210

9- Sum of all interior angles $= (n-2) \times 180$

$= (5-2) \times 180 = 540$

$2t + 91 + 104 + 117 = 540$

$2t = 228 \qquad t = 114°$

10- $7 - 5x \geq -17$

$-5x \geq -17 - 7$

$-5x \geq -24$

$x \geq \frac{-24}{-5}$

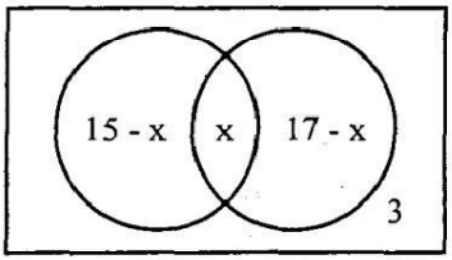
$x \geq 4.8$

$x \in \{1,2,3,4\}$

11- (a) Number of stndents who study both physics and chemistry $= 15 + 17 + 3 - 22$

$= 13$

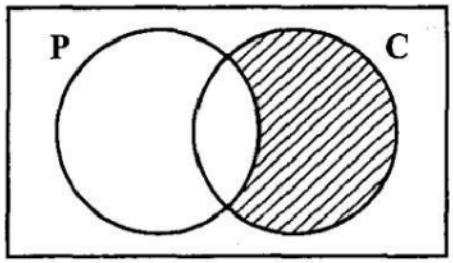
OR



$(15-x) + x + (17-x) + 3 = 22$

$35 - x = 22 \qquad x = 13$

(b)



$P' \cap C$

12- (a) Acceleration $= \frac{\text{Change in Velocity}}{\text{time}}$

$= \frac{14-10}{7-2} = \frac{4}{5} = 0.8 m/s^2$

(b) Distance = Area under the graph

= Area of $\triangle$ + Area of trapezium

$= \frac{1}{2} \times 2 \times 10 + \frac{10+14}{2} \times 5 = 10 + 60 = 70$ m.

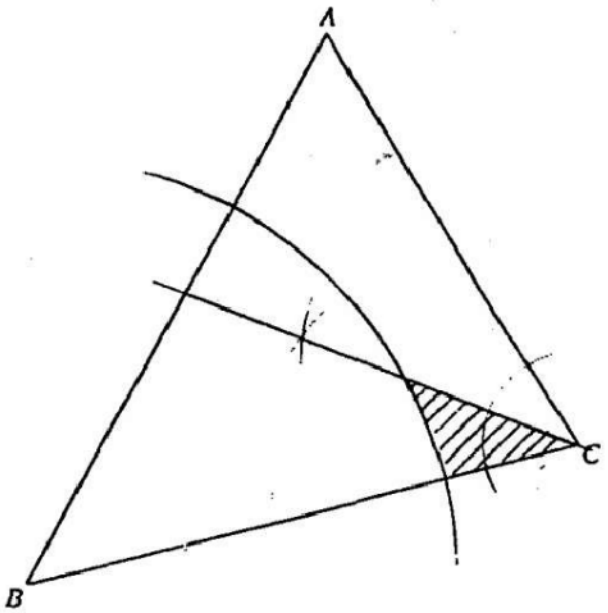
13- (a) (i) From graph at cumulative frequency of 25 height = 35 cm
Lower quartile = 35 cm
(ii) Upper quartile at cumulative frequency of 75
upper quartile = 52
Interquartile range = $52 - 35 = 17$
(b) When height = 25 cm
Cumulative frequency = 10
number of plants with a height greater than 25 cm = $100 - 10 = 90$

14- Bank charges = $\frac{1\frac{1}{2}}{100} \times 250 = 3.75$
number of Drachma received = $(250 - 3.75) \times 485 = 119431.25$
= 119430 to the nearest 10

15- (a) $t^2 - 4 = (t + 2)(t - 2)$
(b) $at^2 - 4a + 2t^2 - 8$
$= a(t^2 - 4) + 2(t^2 - 4) = (t^2 - 4)(a + 2)$
$= (t + 2)(t - 2)(a + 2)$

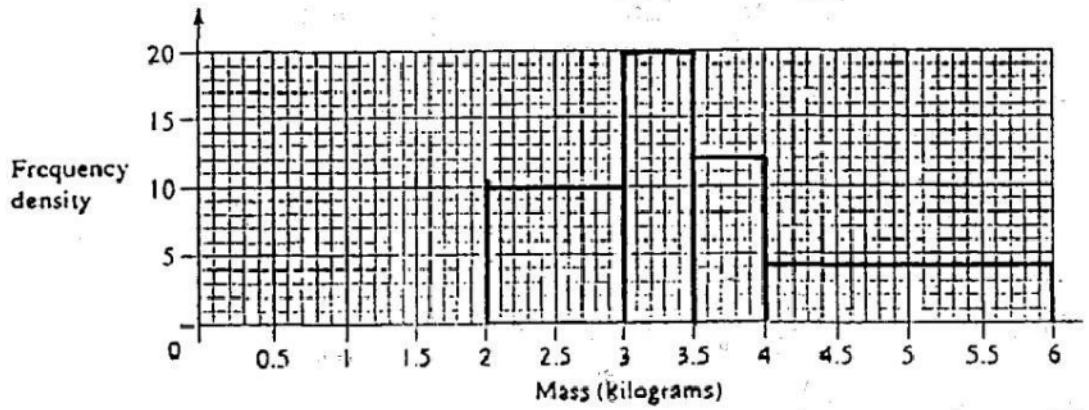
16- (a) $V \propto h^3$
$V = K h^3$
When $h = 3$ $V = 6.75$
$6.75 = K(3)^3$ $K = \frac{6.75}{27} = 0.25$
$V = 0.25 h^3$
(b) $V = 0.25 h^3 = 0.25 \times (2.5)^3 = 3.906\ cm^3$
$= 3.91\ cm^3$

17-



18-

Mass m	Frequency
$2 < m \leq 3$	10
$3 < m \leq 3.5$	$0.5 \times 20 = 10$
$3.5 < m \leq 4$	$0.5 \times 12 = 6$



19- (a) Reflection on the line $y = x$

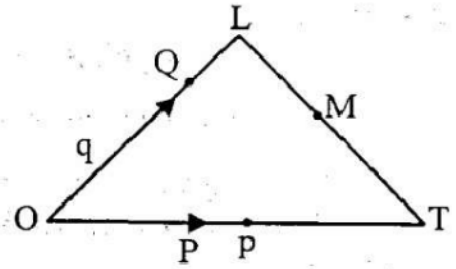
(b) $\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix}$ $\qquad$ $\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix}$

Matrix $= \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

20- (a) (i) OQ : QL = 2 : 1

$\overrightarrow{OQ} = q$

$\overrightarrow{QL} = \frac{1}{2}q$ $\qquad$ $\overrightarrow{OL} = 1\frac{1}{2}q$



(ii) $\overrightarrow{OP} = \overrightarrow{PT}$

$\overrightarrow{OP} = P$

$\therefore \overrightarrow{OT} = 2P$

$\overrightarrow{LT} = -1\frac{1}{2}q + 2P = 2P - 1\frac{1}{2}q$

(b) $\overrightarrow{OM} = \overrightarrow{OL} + \overrightarrow{LM} = \overrightarrow{OL} + \frac{1}{2}\overrightarrow{LT}$

$= 1\frac{1}{2}q + \frac{1}{2}(2p - 1\frac{1}{2}q) = \frac{3}{2}q + P - \frac{3}{4}q = p + \frac{3}{4}q$

OR $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OL} + \overrightarrow{OT})$

$= \frac{1}{2}(\frac{3}{2}q + 2p) = p + \frac{3}{4}q$

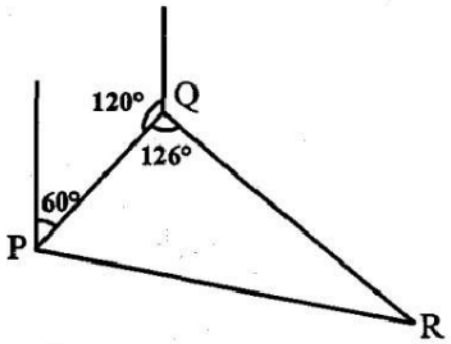
21- (a) $\frac{140}{\sin 31} = \frac{220}{\sin Q}$

$\text{Sin } Q = \frac{220 \sin 31}{140} = 0.8093$

$Q = 54° \text{ or } 180 - 54$

$= 54° \text{ or } 126°$

$\therefore \angle PQR = 126° \text{ (obtuse)}$



(b) Bearing of R from Q

$= 360 - (120 + 126)$

$= 360 - 246$

$= 114°$

22- (a) $f\left(-\frac{3}{4}\right) = 3 - 2\left(-\frac{3}{4}\right) = 3 + \frac{3}{2} = 4\frac{1}{2}$

(b) $g(x) = \frac{x+1}{4}$

$y = \frac{x+1}{4} \qquad 4y = x + 1$

$x = 4y - 1$

$g^{-1}(x) = 4x - 1$

(c) $fg(x) = f(\frac{x+1}{4}) = 3 - 2(\frac{x+1}{4}) = 3 - \frac{x+1}{2} = \frac{6-x-1}{2} = \frac{5-x}{2}$

23- (a) l_1 : $y = 1$

l_2 : using points $(0, 0), (1, 2)$

gradient $m = \frac{2-0}{1-0} = 2$

equation is $y = 2x$

l_3 : line joining $(0, 5)$ and $(5, 0)$ its equation is $x + y = 5$

(b) $y > 1$

$y \leq 2x$

$x + y \leq 5$