June 1997

Paper 4

1. (a) \( \frac{80}{100} \times 200 = 160 \)

   20 children = \( 20 \times 2.50 = 50 \)
   140 adult = \( 140 \times 5 = 700 \)
   total \( 700 + 50 = 750 \)

(b) Sale of children tickets = \( 2.5 \times \)
   Sale of adult tickets = \((200 - x) \times 5 \)
   \( 2.5x + (200 - x)5 = 905 \)
   \( 2.5x + 1000 - 5x = 905 \)
   \( -2.5x = -95 \)
   \( x = 38 \)

(c) (i) \[
\begin{align*}
2 : 3 : 7 & \quad \text{total} \\
? & \quad 10800 \\
\end{align*}
\]

profit = \( \frac{7 \times 10800}{12} = 6300 \)

(ii) \[
I = \frac{PRT}{100} = \frac{6300 \times 5 \times \frac{4}{12}}{100} = £105
\]
2. 

(a) \(BD = 13.9\)

(b) Area of \(\triangle BCD = \frac{1}{2} \times 13.9 \times 2 = 13.9\)

Area of trapezium = \(\frac{8 + 13.9}{2} \times 4 = 43.8\)

Total area of the pentagon = \(13.9 + 43.8 = 57.7\, \text{cm}^2\)

3. (a) (i) Square numbers are 1, 4, 9

\[ \text{Prob.} = \frac{3}{12} = \frac{1}{4} \]

(ii) Prime numbers or numbers less than 6 are 1, 2, 3, 4, 5, 7, 11

\[ \text{Prob.} = \frac{7}{12} \]

(b) (i) \(12 + 9, 9 + 12, 10 + 11, 11 + 10\)

(ii) \(\frac{1}{12} \times \frac{1}{12} \times 4 = \frac{1}{36}\)

(c) 

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<tr>
<td>Frequency</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

(i) the mode = 11
(ii) the median

\[
\text{median order } \frac{30+1}{2} = 15.1
\]

terms numbered 15, 16, 17, 18, 19 are all 10

Median is 10

(iii) the mean \( \frac{\sum fx}{30} = 8.9 \)

(d) (i) area \( = \frac{\theta}{360} \pi r^2 = \frac{30}{360} \times 3.142 \times 10^2 = 26.18 \text{ cm}^2 \)

(ii) \( \frac{\text{Shaded area}}{\text{Area of square}} = \frac{26.18}{30 \times 30} = 0.0291 \)

4. (a) \( \angle ABC = 90 + 25 = 115^\circ \)

(b) (i) \( AC^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \cos 115 = 21.95 = 22 \text{ km} \)

(ii) \( \frac{AC}{\sin B} = \frac{BC}{\sin A} \)

\[ \frac{21.95}{\sin A} = \frac{14}{\sin A} \]

\( \sin A = 0.5779 \)

\( A = 35.3^\circ \)

(iii) Bearing of C from A = 25 + 35.3 = 60.3^\circ

Bearing of A from C = 180 + 60.3 = 240.3^\circ

5. (a) ABC and ADE are similar

(i) \( \frac{AC}{AE} = \frac{BC}{DE} \)

\[ \frac{5}{5+2x} = \frac{x+3}{4x+1} \]

\[(x+3)(5+2x) = 5(4x+1)\]

\[2x^2 + 11x + 15 = 20x + 5\]

\[2x^2 - 9x + 10 = 0\]

(ii) \( 2x^2 - 9x + 10 = (2x - 5)(x - 2) \)

(iii) \( x = \frac{5}{2}, x = 2 \)
(iv) Ratio of sides \( \frac{x + 3}{4x + 1} = \frac{2\sqrt{2} + 3}{4 \times 2\sqrt{2} + 1} = \frac{5\sqrt{2}}{11} = \frac{1}{2} \)

ratio of areas = \( \left( \frac{1}{2} \right)^2 \)

(b) (i) determinant of \( M = (2y + 1)(2y + 3) - y(3y - 4) \)
\[ = 4y^2 + 8y + 3 - 3y^2 + 4y \]
\[ = y^2 + 12y + 3 = 10 \]

\[ y^2 + 12y - 7 = 0 \]
\[ y = \frac{-12 \pm \sqrt{144 - 4 \times 1 \times (-7)}}{2} = \frac{-12 \pm \sqrt{172}}{2} \]
\[ y = 0.557 \quad , \quad -12.557 \]
\[ y = 0.6 \quad \text{or} \quad -12.6 \]

6. (a) \( OC = \sqrt{6^2 - (3.6)^2} = 4.8 \)
\( VC = 6 + 4.8 = 10.8 \)

(b) (i) the volume of the sphere = \( \frac{4}{3} \pi r^3 = \frac{4}{3} \times 3.142 \times 6^3 = 904.896 \)
\[ = 905 \]

(ii) the volume of the cone = \( \frac{1}{3} \pi r^2 h = \frac{1}{3} \times 3.142 \times 3.6^2 \times 10.8 \)
\[ = 146.59 = 147 \]

(iii) percentage of sphere occupied \( \frac{147}{905} = 16.2\% \)
not occupied \( = 100 - 16.20 = 83.8\% \)

(c) (i) \( 2 \pi r = 37.704 \)
\[ \frac{300}{37.628} = 7.957 = 7 \]

(ii) remaining part of revolution = \( 1 - 0.957 = 0.043 \)
\[ \text{Angle} = 0.043 \times 36 = 15.5^\circ \]
7. \( f(x) = x^3 - 12x + 5 \)

(a) \( a = (-1)^3 - 12(-1) + 5 = 16 \)  \( b = (4)^3 - 12(4) + 5 = 21 \)

(c) (i) \( f(x) = 0 \)  \( y = 0 \)  \( x = -3.65, 0.4, 3.25 \)

(ii) \( x^3 - 12x + 10 = 0 \)  \( x^3 - 12x + 5 = -5 \)

\( x = -3.8, 0.9, 2.9 \)

(d) Tangent passes through \( (-3, 16) \) and \( (-1, 29.5) \)

\[
\text{gradient} = \frac{29.5 - 16}{(-1) - (-3)} = \frac{13.5}{2} = 6.75
\]

(iii) To find another point the tangent at which is parallel to the tangent at \( x = -2.5 \) point is \( x = 2.5 \)

( tangent are parallel )
8. (a) (i) B, translation \[
\begin{pmatrix}
3 \\
-3
\end{pmatrix}
\]
(ii) C, reflection on the line \(y = x\)
(iii) D, rotation 90° anticlockwise centre origin
(b) \[
\begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix}
\]
(c) reflection on the y axis.
(d) \[
\begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix}
\]
(e) \(y = x - 3\)

9.

<table>
<thead>
<tr>
<th>Birthday</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheme A</td>
<td>$10</td>
<td>$20</td>
<td>$30</td>
<td>$40</td>
<td>$50</td>
<td>$60</td>
<td>$70</td>
</tr>
<tr>
<td>Scheme B</td>
<td>$1</td>
<td>$2</td>
<td>$3</td>
<td>$8</td>
<td>$16</td>
<td>$32</td>
<td>$64</td>
</tr>
<tr>
<td>Scheme C</td>
<td>$1</td>
<td>$4</td>
<td>$9</td>
<td>$16</td>
<td>$25</td>
<td>$36</td>
<td>$49</td>
</tr>
</tbody>
</table>

(a) (i) his 7th birthday, \(70, 64, 49\).
(ii) his \(n\)th birthday, \(10n, 2^{n-1}, n^2\)

(b) (i) 550, 1023, 385.
(ii) 1710, 262143, 2109 A the smallest.

(iii) A and C an equal when \[5n \ (n + 1) = \frac{n(n+1)(2n+1)}{6}\]

\[30 = 2n + 1 \quad 2n = 29\]
\[n = 14 \frac{1}{2}\]
i.e. up to 14th birthday C is smaller
& starting from 15th birthday A is smaller.
1-(a) Volume = \( \pi r^2 h = 3.142 \times \left( \frac{8}{2} \right)^2 \times 11 = 552.992 = 553 \text{ cm}^3 \)

(b) (i) Length = \( 4 \times 8 = 32 \text{ cm} \)
Width = \( 3 \times 8 = 24 \text{ cm} \)

(ii) Volume of the box = \( 32 \times 24 \times 11 = 8448 \text{ cm}^3 \)
Volume occupied by the tins = \( 12 \times 552.992 = 6635.904 \)
Volume not occupied = \( 8448 - 6635.904 = 1812.096 \)
Percentage not occupied = \( \frac{1812.096}{8448} \times 100 = 21.45\% = 21.5\% \)

(c) Cost Price | Profit | Selling Price
---|---|---
100 | 25 | 125
? | | 0.60

Cost price of one tin = \( \frac{100 \times 0.60}{125} = 0.48 \)

Cost price of a box 12 tins = \( 12 \times 0.48 = $5.76 \)

(d) (i) Selling price for a box = \( 12 \times 0.60 = $7.20 \)
Saving per box = \( 7.20 - 6.49 = $0.71 \)

(ii) Cost price of a box = $5.76
new selling price of a box = $6.49
profit per box = \( 6.49 - 5.76 = 0.73 \)
percentage profit = \( \frac{0.73}{5.76} \times 100 = 12.7\% \)
(a)  
(i)  \[ M(Z) = V \]
(ii) \[ XU = XY = 5 \text{ cm} \]
\[ XV = XZ = 2 \text{ cm} \]
(iii) Scale factor of enlargement \[ \frac{XY}{XV} = \frac{5}{2} = 2.5 \]
\[ XW = XU \times 2.5 = 5 \times 2.5 = 12.5 \text{ cm} \]
\[ VU = YZ = 6 \text{ cm} \]
\[ YW = 6 \times 2.5 = 15 \text{ cm} \]
(v) \[ \angle XYZ = \angle XUV = \angle XWY \]

(b) \[ \cos \angle YXZ = \frac{5^2 + 2^2 - 6^2}{2 \times 5 \times 2} = \frac{-7}{20} = -0.35 \]
\[ \angle YXZ = 110.5^\circ \]

3-(a)  
(i)  11,12,13,14,21,22,23,24,31,32,33,34
(ii) (a) outcomes multiples of 4 are 12,24,32
probability \[ = \frac{3}{12} = \frac{1}{4} \]
(b) no outcome is a multiple of 5
\[ \therefore \text{ probability } = \text{ Zero} \]
(b) Alice's choice  Barbara's choice

(i) Hard  Hard
    \[
    \begin{align*}
    \text{Hard} & : \frac{2}{6} \\
    \text{Soft} & : \frac{4}{6}
    \end{align*}
    \]

    \[
    \begin{align*}
    \text{Hard} & : \frac{4}{5} \\
    \text{Soft} & : \frac{2}{5}
    \end{align*}
    \]

(ii)(a) \(\frac{2}{6} \times \frac{1}{5} = \frac{1}{15}\)

(b) Hard and Soft or Soft and Hard
    \[
    \frac{2}{6} \times \frac{4}{5} + \frac{4}{6} \times \frac{2}{5} = \frac{8}{15}
    \]

(c) Hard and Hard or Soft and Hard
    \[
    \frac{2}{6} \times \frac{1}{5} + \frac{4}{6} \times \frac{2}{5} = \frac{2}{30} + \frac{8}{30} = \frac{1}{3}
    \]

4-(a) \[\angle AOB = \frac{360\degree}{7} = 51.429\degree\]
    \[\angle OAB = \frac{180\degree - 51.429\degree}{2} = 64.29\degree\]

(b)(i) \[\sin \angle DAB = \frac{OX}{OA}\]
    \[OX = 1.5 \sin 64.29\degree = 1.35 \text{cm}\]

(ii) \[AB = 2 \times AX\]
    \[\cos 64.29\degree = \frac{AX}{1.5}\]
    \[AX = 0.65\]
    \[AB = 2 \times 0.65 = 1.30 \text{cm}\]
OR Use Cosine rule

\[ AB^2 = 1.5^2 + 1.5^2 - 2(1.5)(1.5)\cos 51.429 \]

(iii) area of \( \triangle AOB = \frac{1}{2} \text{ base} \times \text{ height} \)

\[ = \frac{1}{2} \times AB \times AX = \frac{1}{2} \times 1.30 \times 1.35 \]
\[ = 0.8775 \approx 0.878 cm^2 \]

(iv) area of the whole face = 7 \times 0.8775 = 6.14 cm^2

(c) Volume = Area \times \text{ thickness}

\[ = 6.14 \times \frac{3}{10} = 1.84 cm^3 \]

5-

(f) all equal.

(g) congruent

(h) radius = 2.6 cm
(b)(i) Line through (3,0) gradient 2, for each 1 unit along x
y increases by 2 i.e. Line passes through (4,2), (5,4) .......
(ii) Line intercepts y axis at -6 equation $y = 2x - 6$
(iii) (0.7, -4.6), (4.3, 2.6)

(c) gradient of tangent $= \frac{2 - (-10)}{4 - 2} = 4$
(d) \[ y = ax^2 + bx \]
\[ x = 4 \quad y = 0 \]
\[ 16a + 4b = 0 \]
\[ b = -4a \]
\[ x = 1 \quad y = -6 \]
\[ -6 = a + b \]
\[ -6 = a - 4a \]
\[ -6 = -3a \]
\[ a = 2 \quad b = -8 \]

(a)(i) Median = 56.5 \approx \$56 \text{ or } \$57
(ii) Upper quartile = \$61
Lower quartile = \$53
(iii) 60th percentile = \$58
(b)(i) Interquartile range = Upper quartile – Lower quartile
= 61 – 53 = $8

(ii) Percentage = $\frac{8}{50} \times 100 = 16\%$

(c)(i) Weekly amount $x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>Frequency</th>
<th>$x$</th>
<th>$fx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40 &lt; x \leq 50$</td>
<td>14</td>
<td>45</td>
<td>630</td>
</tr>
<tr>
<td>$50 &lt; x \leq 60$</td>
<td>72 – 14 = 58</td>
<td>55</td>
<td>3190</td>
</tr>
<tr>
<td>$60 &lt; x \leq 70$</td>
<td>92 – 72 = 20</td>
<td>65</td>
<td>1300</td>
</tr>
<tr>
<td>$70 &lt; x \leq 80$</td>
<td>98 – 92 = 6</td>
<td>75</td>
<td>450</td>
</tr>
<tr>
<td>$80 &lt; x \leq 90$</td>
<td>100 – 98 = 2</td>
<td>85</td>
<td>$170$</td>
</tr>
</tbody>
</table>

(ii) Modal class is $50 < x \leq 60$

(iii) Mean = $\frac{\sum fx}{\sum f} = \frac{5740}{100} = 57.4$

(iv) Using smaller class intervals i.e. $40 – 42$, $42 – 44$, ..... Or $40 – 45$, $45 – 50$ etc.

8- (b)(i) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 6 \\ 2 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 6 \\ 6 & 8 & 16 \end{pmatrix}$

(ii) Area of $S$ is the same as $T$
which equal = $\frac{1}{2} \times 2 \times 4 = 4$

(iii) transformation is a shear parallel to the $y$ axis (y axis invariant)

(c)(i) $M = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

$M^{-1} = \frac{1}{1} \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$

(ii) Image of $S$ under the transformation $M^{-1}$ is $T$
9- (a) \[ 5^2 + 12^2 = 25 + 144 = 169 = 13^2 \]
(b) \[ 24^2 = 576 \]
\[ 25^2 = 625 \]
\[ 25^2 - 24^2 = 625 - 576 = 49 = 7^2 \]
Pythagorean triple is 7, 24, 25

(c) (i) \[ y^2 = x^2 - (x-2)^2 \]
\[ = x^2 - (x^2 - 4x + 4) \]
\[ = 4x - 4 \]
\[ y = \sqrt{4x - 4} \]

(ii) \[ x = 50 \]
\[ y = \sqrt{4 \times 50 - 4} = \sqrt{196} = 14 \]
\[ x - 2 = 50 - 2 = 48 \]
other two numbers are 48, 14

(iii) \[ x = 101 \]
\[ y = \sqrt{4 \times 101 - 4} = 20 \]
\[ 101 - 2 = 99 \]
other two numbers are 99, 20

(iv) since \[ y = \sqrt{4x - 4} = \sqrt{4(x-1)} = 2\sqrt{x-1} \]

In order to get y whole number, x should be taken such that \((x-1)\) is a perfect square.

Possible values of x are 9 + 1 = 10, or 16 + 1 = 17, 25 + 1 = 26, 36 + 1 = 37

for each x, x - 2 and y can be calculated

i.e. \[ x = 10 \]
\[ x - 2 = 8 \]
\[ y = 2\sqrt{10-2} = 6 \] \{ 6, 8, 10 \}
\[ x = 17 \]
\[ x - 2 = 15 \]
\[ y = 2\sqrt{17-1} = 8 \] \{ 8, 15, 17 \}
\[ x = 26 \]
\[ x - 2 = 24 \]
\[ y = 2\sqrt{26-1} = 10 \] \{ 10, 24, 26 \}
\[ x = 37 \]
\[ x - 2 = 35 \]
\[ y = 2\sqrt{37-1} = 12 \] \{ 12, 35, 37 \}

Any one set is a possible answer.