June 1994

Paper 4

1- (a) Each exterior angle $= \frac{360}{5} = 72$

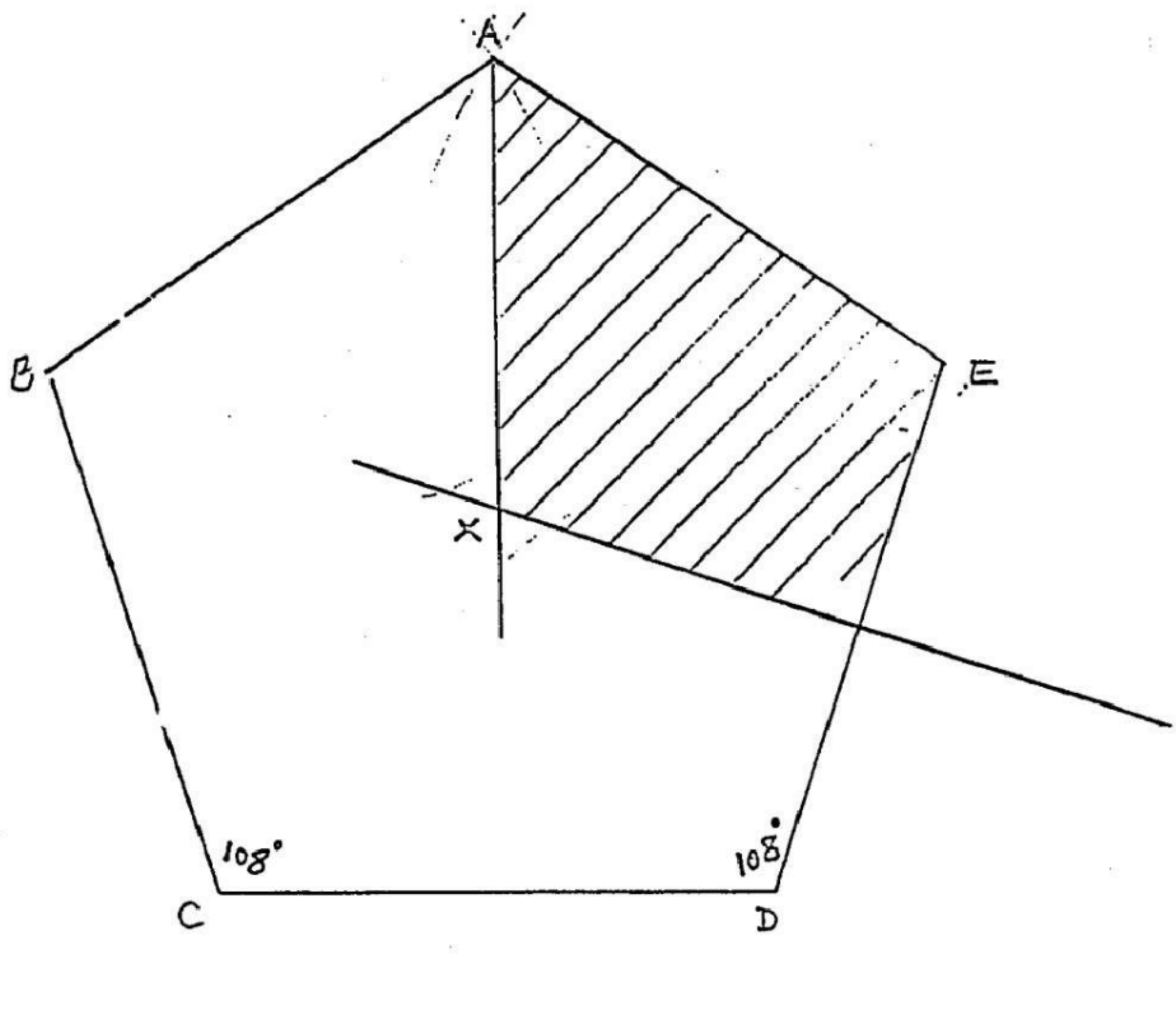
Each interior angle $= 180 - 72 = 108°$

<u>OR</u> Sum of all angles $= (2n - 4) \times 90$

$= (10 - 4) \times 90 = 540$

each interior angle $= \frac{540}{5} = 108°$

(b)



(iii) AX = 6.6 cm

2- (a) (i) speed $= \frac{80}{10} = 8$ m/s

(ii) speed $= \frac{8 \times 3600}{1000} = 28.8$ km/h

(b) total time $= 10.5 + 13 + \frac{120}{8.5}$

$= 37.6$ s

overall average speed $= \frac{80+100+120}{37.6}$

$= 7.98$ m/s

(c) $\text{Cos BAC} = \frac{80^2 + 120^2 - 100^2}{2 \times 80 \times 120} = 0.5625$

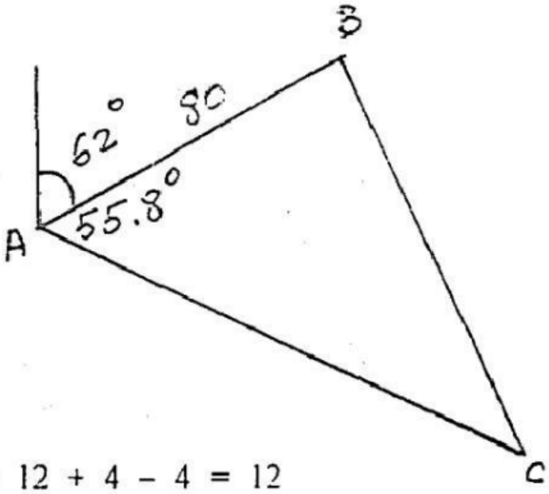
$\angle BAC = 55.8°$

(d) Bearing of C from A

$= 55.8 + 62 = 117.8° = 118°$

Bearing of A from C

$= 180 + 118 = 298°$



3- (a) $f(x) = 3x^2 - 2x - 4$

$f(-2) = 3(4) - 2(-2) - 4 = 12 + 4 - 4 = 12$

(b) $f(x) = -3$

$3x^2 - 2x - 4 = -3$

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = -\frac{1}{3}$ or $x = 1$

(c) $f(x) = 0$ $3x^2 - 2x - 4 = 0$

$x = \frac{2 \pm \sqrt{4 - 4 \times 3(-4)}}{6}$

$\frac{2 \pm \sqrt{52}}{6} = 1.54$ or -0.87

(d) $g(x) = 2g(x) - 1$

$4 - 3x = 2(4 - 3x) - 1$

$4 - 3x = 8 - 6x - 1$

$4 - 3x - 8 + 6x + 1 = 0$

$3x - 3 = 0$

$x = 1$

(e) $g(x) = 4 - 3x$

$y = 4 - 3x$

$3x = 4 - y$

$x = \frac{4-y}{3}$

$g^{-1}(x) = \frac{4-x}{3}$

(f) (i) $y = f(x)$ graph B

(ii) $y = g(x)$ graph C

4- (a) No of teachers x , No of students y

$\therefore \quad 24x + 20y \geq 240$

$\div 4 \quad 6x + 5y \geq 60$

(b) $x + y \leq 13$, $x \geq 4$, $y \geq 3$

(c) $6x + 5y = 60 \quad x = 0 \quad y = 12$, $y = 0 \quad x = 10$

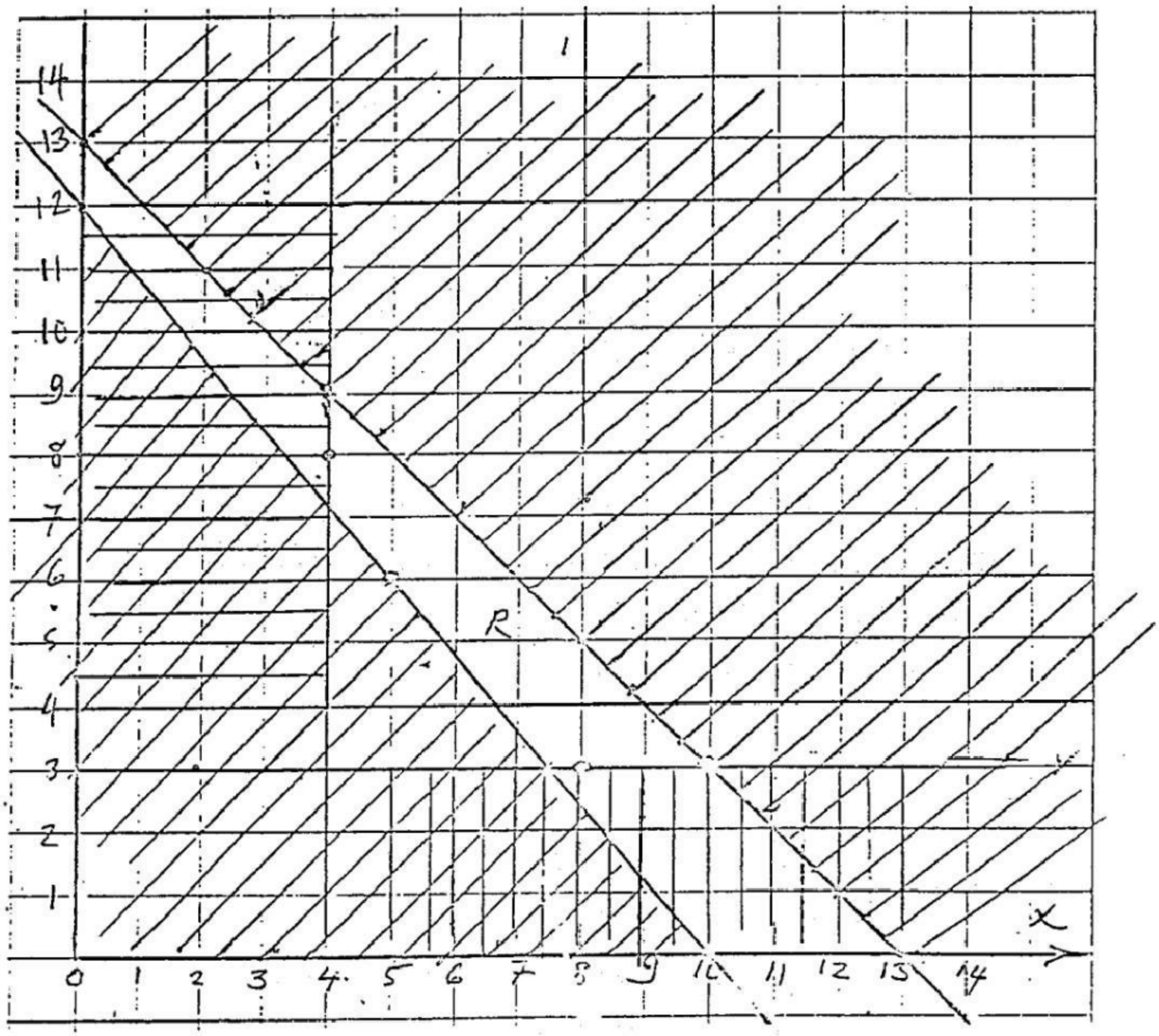
$x + y = 13 \quad x = 0 \quad y = 13$, $y = 0 \quad x = 13$

(d) The region satisfying all inequelities are marked R

To find the answers to (i) and (ii) we test the corners of the quad R or points closer to corners, so we check points (4 , 8) , (4 , 9) , (5 , 6) , (8 , 3) , (10 , 3) .

(i) Least x + y is for (5 , 6) and (8 , 3) which is equal 11.

(ii) Greatest value of 24x + 20y is for (10 , 3) and equal 300 kg.


14
13
12
11
10
9
8
7
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5
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3
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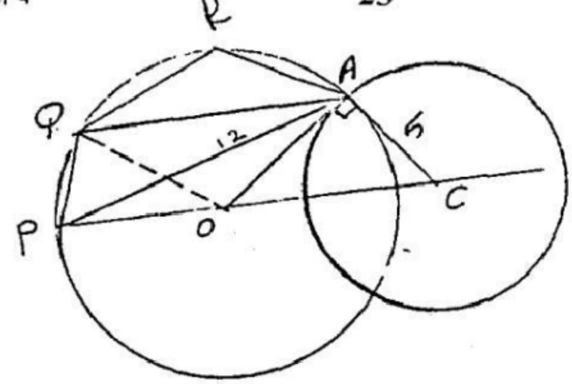
12
13
14
x
R



5- (a) $\overline{OC}^2 = 12^2 + 5^2 = 169$

$\therefore$ OC = 13

(b) circle through O , A and C

has OC as diameter = 13

its radius = $\frac{13}{2} = 6\frac{1}{2}$ cm

(c) Sin < AOC = $\frac{5}{13}$ or tan < ADC = $\frac{5}{12}$

< AOC = 22.6°

(d) < APC = ½ < AOC = $\frac{22.6}{2}$ = 11.3°

(e) < OAQ = < AOC = 22.6°

OA = OQ

$\therefore$ < OQA = < OAQ

$\therefore$ < AOQ = 180 − 2 (22.6) = 134.8°

(f) < APQ = ½ < AOQ = ½ x 134.8 = 67.4°

(g) APQR is a cyclic quad

< QRA = 180 − < APQ

= 180 − 67.4 = 112.6°

6- (b) (i) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 5 \\ 1 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 5 \\ 2 & 5 & 5 \end{pmatrix}$

(ii) transformation is the reflection on the line y = x

(c) (ii) For reflection on the line y = − x

point (1 , 0) is reflected into (0 , − 1)

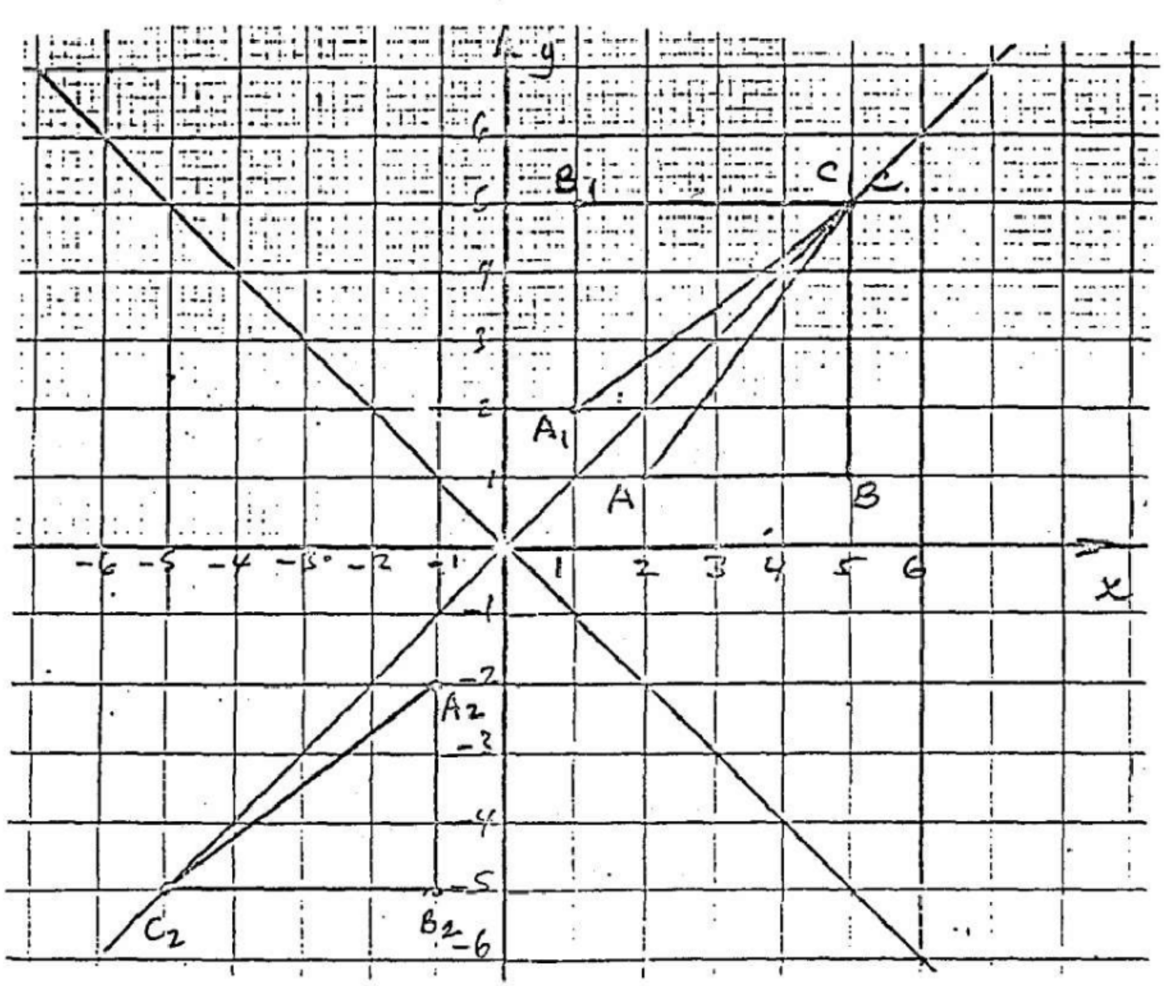
point (0 , 1) is reflected into (− 1 , 0)

Matrix of trasformation is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(d) (i) transformation which maps $A_1B_1C_1$ to $A_2B_2C_2$ is a rotation by 180° centre origin or enlargement by − 1 centre origin.

point (1 , 0) → (− 1 , 0)

(0 , 1) → (0 , − 1) matrix is $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$



7. (a)

Height	Frequency
$0 < h \le 5$	20
$5 < h \le 10$	40
$10 < h \le 15$	60
$15 < h \le 25$	80
$25 < h \le 50$	50

(b)

Mid Interval x	Frequency f	fx
2.5	20	50
7.5	40	300
12.5	60	750
20	80	1600
37.5	50	1875
	250	4575

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{4575}{250} = 18.3$$

(c)

height h	Cummulative frequency
≤ 5	20
≤ 10	60
≤ 15	120
≤ 25	200
≤ 50	250

(d) (i) class interval $15 < h \le 25$

$$\text{Median} = 15 + \frac{125-120}{200-120} \times (25 - 15)$$

$$= 15 + \frac{5}{80} \times 10 = 15\frac{5}{8} = 15.6$$

(e) $\text{probability} = \frac{250-60}{250} = \frac{190}{250} = \frac{19}{25}$

(f) probability = $\frac{190}{250} \times \frac{189}{249} = 0.577$

8- (a) Length = $\frac{7.56}{0.42}$ = 18

(b) mass = 7.56 x 0.88 = 6.65 g

(c) $0.5\ m^3 = 0.5 \times 10^6\ cm^3$

no. of prisms = $\frac{75}{100} \times \frac{0.5 \times 10^6}{7.56}$

= 49603

= 50000 to the nearest thousand

(d) (i) area of Δ OAB = $\frac{1}{6} \times 0.42 = 0.07\ cm^2$

(ii) Equilateral.

(iii) Area = $\frac{1}{2}x^2 \sin 60$

$\frac{1}{2}x^2 \times 0.866 = 0.07$

$x^2 = \frac{0.14}{0.866}$

x = 0.402 cm

= 4 mm

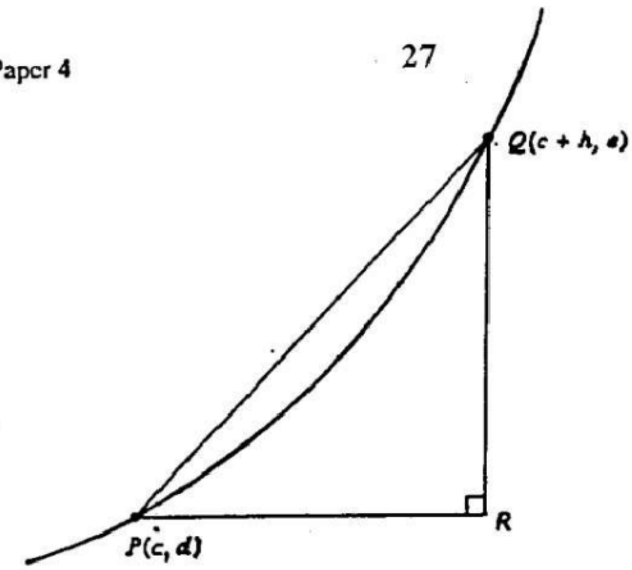
9- (a) gradient = $\frac{24.5-18}{3.5-3} = \frac{6.5}{0.5} = 13$

(b) (i) $y = 2x^2$ P is (c , d)

$d = 2c^2$

(ii) Q is (c + h , e)

$e = 2(c+h)^2$



(iii) PR = h

QR = e − d

$= 2(c + h)^2 - 2c^2$

$= 2(c^2 + 2ch + h^2) - 2c^2$

(iv) gradient $= \dfrac{QR}{PR} = \dfrac{4ch + 2h^2}{h}$

$= 4c + 2h$

(v) P is (c , d) which is (3 , 18)

i.e. c = 3

Q is (c + h , e) which is (3.5 , 24.5)

$\therefore$ h = 0.5

gradient = 4c + 2h

$= 4 \times 3 + 2(0.5) = 13$

(vi) c = 3 h = 0.1

gradient $= 4 \times 3 + 2 \times 0.1$

= 12.2

(vii) (a) h approaches zero

(b) gradient $= 4c = 4 \times 3 = 12$

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Nov. 1994

Paper 4

1. (a) (i) Amount divided between them $= \frac{40}{100} \times 9000 = 3600$

Amount Alexis receives $= \frac{5}{9} \times 3600 = \$\,2000$

Amount Biatriz receives $= \frac{3}{9} \times 3600 = \$\,1200$

Amount Carlos receives $= \frac{1}{9} \times 3600 = \$\,400$

(ii) Carlos receives $\frac{1}{9}$ of the Amount

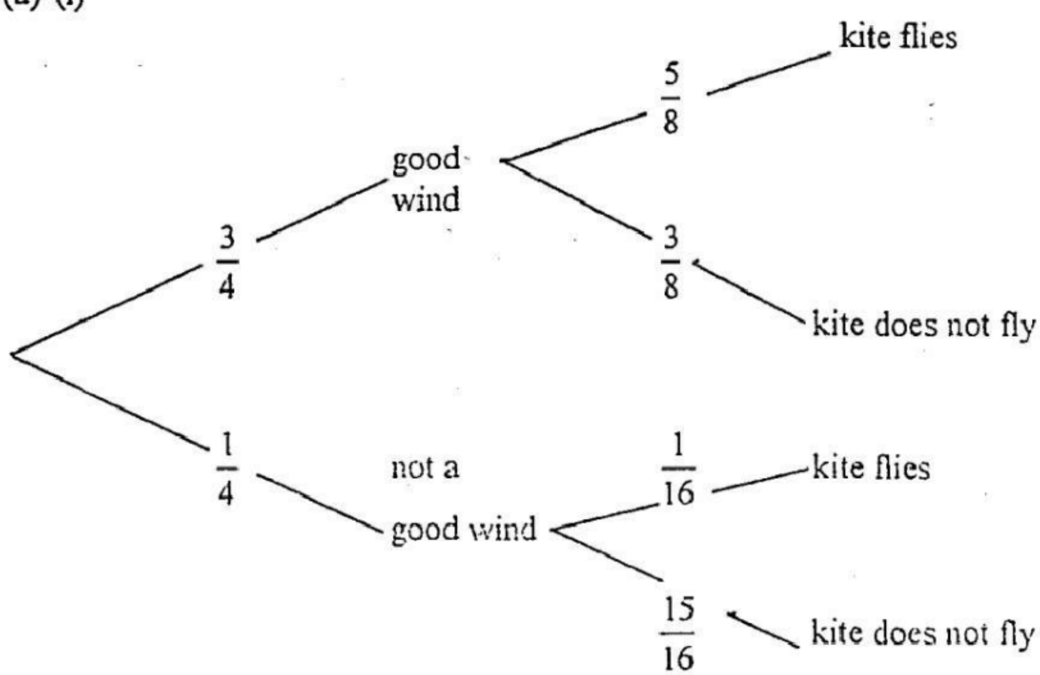
Amount divided $= 9 \times 420 = 3780$

Income $= \frac{3780 \times 100}{40} = \$\,9450$

(b) Interest $I = \frac{PRT}{100}$

$$I = \frac{16000 \times 12 \times \frac{6}{12}}{100} = \$\,960$$

2. (a) (i)



(ii) Prob. of a good wind and the kite flying

$$= \frac{3}{4} x \frac{5}{8} = \frac{15}{32}$$

(iii) Prob. that the kite does not fly

$$= \frac{3}{4} x \frac{3}{8} + \frac{1}{4} x \frac{15}{16} = \frac{9}{32} + \frac{15}{64} = \frac{33}{64}$$

(b) Prob. that the kite stick in a tree

$$= \frac{3}{4} x \frac{5}{8} x \frac{1}{2} + \frac{1}{4} x \frac{1}{16} x \frac{1}{2}$$

$$= \frac{15}{64} + \frac{1}{128} = \frac{31}{128}$$

(c) (i) mode is the most frequent wind strength, therefore the mode is 7. To find the median construct the following table :

wind strength	1	2	3	4	5	6	7	8	9
frequency	3	5	6	8	6	7	9	5	1
cummulative freq.	3	8	14	22	28	35	44	49	50

order of median $= \frac{50}{2} = 25$ (or $\frac{50+1}{2} = 25.5$)

from the above table this term number 25 (or 25.5) lies within the group of wind strength of 5.

Therefore the median is 5.

(ii) Mean = (1x3+2x5+3x6+4x8+5x6+6x7+7x9+8x5+9x1) ÷ 50

= 247 ÷ 50 = 4.94

(iii) Number of days for which the wind strength x given by

$3 \leq x \leq 7$ is equal to $6+8+6+7+9 = 36$

Prbability of a good wind $= \frac{36}{50} = \frac{18}{25}$

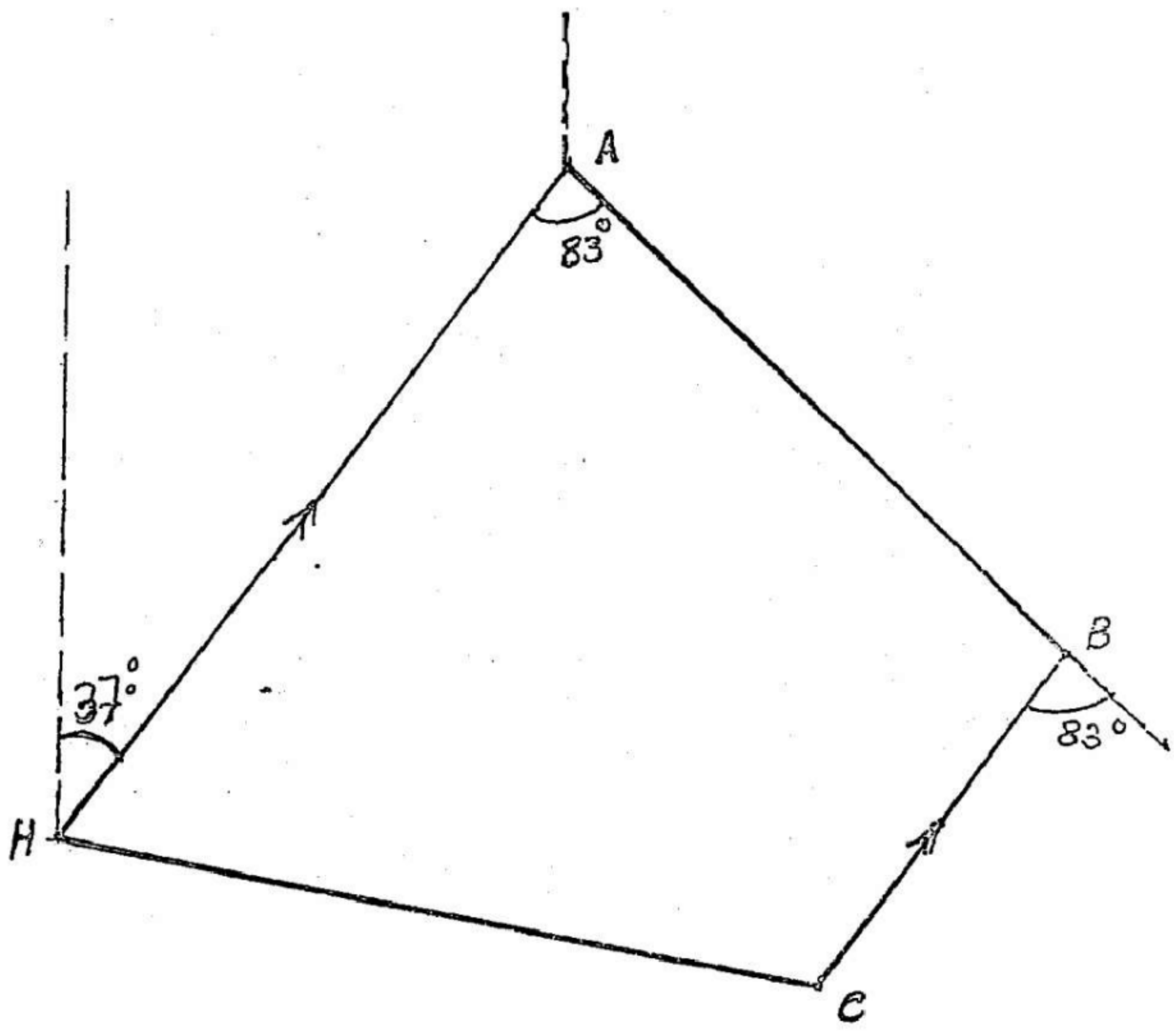
3. (a) (i) Bearing of B from A = 360 − (180 − 37) − 83 = 134°

(ii) Bearing of C from B = 134 + 83 = 217°

(b) Using a scale of 1 cm = 10 km

120 km ⇒ 12 cm.

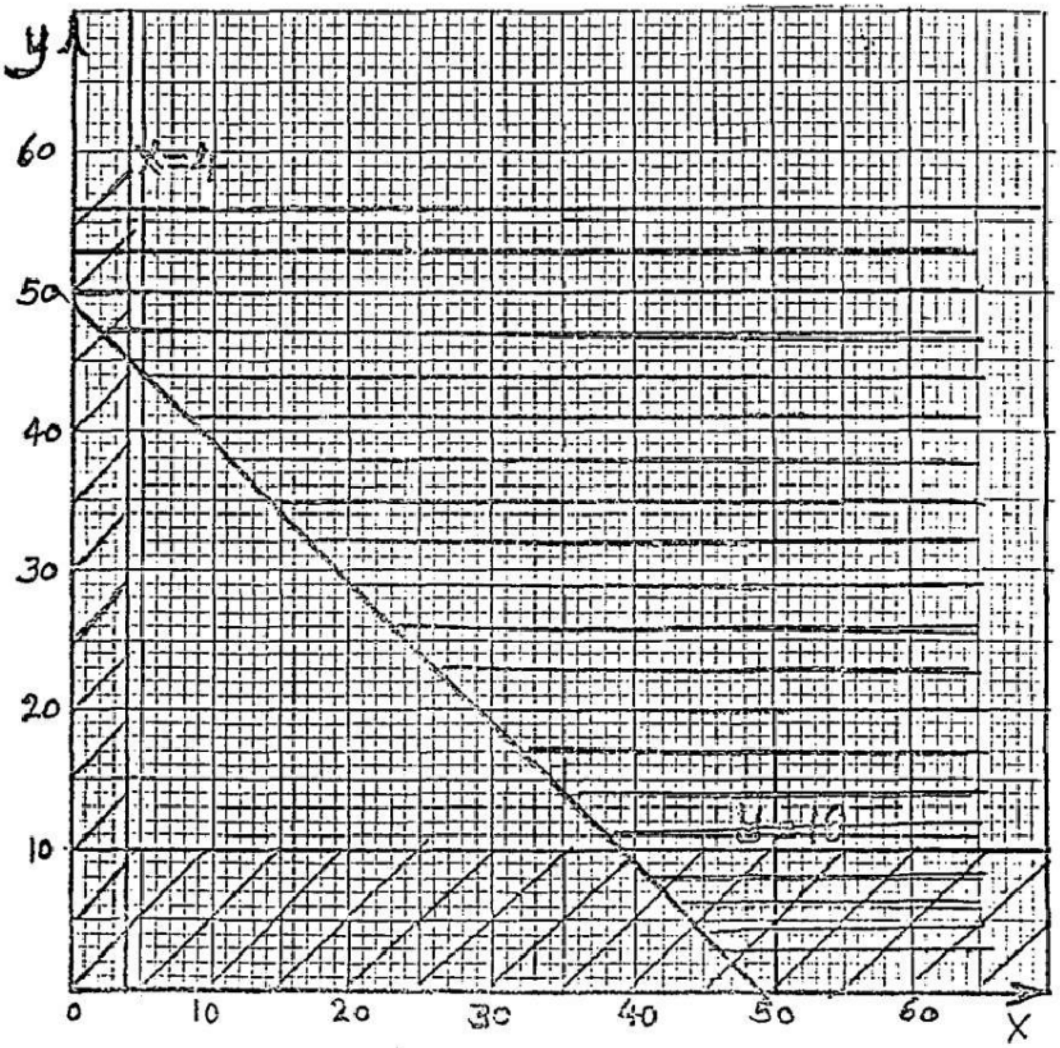
100 km ⇒ 10 cm.



Distance CH = 11 x 10 = 110 km

4. (b) $x + Y \leq 49$

$x = 0 \quad y = 49 \quad , \quad y = 0 \quad x = 49$



(d) Profit = 100 x + 50 y

for the corner points

(39 , 10) profit = 3900 + 500 = 4400

(10 , 39) profit = 1000 + 1950 = 2950

Maximum profit = $ 4400

5. (a) (i) $3xa + 6xb - 9xc = 3x(a + 2b - 3c)$

(ii) $x^2 - 10x - 24 = (x + 2)(x - 12)$

(iii) $10x^2 - 7x + 1 = (2x - 1)(5x - 1)$

(b) $y = \frac{a}{x} + bx$

(i) $x = 1, y = 2$ $\quad\therefore\quad 2 = a + b$

$x = 2, y = -5$ $\quad\therefore\quad -5 = \frac{a}{2} + 2b \quad x-2$

$$10 = -a - 4b$$
$$2 = a + b$$
$$12 = -3b \quad\Rightarrow\quad b = -4$$
$$\therefore\ a = 2 - b = 2 + 4 = 6$$
$$a = 6 \quad \text{and} \quad b = -4$$

(ii) $y = 16$ $\quad 16 = \frac{6}{x} - 4x$

$$16x = 6 - 4x^2$$
$$4x^2 + 16x - 6 = 0$$
$$2x^2 + 8x - 3 = 0$$
$$x = \frac{-8 \pm \sqrt{8^2 - 4x2x-3}}{2x2}$$
$$= \frac{-8 \pm \sqrt{88}}{4} = \frac{-8 \pm 9.3808}{4}$$
$$= 0.35 \quad \text{or} \quad -4.35$$

6. (a) (i) $BC^2 = BA^2 + AC^2 - 2\,BA \times AC\ \cos A$

$$= 7^2 + 9^2 - 2 \,.\, 7 \,.\, 9 \times \cos 120°$$
$$= 193$$
$$BC = 13.9 \text{ cm}$$

(ii) $\frac{BC}{\sin A} = \frac{CA}{\sin B}$ sine rule

$$\frac{13.9}{\sin 120} = \frac{9}{\sin B}$$
$$\sin B = \frac{9 \times \sin 120}{13.9}$$
$$B = 34.1°$$

(b) (i) angle OAS $= \frac{120}{2} = 60°$

from (a) above, angle B $= 34.1°$

angle OBS $= \frac{34.1}{2} = 17.1°$

(ii) $\tan 60° = \frac{r}{AS} \Rightarrow AS = \frac{r}{\tan 60°}$

(iii) $\tan 17.1° = \frac{r}{BS} \Rightarrow BS = \frac{r}{\tan 17.1°}$

(iv) $AS + BS = AB = 7$

$$\frac{r}{\tan 60°} + \frac{r}{\tan 17.1°} = 7$$

$$\frac{r}{1.732} + \frac{r}{0.307} = 7$$

$$r\left(\frac{1}{1.732} + \frac{1}{0.307}\right) = 7$$

$$r \times 3.835 = 7$$

$$r = 1.83 \text{ cm}$$

7. (a) (i) $13^2 - 5^2 = 169 - 25 = 144$

$\sqrt{144} = 12$ $\therefore$ $CD = 2 \times 12 = 24$ cm

(ii) $\text{Cos } x = \frac{5}{13}$ $x = 67.4$

$< COD = 2 \times 67.4 = 135°$

(iii) arc CBD $= \frac{135}{360} \times 2 \times 3.142 \times 13 = 30.6$ cm

(iv) distance CD round the semicircle

$= \pi r = \pi \times 12 = 37.7$ cm

(b) (i) Area above the water level $= 2\pi r(r - h)$

$= 2\pi \times 13(13 - 5) = 654$ cm

(ii) total surface area $= 4\pi r^2 = 4 \times 3.142 \times 13^2 = 2124$

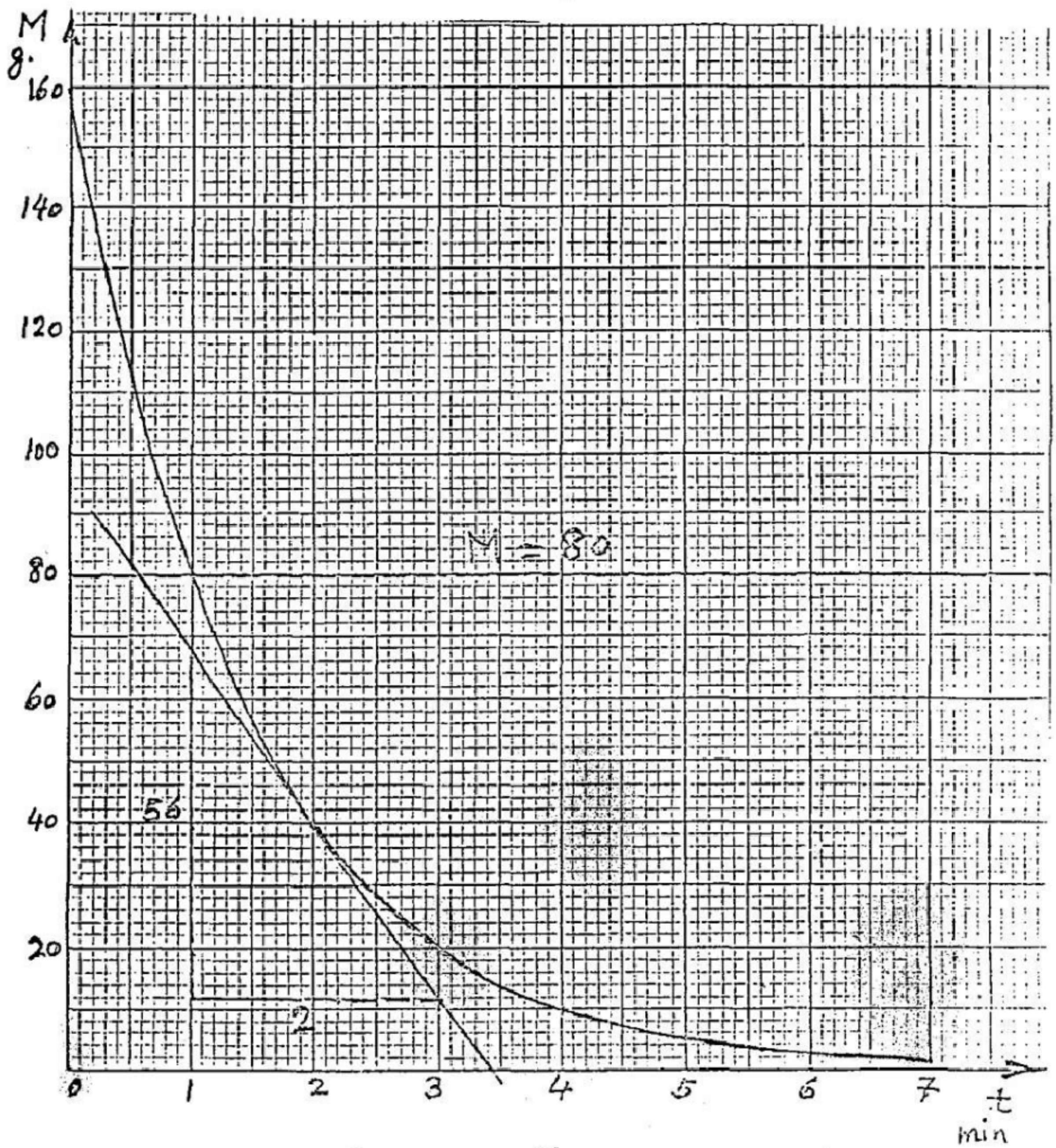
percentage $= \frac{654}{2124} \times 100 = 30.8$ %

8. (a) $M = 160 \times 2^{-t}$

$t = 0$ $M = 160 \times 2^0 = 160$, $p = 160$

$t = 4$ $M = 160 \times 2^{-4} = \frac{160}{16} = 10$, $q = 10$

$t = 6$ $M = 160 \times 2^{-6} = \frac{160}{64} = 2.5$, $r = 2.5$



rate of change = gradient = $\frac{-56}{2} = -28$ grams per min.

(b) (i) $m = 160 - M$

when $m = M$ $\therefore$ $2M = 160$

$M = 80$

from graph $t = 1$ min.

(ii) reflection on the line $M = 80$

9. (a) (i) Translation of $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

(ii) Enlargement by factor 3 Centre the origin.

(iii) Rotation by 90° anticlockwise centre the origin.

(iv) Stretch along the y-axis factor 4.

(v) Shear parallel to the x-axis.

(b) Shapes B , D and E

(c) matrix of stretch $= \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$

(d) matrix which trasnform F onto A is the inverse of

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \text{ i.e. } \frac{1}{1}\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$$

10. (a) $a = \dfrac{6.8+6.9+7+7.1+7.2}{5} = 7$

$b = 7 \times 1.8 = 12.6$

$c = \dfrac{4.7+4.9+5.1+5.1+5.2}{5} = 5$

$d = 5 \times 2.3 = 11.5$

(b) $\dfrac{7.3+7.6+7.7+8+x}{5} \times 2.2 = 16.5$

$30.6 + x = \dfrac{16.5 \times 5}{2.2} = 37.5$

$x = 37.5 - 30.6 = 6.9$

(c) Since the mean is 7.2 and all the known marks are less than 7.2, this means y and z an greater than 7.2.
Assuring that $y < z$, this means z is the largest and 7 is the least.
Deleting these two marks

$$\therefore \frac{7.1+7.1+7.1+7.1+y}{5} = 7.2$$

$$y = 5 \times 7.2 - 4 \times 7.1$$
$$= 36 - 28.4 = 7.6$$

and z is any value greater than 7.6
A possible pair of values for y and z is 7.6 , 7.7
(7.6 and any number greater than 7.6)