3 Indices and Standard Form

3.1 Index Notation

Here we revise the use of index notation. You will already be familiar with the notation for squares and cubes

$$\boxed{\begin{aligned} a^2 &= a \times a \\ a^3 &= a \times a \times a \end{aligned}}\ , \text{ and}$$

this is generalised by defining:

$$\boxed{a^n = \underbrace{a \times a \times \ldots \times a}_{n \text{ of these}}}$$

Example 1

Calculate the value of:

(a) 5^2 (b) 2^5 (c) 3^3 (d) 10^4

Solution

(a) $5^2 = 5 \times 5$
$= 25$

(b) $2^5 = 2 \times 2 \times 2 \times 2 \times 2$
$= 32$

(c) $3^3 = 3 \times 3 \times 3$
$= 27$

(d) $10^4 = 10 \times 10 \times 10 \times 10$
$= 10\,000$

Example 2

Copy each of the following statements and fill in the missing number or numbers:

(a) $2^{\square} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

(b) $9 = 3^{\square}$

(c) $1000 = 10^{\square}$

(d) $5^3 = \square \times \square \times \square$

Solution

(a) $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

(b) $9 = 3 \times 3 = 3^2$

(c) $1000 = 10 \times 10 \times 10 = 10^3$

(d) $5^3 = 5 \times 5 \times 5$

Example 3

(a) Determine 2^5.

(b) Determine 2^3.

(c) Determine $2^5 \div 2^3$.

(d) Express your answer to (c) in index notation.

Solution

(a) $2^5 = 32$

(b) $2^3 = 8$

(c) $2^5 \div 2^3 = 32 \div 8$
$= 4$

(d) $4 = 2^2$

Exercises

1. Calculate:

(a) 2^3 (b) 10^2 (c) 3^2

(d) 10^3 (e) 9^2 (f) 3^3

(g) 2^4 (h) 3^4 (i) 7^2

2. Copy each of the following statements and fill in the missing numbers:

(a) $10 \times 10 \times 10 \times 10 \times 10 = 10^{\square}$

(b) $3 \times 3 \times 3 \times 3 = 3^{\square}$

(c) $7 \times 7 \times 7 \times 7 \times 7 = 7^{\square}$

(d) $8 \times 8 \times 8 \times 8 \times 8 = 8^{\square}$

(e) $5 \times 5 = 5^{\square}$

(f) $19 \times 19 \times 19 \times 19 = 19^{\square}$

(g) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^{\square}$

(h) $11 \times 11 \times 11 \times 11 \times 11 \times 11 = 11^{\square}$

3. Copy each of the following statements and fill in the missing numbers:

(a) $8 = 2^{\square}$

(b) $81 = 3^{\square}$

(c) $100 = 10^{\square}$

(d) $81 = 9^{\square}$

(e) $125 = 5^{\square}$

(f) $1\,000\,000 = 10^{\square}$

(g) $216 = 6^{\square}$

(h) $625 = 5^{\square}$

4. Is 10^2 bigger than 2^{10} ?

5. Is 3^4 bigger than 4^3 ?

6. Is 5^2 bigger than 2^5 ?

7. Copy each of the following statements and fill in the missing numbers:

(a) $49 = \square^{2}$

(b) $64 = \square^{3}$

(c) $64 = \square^{6}$

(d) $64 = \square^{2}$

(e) $100\,000 = \square^{5}$

(f) $243 = \square^{5}$

8. Calculate:

(a) $2^2 + 2^3$

(b) $2^2 \times 2^3$

(c) $3^2 + 2^2$

(d) $3^2 \times 2^2$

(e) $2^3 \times 10^3$

(f) $10^3 + 2^5$

9. Calculate:

(a) $(3+2)^4$ (b) $(3-2)^4$

(c) $(7-4)^3$ (d) $(7+4)^3$

10. Writing your answers in index form, calculate:

(a) $10^2 \times 10^3$ (b) $2^3 \times 2^7$

(c) $3^4 \div 3^2$ (d) $2^5 \div 2^2$

(e) $10^6 \div 10^2$ (f) $5^4 \div 5^2$

11. (a) Without using a calculator, write down the values of k and m.

$$64 = 8^2 = 4^k = 2^m$$

(b) Complete the following:

$$2^{15} = 32\ 768$$

$$2^{14} = \boxed{}$$

(KS3/99Ma/Tier 5-7/P1)

3.2 Laws of Indices

There are three rules that should be used when working with indices:

When m and n are positive integers,

1. $a^m \times a^n = a^{m+n}$

2. $a^m \div a^n = a^{m-n}$ or $\dfrac{a^m}{a^n} = a^{m-n}$ $(m \geq n)$

3. $\left(a^m\right)^n = a^{m \times n}$

These three results are logical consequences of the definition of a^n, but really need a formal proof. You can 'verify' them with particular examples as below, but this is not a proof:

$$\begin{aligned} 2^7 \times 2^3 &= (2\times2\times2\times2\times2\times2\times2)\times(2\times2\times2) \\ &= 2\times2\times2\times2\times2\times2\times2\times2\times2\times2 \\ &= 2^{10} \qquad \text{(here } m=7,\ n=3 \text{ and } m+n=10\text{)} \end{aligned}$$

or,

$$2^7 \div 2^3 = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$$

$$= 2 \times 2 \times 2 \times 2$$

$$= 2^4 \qquad \text{(again } m = 7,\ n = 3 \text{ and } m - n = 4\text{)}$$

Also, $\left(2^7\right)^3 = 2^7 \times 2^7 \times 2^7$

$$= 2^{21} \quad \text{(using rule 1)} \qquad \text{(again } m = 7,\ n = 3 \text{ and } m \times n = 21\text{)}$$

The proof of the first rule is given below:

Proof

$$a^m \times a^n = \underbrace{a \times a \times \ldots \times a}_{m \text{ of these}} \times \underbrace{a \times a \times \ldots \times a}_{n \text{ of these}}$$

$$= \underbrace{a \times a \times \ldots \times a \times a \times a \ldots \times a}_{(m + n) \text{ of these}}$$

$$= a^{m+n}$$

The second and third rules can be shown to be true for all positive integers m and n in a similar way.

We can see an important result using rule 2:

$$\frac{x^n}{x^n} = x^{n-n} = x^0$$

but $\frac{x^n}{x^n} = 1$, so

$$\boxed{x^0 = 1}$$

This is true for any non-zero value of x, so, for example, $3^0 = 1$, $27^0 = 1$ and $1001^0 = 1$.

Example 1

Fill in the missing numbers in each of the following expressions:

(a) $2^4 \times 2^6 = 2^{\square}$ (b) $3^7 \times 3^9 = 3^{\square}$

(c) $3^6 \div 3^2 = 3^{\square}$ (d) $\left(10^4\right)^3 = 10^{\square}$

Solution

(a) $$2^4 \times 2^6 = 2^{4+6} = 2^{10}$$

(b) $$3^7 \times 3^9 = 3^{7+9} = 3^{16}$$

(c) $$3^6 \div 3^2 = 3^{6-2} = 3^4$$

(d) $$\left(10^4\right)^3 = 10^{4 \times 3} = 10^{12}$$

Example 2

Simplify each of the following expressions so that it is in the form a^n, where n is a number:

(a) $a^6 \times a^7$ (b) $\dfrac{a^4 \times a^2}{a^3}$ (c) $\left(a^4\right)^3$

Solution

(a) $$a^6 \times a^7 = a^{6+7} = a^{13}$$

(b) $$\frac{a^4 \times a^2}{a^3} = \frac{a^{4+2}}{a^3} = \frac{a^6}{a^3} = a^{6-3} = a^3$$

(c) $$\left(a^4\right)^3 = a^{4 \times 3} = a^{12}$$

Exercises

1. Copy each of the following statements and fill in the missing numbers:

(a) $2^3 \times 2^7 = 2^{\square}$ (b) $3^6 \times 3^5 = 3^{\square}$

(c) $3^7 \div 3^4 = 3^{\square}$ (d) $8^3 \times 8^4 = 8^{\square}$

(e) $\left(3^2\right)^5 = 3^{\square}$ (f) $\left(2^3\right)^6 = 2^{\square}$

(g) $\dfrac{3^6}{3^2} = 3^{\square}$ (h) $\dfrac{4^7}{4^2} = 4^{\square}$

2. Copy each of the following statements and fill in the missing numbers:

(a) $a^3 \times a^2 = a^{\square}$ (b) $b^7 \div b^2 = b^{\square}$

(c) $\left(b^2\right)^5 = b^{\square}$ (d) $b^6 \times b^4 = b^{\square}$

(e) $\left(z^3\right)^9 = z^{\square}$ (f) $\dfrac{q^{16}}{q^7} = q^{\square}$

3. Explain why $9^4 = 3^8$.

4. Calculate:

(a) $3^0 + 4^0$ (b) $6^0 \times 7^0$

(c) $8^0 - 3^0$ (d) $6^0 + 2^0 - 4^0$

5. Copy each of the following statements and fill in the missing numbers:

(a) $3^6 \times 3^{\square} = 3^{17}$ (b) $4^6 \times 4^{\square} = 4^{11}$

(c) $\dfrac{a^6}{a^{\square}} = a^4$ (d) $\left(z^{\square}\right)^6 = z^{18}$

(e) $\left(a^{19}\right)^{\square} = a^{95}$ (f) $p^{16} \div p^{\square} = p^7$

(g) $\left(p^{\square}\right)^8 = p^{40}$ (h) $q^{13} \div q^{\square} = q$

6. Calculate:

(a) $\dfrac{2^3}{2^2} + 3^0$ (b) $\dfrac{3^4}{3^3} - 3^0$

(c) $\dfrac{5^4}{5^2} + \dfrac{6^2}{6}$ (d) $\dfrac{7^7}{7^5} - \dfrac{5^9}{5^7}$

(e) $\dfrac{10^8}{10^5} - \dfrac{5^6}{5^3}$ (f) $\dfrac{4^{17}}{4^{14}} - \dfrac{4^{13}}{4^{11}}$

7. Fill in the missing numbers in each of the following expressions:

(a) $8^2 = 2^{\square}$ (b) $81^3 = 9^{\square} = 3^{\square}$

(c) $25^6 = 5^{\square}$ (d) $4^7 = 2^{\square}$

(e) $125^4 = 5^{\square}$ (f) $1000^6 = 10^{\square}$

(g) $81 = \square^4$ (h) $256 = \square^4 = \square^8$

8. Fill in the missing numbers in each of the following expressions:

(a) $8 \times 4 = 2^{\square} \times 2^{\square}$
$= 2^{\square}$

(b) $25 \times 625 = 5^{\square} \times 5^{\square}$
$= 5^{\square}$

(c) $\dfrac{243}{9} = \dfrac{3^{\square}}{3^{\square}}$
$= 3^{\square}$

(d) $\dfrac{128}{16} = \dfrac{2^{\square}}{2^{\square}}$
$= 2^{\square}$

9. Is each of the following statements true or false?

(a) $3^2 \times 2^2 = 6^4$ (b) $5^4 \times 2^3 = 10^7$

(c) $\dfrac{6^8}{2^8} = 3^8$ (d) $\dfrac{10^8}{5^6} = 2^2$

10. Copy and complete each expression:

(a) $\left(2^6 \times 2^3\right)^4 = \left(2^{\square}\right)^4 = 2^{\square}$

(b) $\left(\dfrac{3^6}{3^2}\right)^5 = \left(3^{\square}\right)^5 = 3^{\square}$

(c) $\left(\dfrac{2^3 \times 2^4}{2^7}\right)^4 = \left(2^{\square}\right)^4 = 2^{\square}$

(d) $\left(\dfrac{3^2 \times 9}{3^3}\right)^4 = \left(3^{\square}\right)^4 = 3^{\square}$

(e) $\left(\dfrac{6^2 \times 6^8}{6^3}\right)^4 = \left(6^{\square}\right)^4 = 6^{\square}$

(f) $\left(\dfrac{7^8}{7^2 \times 7^3}\right)^5 = \left(7^{\square}\right)^5 = 7^{\square}$

3.3 Negative Indices

Using negative indices produces fractions. In this section we practice working with negative indices. From our work in the last section, we see that

$$a^2 \div a^3 = a^{2-3} = a^{-1}$$

but we know that

$$a^2 \div a^3 = \frac{a \times a}{a \times a \times a} = \frac{1}{a}, \text{ a fraction.}$$

So clearly,

$$\boxed{a^{-1} = \frac{1}{a}}$$

In same way,

$$\boxed{\begin{aligned} a^{-2} &= \frac{1}{a^2} \\ &= \frac{1}{a \times a} \\ a^{-3} &= \frac{1}{a^3} \\ &= \frac{1}{a \times a \times a} \end{aligned}}$$

and, in general,

$$a^{-n} = \frac{1}{a^n}$$

for positive integer values of n. The three rules at the start of section 3.2 can now be used for any integers m and n, not just for positive values.

Example 1

Calculate, leaving your answers as fractions:

(a) 3^{-2} (b) $2^{-1} - 4^{-1}$ (c) 5^{-3}

Solution

(a) $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

(b) $2^{-1} - 4^{-1} = \frac{1}{2} - \frac{1}{4}$

$= \frac{1}{4}$

(c) $5^{-3} = \frac{1}{5^3}$

$= \frac{1}{125}$

Example 2

Simplify:

(a) $\frac{6^7}{6^9}$ (b) $6^4 \times 6^{-3}$ (c) $\left(10^2\right)^{-3}$

Solution

(a) $\frac{6^7}{6^9} = 6^{7-9}$

$= 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

(b) $6^4 \times 6^{-3} = 6^{4+(-3)}$

$= 6^{4-3} = 6^1 = 6$

(c) $\left(10^{2}\right)^{-3} = 10^{-6}$

$= \dfrac{1}{10^{6}}$

$= \dfrac{1}{1000\,000}$

Exercises

1. Write the following numbers as fractions *without using any indices*:

(a) 4^{-1} (b) 2^{-3} (c) 10^{-3}

(d) 7^{-2} (e) 4^{-3} (f) 6^{-2}

2. Copy the following expressions and fill in the missing numbers:

(a) $\dfrac{1}{49} = \dfrac{1}{7^{\square}} = 7^{\square}$ (b) $\dfrac{1}{100} = \dfrac{1}{10^{\square}} = 10^{\square}$

(c) $\dfrac{1}{81} = \dfrac{1}{9^{\square}} = 9^{\square}$ (d) $\dfrac{1}{16} = \dfrac{1}{2^{\square}} = 2^{\square}$

(e) $\dfrac{1}{10\,000\,000} = \dfrac{1}{10^{\square}} = 10^{\square}$ (f) $\dfrac{1}{1024} = \dfrac{1}{2^{\square}} = 2^{\square}$

3. Calculate:

(a) $4^{-1} + 3^{-1}$ (b) $6^{-1} + 2^{-1}$

(c) $5^{-1} - 10^{-1}$ (d) $10^{-2} - 10^{-3}$

(e) $4^{-1} - 10^{-1}$ (f) $6^{-1} + 7^{-1}$

4. Simplify the following expressions giving your answers in the form of a number to a power:

(a) $4^{7} \times 4^{-6}$ (b) $5^{7} \times 5^{-3}$

(c) $\dfrac{7^{4}}{7^{-6}}$ (d) $\left(3^{2}\right)^{-4}$

(e) $\left(6^{-2}\right)^{-3}$ (f) $8^{4} \times 8^{-9}$

(g) $\dfrac{7^{2}}{7^{-2}}$ (h) $\dfrac{8^{9}}{8^{-9}}$

5. Copy each of the following expressions and fill in the missing numbers;

(a) $\frac{1}{9} = 3^{\square}$ (b) $\frac{1}{100} = 10^{\square}$

(c) $\frac{1}{125} = 5^{\square}$ (d) $\frac{5}{5^4} = 5^{\square}$

(e) $\frac{6^2}{6^3} = 6^{\square}$ (f) $\frac{2^2}{2^{10}} = 2^{\square}$

6. Simplify the following expressions:

(a) $\frac{x^8}{x^3}$ (b) $\frac{x^7}{x^9}$

(c) $\frac{x^4}{x^8}$ (d) $\left(x^6\right)^{-4}$

(e) $\left(\frac{1}{x^2}\right)^4$ (f) $\left(x^{-8}\right)^3$

7. Copy and complete the following statements:

(a) $0.1 = 10^{\square}$ (b) $0.25 = 2^{\square}$

(c) $0.0001 = 10^{\square}$ (d) $0.2 = 5^{\square}$

(e) $0.001 = 10^{\square}$ (f) $0.02 = 50^{\square}$

8. Copy the following expressions and fill in the missing numbers:

(a) $\frac{x^4}{x^{\square}} = x^2$ (b) $x^6 \times x^{\square} = x^2$

(c) $x^9 \times x^{\square} = x^2$ (d) $\frac{x^7}{x^{\square}} = x^{-2}$

(e) $\frac{x^3}{x^{\square}} = x^4$ (f) $\left(x^3\right)^{\square} = x^{-6}$

9. Copy the following expressions and fill in the missing numbers:

(a) $\frac{1}{8} = 2^{\square}$ (b) $\frac{1}{25} = 5^{\square}$

(c) $\frac{1}{81} = 9^{\square}$ (d) $\frac{1}{10\,000} = 10^{\square}$

10. If $a = b^3$ and $b = \frac{1}{c^2}$, express a as a power of c, without having any fractions in your final answer.

3.4 Standard Form

Standard form is a convenient way of writing very large or very small numbers. It is used on a scientific calculator when a number is too large or too small to be displayed on the screen.

Before using standard form, we revise multiplying and dividing by powers of 10.

Example 1

Calculate:

(a) 3×10^4 (b) 3.27×10^3

(c) $3 \div 10^2$ (d) $4.32 \div 10^4$

Solution

(a) $3 \times 10^4 = 3 \times 10\,000$
$= 30\,000$

(b) $3.27 \times 10^3 = 3.27 \times 1000$
$= 3270$

(c) $3 \div 10^2 = \frac{3}{100}$
$= 0.03$

(d) $4.32 \div 10^4 = \dfrac{4.32}{10\,000}$

$= \dfrac{432}{1\,000\,000}$

$= 0.000432$

These examples lead to the approach used for standard form, which is a reversal of the approach used in Example 1.

> In *standard form*, numbers are written as
>
> $$a \times 10^n$$
>
> where $1 \le a < 10$ and n is an integer.

Example 2

Write the following numbers in standard form:

(a) 5720

(b) 7.4

(c) 473 000

(d) 6 000 000

(e) 0.09

(f) 0.000621

Solution

(a) $5720 = 5.72 \times 1000$

$= 5.72 \times 10^3$

(b) $7.4 = 7.4 \times 1$

$= 7.4 \times 10^0$

(c) $473\,000 = 4.73 \times 100\,000$

$= 4.73 \times 10^5$

(d) $6\,000\,000 = 6 \times 1\,000\,000$

$= 6 \times 10^6$

(e) $0.09 = \dfrac{9}{100}$

$= 9 \div 10^2$

$= 9 \times 10^{-2}$

(f) $0.000621 = \dfrac{6.21}{10\,000}$

$= \dfrac{6.21}{10^{4}}$

$= 6.21 \times 10^{-4}$

Example 3

Calculate:

(a) $\left(3 \times 10^{6}\right) \times \left(4 \times 10^{3}\right)$

(b) $\left(6 \times 10^{7}\right) \div \left(5 \times 10^{-2}\right)$

(c) $\left(3 \times 10^{4}\right) + \left(2 \times 10^{5}\right)$

Solution

(a) $\left(3 \times 10^{6}\right) \times \left(4 \times 10^{3}\right) = (3 \times 4) \times \left(10^{6} \times 10^{3}\right)$

$= 12 \times 10^{9}$

$= 1.2 \times 10^{1} \times 10^{9}$

$= 1.2 \times 10^{10}$

(b) $\left(6 \times 10^{7}\right) \div \left(5 \times 10^{-2}\right) = (6 \div 5) \times \left(10^{7} \div 10^{-2}\right)$

$= 1.2 \times 10^{9}$

(c) $\left(3 \times 10^{4}\right) + \left(2 \times 10^{5}\right) = 30\,000 + 200\,000$

$= 230\,000$

$= 2.3 \times 10^{5}$

Note on Using Calculators

Your calculator will have a key EE or EXP for entering numbers in standard form.

For example, for 3.2×10^{7}, press

3 · 2 EXP 7

which will appear on your display like this:

3.2 07

Some calculators also display the ' $\times 10$ ' part of the number, but not all do. You need to find out what your calculator displays. Remember, you must always write the ' $\times 10$ ' part when you are asked to give an answer in standard form.

Exercises

1. Calculate:

   (a) 6.21×1000 (b) 8×10^{3} (c) 4.2×10^{2}

   (d) $3 \div 1000$ (e) $6 \div 10^{2}$ (f) $3.2 \div 10^{3}$

   (g) 6×10^{-3} (h) 9.2×10^{-1} (i) 3.6×10^{-2}

2. Write each of the following numbers in standard form:

   (a) 200 (b) 8000

   (c) 9 000 000 (d) 62 000

   (e) 840 000 (f) 12 000 000 000

   (g) 61 800 000 000 (h) 3 240 000

3. Convert each of the following numbers from standard form to the normal decimal notation:

   (a) 3×10^{4} (b) 3.6×10^{4} (c) 8.2×10^{3}

   (d) 3.1×10^{2} (e) 1.6×10^{4} (f) 1.72×10^{5}

   (g) 6.83×10^{4} (h) 1.25×10^{6} (i) 9.17×10^{3}

4. Write each of the following numbers in standard form:

   (a) 0.0004 (b) 0.008

   (c) 0.142 (d) 0.0032

   (e) 0.00199 (f) 0.000000062

   (g) 0.0000097 (h) 0.0000000000021

5. Convert the following numbers from standard form to the normal decimal format:

(a) 6×10^{-2} (b) 7×10^{-1} (c) 1.8×10^{-3}

(d) 4×10^{-3} (e) 6.2×10^{-3} (f) 9.81×10^{-4}

(g) 6.67×10^{-1} (h) 3.86×10^{-5} (i) 9.27×10^{-7}

6. Without using a calculator, determine:

(a) $\left(4 \times 10^{4}\right) \times \left(2 \times 10^{5}\right)$ (b) $\left(2 \times 10^{6}\right) \times \left(3 \times 10^{5}\right)$

(c) $\left(6 \times 10^{4}\right) \times \left(8 \times 10^{-9}\right)$ (d) $\left(3 \times 10^{-8}\right) \times \left(7 \times 10^{-4}\right)$

(e) $\left(6.1 \times 10^{6}\right) \times \left(2 \times 10^{-5}\right)$ (f) $\left(3.2 \times 10^{-5}\right) \times \left(4 \times 10^{-9}\right)$

7. Without using a calculator, determine:

(a) $\left(9 \times 10^{7}\right) \div \left(3 \times 10^{4}\right)$ (b) $\left(8 \times 10^{5}\right) \div \left(2 \times 10^{-2}\right)$

(c) $\left(6 \times 10^{-2}\right) \div \left(2 \times 10^{-3}\right)$ (d) $\left(6 \times 10^{4}\right) \div \left(3 \times 10^{-6}\right)$

(e) $\left(4.8 \times 10^{12}\right) \div \left(1.2 \times 10^{3}\right)$ (f) $\left(3.6 \times 10^{8}\right) \div \left(9 \times 10^{3}\right)$

8. Without a calculator, determine the following, giving your answers in both normal and standard form::

(a) $\left(6 \times 10^{5}\right) + \left(3 \times 10^{6}\right)$ (b) $\left(6 \times 10^{2}\right) + \left(9 \times 10^{3}\right)$

(c) $6 \times 10^{5} - 1 \times 10^{4}$ (d) $8 \times 10^{-2} + 9 \times 10^{-3}$

(e) $6 \times 10^{-4} + 8 \times 10^{-3}$ (f) $6 \times 10^{-4} - 3 \times 10^{-5}$

9. Use a calculator to determine:

(a) $\left(3.4 \times 10^{6}\right) \times \left(2.1 \times 10^{4}\right)$ (b) $\left(6 \times 10^{21}\right) \times \left(8.2 \times 10^{-11}\right)$

(c) $\left(3.6 \times 10^{5}\right) \times \left(4.5 \times 10^{7}\right)$ (d) $\left(8.2 \times 10^{11}\right) \div \left(4 \times 10^{-8}\right)$

(e) $\left(1.92 \times 10^{6}\right) \times \left(3.2 \times 10^{-11}\right)$ (f) $\left(6.2 \times 10^{14}\right)^{3}$

10. The radius of the earth is 6.4×10^{6} m. Giving your answers in standard form, correct to 3 significant figures, calculate the circumference of the earth in:

(a) m (b) cm (c) mm (d) km

11. Sir Isaac Newton (1642-1727) was a mathematician, physicist and astronomer.

In his work on the gravitational force between two bodies he found that he needed to multiply their masses together.

(a) Work out the value of the mass of the Earth multiplied by the mass of the Moon. Give your answer in standard form.

Mass of Earth	$= 5.98 \times 10^{24}$ kg
Mass of Moon	$= 7.35 \times 10^{22}$ kg

Newton also found that he needed to work out the square of the distance between the two bodies.

(b) Work out the square of the distance between the Earth and the Moon. Give your answer in standard form.

Distance between Earth and Moon	$= 3.89 \times 10^{5}$ km

Newton's formula to calculate the gravitational force (F) between two bodies is $F = \dfrac{G m_1 m_2}{R^2}$ where

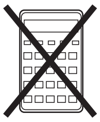
G is the gravitational constant, m_1 and m_2 are the masses of the two bodies, and R is the distance between them.

(c) Work out the gravitational force (F) between the Sun and the Earth using the formula $F = \dfrac{G m_1 m_2}{R^2}$ with information in the box below.

Give your answer in standard form.

$m_1 m_2$	$= 1.19 \times 10^{55}$ kg^2
R^2	$= 2.25 \times 10^{16}$ km^2
G	$= 6.67 \times 10^{-20}$

(KS3/95/Ma/Levels 6-8/P1)



12. (a) Which of these statements is true?

(i) 4×10^3 is a larger number than 4^3.

(ii) 4×10^3 is the same size as 4^3.

(iii) 4×10^3 is a smaller number than 4^3.

Explain your answer.

(b) One of the numbers below has the same value as 3.6×10^4. Write down the number.

36^3 36^4 $(3.6 \times 10)^4$ 0.36×10^3 0.36×10^5

(c) One of the numbers below has the same value as 2.5×10^{-3}. Write down the number.

25×10^{-4} 2.5×10^3 -2.5×10^3 0.00025 2500

(d) $\left(2 \times 10^2\right) \times \left(2 \times 10^2\right)$ can be written more simply as 4×10^4.

Write the following values as simply as possible:

(i) $\left(3 \times 10^2\right) \times \left(2 \times 10^{-2}\right)$

(ii) $\dfrac{6 \times 10^8}{2 \times 10^4}$

(KS3/98/Ma/Tier 6-8/P1)

3.5 Fractional Indices

Indices that are fractions are used to represent square roots, cube roots and other roots of numbers.

$a^{\frac{1}{2}}$	$=$	$\sqrt{a}$	for example,	$9^{\frac{1}{2}}$	$=$	3
$a^{\frac{1}{3}}$	$=$	$\sqrt[3]{a}$	for example,	$8^{\frac{1}{3}}$	$=$	2
$a^{\frac{1}{4}}$	$=$	$\sqrt[4]{a}$	for example,	$625^{\frac{1}{4}}$	$=$	5
$a^{\frac{1}{n}}$	$=$	$\sqrt[n]{a}$				

Example 1

Calculate:

(a) $81^{\frac{1}{2}}$ (b) $1000^{\frac{1}{3}}$ (c) $4^{-\frac{1}{2}}$

Solution

(a) $81^{\frac{1}{2}} = \sqrt{81}$

$= 9$

(b) $1000^{\frac{1}{3}} = \sqrt[3]{1000}$

$= 10$

(c) $4^{-\frac{1}{2}} = \dfrac{1}{4^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{4}}$

$= \dfrac{1}{2}$

Exercises

1. Calculate:

(a) $49^{\frac{1}{2}}$ (b) $64^{\frac{1}{2}}$ (c) $16^{\frac{1}{2}}$

(d) $81^{-\frac{1}{2}}$ (e) $100^{-\frac{1}{2}}$ (f) $25^{-\frac{1}{2}}$

(g) $9^{\frac{1}{2}}$ (h) $36^{-\frac{1}{2}}$ (i) $144^{\frac{1}{2}}$

2. Calculate:

(a) $8^{\frac{1}{3}}$ (b) $8^{-\frac{1}{3}}$ (c) $125^{\frac{1}{3}}$

(d) $64^{-\frac{1}{3}}$ (e) $216^{\frac{1}{3}}$ (f) $1000\,000^{-\frac{1}{3}}$

3. Calculate:

(a) $32^{\frac{1}{5}}$ (b) $64^{-\frac{1}{2}}$ (c) $10\,000^{\frac{1}{4}}$

(d) $81^{-\frac{1}{4}}$ (e) $625^{\frac{1}{4}}$ (f) $100\,000^{-\frac{1}{5}}$

4. Calculate:

(a) $\left(\dfrac{4 \times 8}{2}\right)^{\frac{1}{2}}$ (b) $\left(\dfrac{9 \times 27}{3}\right)^{\frac{1}{4}}$

(c) $\left(\dfrac{125 \times 5}{25}\right)^{\frac{1}{2}}$ (d) $\left(\dfrac{625}{5}\right)^{-\frac{1}{3}}$

5. Is each of the following statements *true* or *false*:

(a) $16^{\frac{1}{2}} = 8$ (b) $16^{\frac{1}{4}} = 2$

(c) $81^{\frac{1}{3}} = 9$ (d) $\left(\dfrac{1}{100}\right)^{-\frac{1}{2}} = 10$

6. Simplify:

(a) $\left(x^9\right)^{\frac{1}{3}}$ (b) $\left(a^{10}\right)^{-\frac{1}{2}}$

(c) $\dfrac{a}{a^{\frac{1}{2}}}$ (d) $\dfrac{a^{\frac{1}{2}}}{a}$

7. Simplify:

(a) $\dfrac{x^{\frac{3}{2}}}{x}$ (b) $\dfrac{x}{x^{\frac{3}{2}}}$

(c) $\dfrac{a^{\frac{1}{3}}}{a}$ (d) $\dfrac{a^{\frac{1}{3}}}{a^{\frac{1}{2}}}$

8. Calculate:

(a) $4^{-\frac{1}{2}} + 4^{\frac{1}{2}}$ (b) $\left(9^0 + 9^{\frac{1}{2}}\right)^{\frac{1}{2}}$

(c) $\left(256^{\frac{1}{2}}\right)^{\frac{1}{2}}$ (d) $\left(9 - 9^0\right)^{\frac{1}{3}}$