I.G.C.S.E. Solving Quadratic Equations

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Question 1

Show the following quadratic equations by factorisation showing clearly all working out.

a.	$x^2+7x+12=0$	**b.**	$x^2-8x+12=0$	**c.**	$x^2+3x-54=0$
d.	$3x^2+5x+2=0$	**e.**	$14x^2-29x+12=0$	**f.**	$15x^2-17x-18=0$
g.	$4x^2-20x+25=0$	**h.**	$5x^2-2x=0$	**i.**	$x^2-81=0$
j.	$25x^2-4=0$	**k.**	$49x^2=\frac{1}{4}$		

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Solution to question 1

a. $x^2+7x+12=0$

product = 12
sum = 7
factors = 3, 4

$\Rightarrow x^2+3x+4x+12=0$

$\Rightarrow x(x+3)+4(x+3)=0$

$\Rightarrow (x+3)(x+4)=0$

either $x+3=0 \Rightarrow x=-3$

or $x+4=0 \Rightarrow x=-4$

$x=-4$ or $x=-3$

b. $x^2-8x+12=0$

product = 12
sum = –8
factors = –2, – 6

$\Rightarrow x^2-2x-6x+12=0$

$\Rightarrow x(x-2)-6(x-2)=0$

$\Rightarrow (x-2)(x-6)=0$

either $x-2=0 \Rightarrow x=2$

or $x-6=0 \Rightarrow x=6$

$x=2$ or $x=6$

c. $x^2+3x-54=0$

product = –54
sum = 3
factors = –6, 9

$\Rightarrow x^2-6x+9x+54=0$

$\Rightarrow x(x-6)+9(x-6)=0$

$\Rightarrow (x-6)(x+9)=0$

either $x-6=0 \Rightarrow x=6$

or $x+9=0 \Rightarrow x=-9$

$x=-9$ or $x=6$

d. $3x^2+5x+2=0$

product = 6
sum = 5
factors = 2, 3

$\Rightarrow 3x^2+2x+3x+2=0$

$\Rightarrow x(3x+2)+(3x+2)=0$

$\Rightarrow (3x+2)(x+1)=0$

either $3x+2=0 \Rightarrow 3x=-2 \Rightarrow x=-\frac{2}{3}$

or $x+1=0 \Rightarrow x=-1$

$x=-1$ or $x=-\frac{2}{3}$

e. $14x^2-29x+12=0$

product = 168
sum = –29
factors = –21, –8

$\Rightarrow 14x^2-21x-8x+12=0$

$\Rightarrow 7x(2x-3)-4(2x-3)=0$

$\Rightarrow (2x-3)(7x-4)=0$

either $2x-3=0 \Rightarrow 2x=3 \Rightarrow x=\frac{3}{2}$

or $7x-4=0 \Rightarrow 7x=4 \Rightarrow x=\frac{4}{7}$

$x=\frac{4}{7}$ or $x=\frac{3}{2}$

f. $15x^2-17x-18=0$

product = –270
sum = –17
factors = –27,10

$\Rightarrow 15x^2-27x+10x-18=0$

$\Rightarrow 3x(5x-9)+2(5x-9)=0$

$\Rightarrow (5x-9)(3x+2)=0$

either $5x-9=0 \Rightarrow 5x=9 \Rightarrow x=\frac{9}{5}$

or $3x+2=0 \Rightarrow 3x=-2 \Rightarrow x=-\frac{2}{3}$

$x=-\frac{2}{3}$ or $x=\frac{9}{5}$

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g. $4x^2 - 20x + 25 = 0$

product = 100

sum = –20

factors = –10, –10

$\Rightarrow 4x^2 - 10x - 10x + 25 = 0$

$\Rightarrow 2x(2x-5) - 5(2x-5) = 0$

$\Rightarrow (2x-5)(2x-5) = 0$

$\Rightarrow (2x-5)^2 = 0$

$\Rightarrow \ 2x - 5 = 0 \Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2}$

h. $5x^2 - 2x = 0$

$x(5x-2) = 0$

either $x = 0$

or $\quad 5x - 2 = 0 \Rightarrow 5x = 2 \Rightarrow x = \frac{2}{5}$

$x = 0$ or $x = \frac{2}{5}$

i. $x^2 - 81 = 0$

Using $a^2 - b^2 = (a+b)(a-b)$

We have

$(x+9)(x-9) = 0$

either $x + 9 = 0 \Rightarrow x = -9$

or $\quad x - 9 = 0 \Rightarrow x = 9$

$x = -9$ or $x = 9$

j. $25x^2 - 4 = 0$

Using $a^2 - b^2 = (a+b)(a-b)$

We have

$(5x+2)(5x-2) = 0$

either $5x + 2 = 0 \Rightarrow x = -\frac{2}{5}$

or $\quad 5x - 2 = 0 \Rightarrow x = \frac{2}{5}$

$x = -\frac{2}{5}$ or $x = \frac{2}{5}$

k. $49x^2 = \frac{1}{4}$

$49x^2 - \frac{1}{4} = 0$

Using $a^2 - b^2 = (a+b)(a-b)$

We have

$\left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right) = 0$

either $7x + \frac{1}{2} = 0 \Rightarrow 7x = -\frac{1}{2} \Rightarrow x = -\frac{1}{14}$

or $\quad 7x - \frac{1}{2} = 0 \Rightarrow 7x = \frac{1}{2} \Rightarrow x = \frac{1}{14}$

$x = -\frac{1}{14}$ or $x = \frac{1}{14}$

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Question 2

Solve the following by using the quadratic formula, giving your answers correct to two decimal places.

a. $2x^2 + 5x + 1 = 0$ **b.** $4x^2 - 3x - 2 = 0$ **c.** $3 + 4x - 2x^2 = 0$

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Solution to question 2

a. $2x^2+5x+1=0$

$ax^2+bx+c=0$ $\qquad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=2, b=5$ and $c=1$

$$x=\frac{-5\pm\sqrt{5^2-4(2)(1)}}{2(2)}=\frac{-5\pm\sqrt{25-8}}{4}=\frac{-5\pm\sqrt{17}}{4}$$

$$x=\frac{-5-\sqrt{17}}{4} \text{ or } x=\frac{-5+\sqrt{17}}{4}$$

$x=-2.28$ or $x=-0.22$

b. $4x^2-3x-2=0$

$ax^2+bx+c=0$ $\qquad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=4, b=-3$ and $c=-2$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-2)}}{2(4)}=\frac{3\pm\sqrt{9+32}}{8}=\frac{3\pm\sqrt{41}}{8}$$

$$x=\frac{3-\sqrt{41}}{8} \text{ or } x=\frac{3+\sqrt{41}}{8}$$

$x=-0.43$ or $x=1.18$

c. $3+4x-2x^2=0$

$-2x^2+4x+3=0$ (rewrite in the equation in the form $ax^2+bx+c=0$)

$ax^2+bx+c=0$

$a=-2, b=4$ and $c=3$

$$x=\frac{-4\pm\sqrt{4^2-4(-2)(3)}}{2(-2)}=\frac{-4\pm\sqrt{16+24}}{-4}=\frac{-4\pm\sqrt{40}}{-4}$$

$$x=\frac{-4-\sqrt{40}}{-4} \text{ or } x=\frac{-4+\sqrt{40}}{-4}$$

$x=2.58$ or $x=-0.58$

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Question 3

Solve the following equations giving your answers correct to two decimal places where necessary.

a. $3x(x+2)=(x-1)^2+5$ **b.** $x-7=-\frac{12}{x}$ **c.** $\frac{2}{x-2}+\frac{2}{x+2}=3$

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Solution to question 3

a. $3x(x+2) = (x-1)^2 + 5$

$3x^2 + 6x = x^2 - 2x + 1 + 5$

$2x^2 + 8x - 6 = 0$

$x^2 + 4x - 3 = 0$

$ax^2 + bx + c = 0$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$

$= \dfrac{-4 \pm \sqrt{16 + 12}}{2} = \dfrac{-4 \pm \sqrt{28}}{2}$

$x = \dfrac{-4 \pm \sqrt{28}}{2}$ or $x = \dfrac{-4 \pm \sqrt{28}}{2}$

$x = -4.65$ or $x = 0.65$

b. $x - 7 = -\dfrac{12}{x}$

$x^2 - 7x = -12 \quad (\times x)$

$x^2 - 7x + 12 = 0$

product = 12

sum = –7

factors = –3, – 4

$\Rightarrow x^2 - 4x - 3x + 12 = 0$

$\Rightarrow x(x-4) - 3(x-4) = 0$

$\Rightarrow (x-4)(x-3) = 0$

either $x - 4 = 0 \Rightarrow x = 4$

or $x - 3 = 0 \Rightarrow x = 3$

$x = 3$ or $x = 4$

c. $\dfrac{2}{x-2} + \dfrac{2}{x+2} = 3$

$\dfrac{2(x+2) + 2(x-2)}{(x-2)(x+2)} = 3$

$2x + 4 + 2x - 4 = 3(x-2)(x+2)$

$4x = 3(x^2 - 4)$

$4x = 3x^2 - 12$

$3x^2 - 4x - 12 = 0$

$ax^2 + bx + c = 0$

$a = 3, b = -4$ and $c = -12$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-12)}}{2(3)}$

$= \dfrac{4 \pm \sqrt{16 + 144}}{3} = \dfrac{4 \pm \sqrt{160}}{3}$

$x = \dfrac{4 - \sqrt{160}}{3}$ or $x = \dfrac{4 + \sqrt{160}}{3}$

$x = -1.44$ or $x = 2.77$

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