I.G.C.S.E. Sine and Cosine Rules

Index:

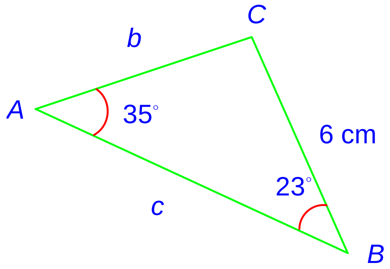
Please click on the question number you want

You can access the solutions from the end of each question

Question 1

For each of the following find each side marked with the letter.

a.



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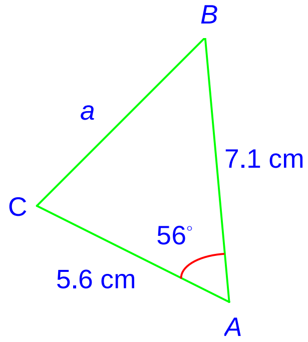
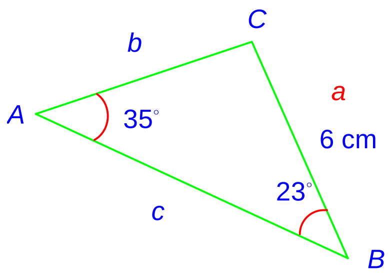


Click here to read the solution to this question

Click here to return to the index

Solution to question 1

a.



$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

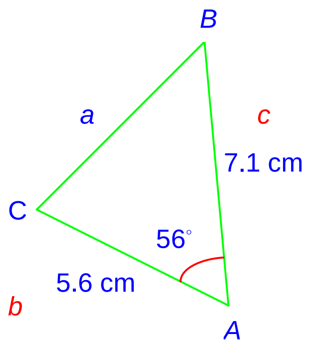
Using $\frac{b}{\sin B}=\frac{a}{\sin A}$ we have

$$\frac{b}{\sin 23^\circ}=\frac{6}{\sin 35^\circ}\Rightarrow b=\frac{6\sin 23^\circ}{\sin 35^\circ}=4.09\,\text{cm}$$

$$C=180^\circ-35^\circ-23^\circ=122^\circ$$

$$\frac{c}{\sin C}=\frac{a}{\sin A}\Rightarrow\frac{c}{\sin 122^\circ}=\frac{6}{\sin 35^\circ}\Rightarrow c=\frac{6\sin 122^\circ}{\sin 35^\circ}=8.87\,\text{cm}$$

b.



Here we have two sides and the included angle so we have to use the cosine rule.

$$a^2=b^2+c^2-2bc\cos A$$
$$=5.6^2+7.1^2-2(5.6)(7.1)\cos 56^\circ$$
$$=37.302\ldots$$
$$a=\sqrt{37.302\ldots}$$
$$=6.11\text{cm}$$

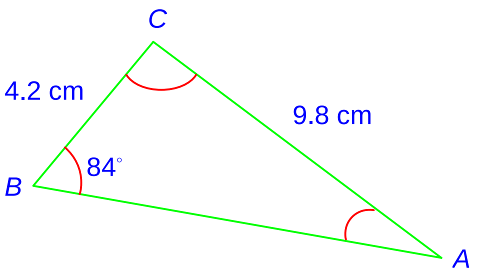
Click here to read the question again

Click here to return to the index

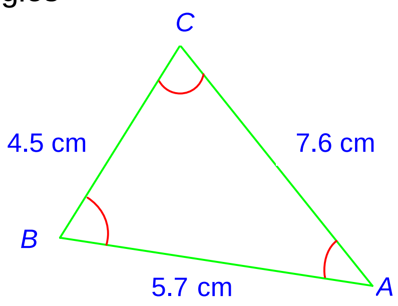
Question 2

In each of the following triangles find the missing angles

a.



b.

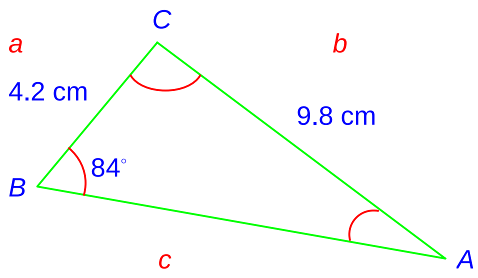


Click here to read the solution to this question

Click here to return to the index

Solution to question 2

a.



$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

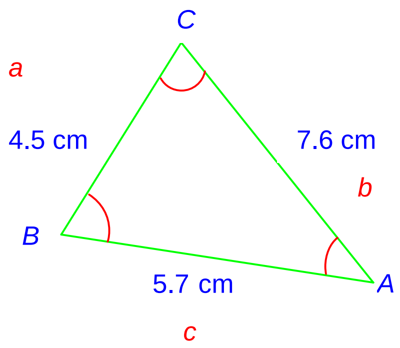
Using $\frac{\sin A}{a}=\frac{\sin B}{b}$ we have

$$\frac{\sin A}{4.2}=\frac{\sin 84^\circ}{9.8} \Rightarrow \sin A=\frac{4.2\sin 84^\circ}{9.8}$$

$$A=\sin^{-1}\left(\frac{4.2\sin 84^\circ}{9.8}\right)=25.2^\circ$$

$$C=180^\circ-84^\circ-25.2^\circ=70.8^\circ$$

b.



Here we have three sides and we are required to find an angle, so we use the cosine rule.

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$=\frac{7.6^2+5.7^2-4.5^2}{2(7.6)(5.7)}$$

$$A=\cos^{-1}\left[\frac{7.6^2+5.7^2-4.5^2}{2(7.6)(5.7)}\right]$$

$$=36.1^\circ$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$

$$=\frac{4.5^2+5.7^2-7.6^2}{2(4.5)(5.7)}$$

$$B=\cos^{-1}\left[\frac{4.5^2+5.7^2-7.6^2}{2(4.5)(5.7)}\right]$$

$$=95.6^\circ$$

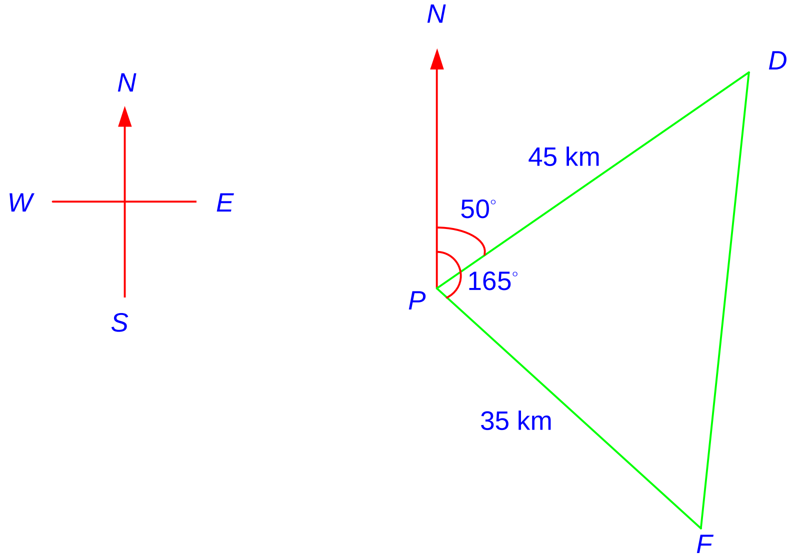
$$C=180^\circ-36.1^\circ-95.6^\circ=48.3^\circ$$

Click here to read the question again

Click here to return to the index

Question 3

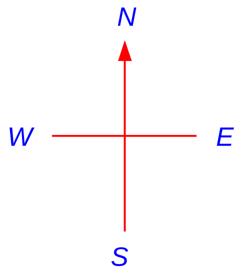
A destroyer *D* and a frigate *F* leave a port *P* at the same time. The diagram shows the destroyer, sailing 45 km on a bearing of 050° and the frigate sailing 35 km on a bearing of 165°. How far are the ships apart?

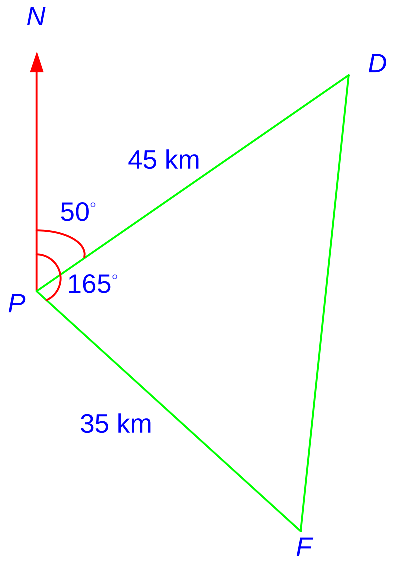


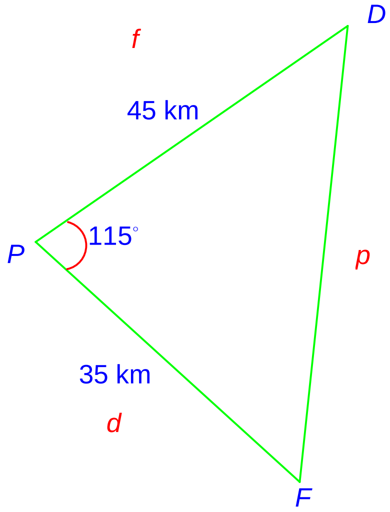
Click here to read the solution to this question

Click here to return to the index

Solution to question 3







We have to calculate *DF*. Drawing triangle *PDF*, we have $\hat{P} = 165^\circ - 50^\circ = 115^\circ$. Using the cosine rule as we have two sides and the included angle.

$$p^2 = d^2 + f^2 - 2df\cos P$$
$$= 35^2 + 45^2 - 2(35)(45)\cos 115^\circ$$
$$= 4581.247\ldots$$
$$p = \sqrt{4581.247\ldots}$$
$$= 67.7\text{km}$$

Click here to read the question again

Click here to return to the index

Question 4

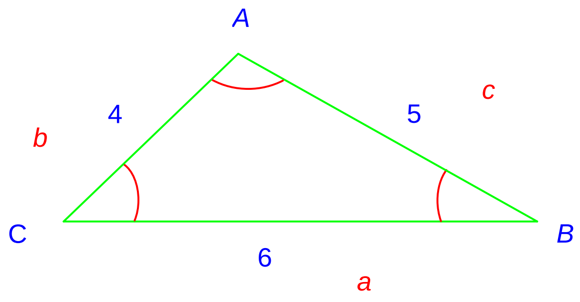
Find all the angles of a triangle whose sides are in the ratio 4 : 5 : 6.

Click here to read the solution to this question

Click here to return to the index

Solution to question 4

First draw a diagram and label the sides.



Now calculating angle $\hat{A}$ and $\hat{B}$ using the cosine rule, we have

$$\begin{aligned}\cos\hat{A} &= \frac{b^2+c^2-a^2}{2bc}\\ &= \frac{4^2+5^2-6^2}{2(4)(5)}\\ &= \frac{5}{40}\\ &= \frac{1}{8}\\ \hat{A} &= \cos^{-1}\left(\frac{1}{8}\right)\\ &= 82.8^\circ\end{aligned}$$

$$\begin{aligned}\cos\hat{B} &= \frac{a^2+c^2-b^2}{2ac}\\ &= \frac{6^2+5^2-4^2}{2(6)(5)}\\ &= \frac{45}{60}\\ &= \frac{3}{4}\\ \hat{A} &= \cos^{-1}\left(\frac{3}{4}\right)\\ &= 41.4^\circ\end{aligned}$$

Now using that the angle sum of a triangle is 180°, we have

$$\hat{C} = 180^\circ - 82.8^\circ - 41.4^\circ = 55.8^\circ$$

Click here to read the question again

Click here to return to the index

Question 5

From a lighthouse *L* an oil tanker *A* is 12 km away on a bearing of 126° and a car ferry *C* is 15 km away on a bearing of 210°. Draw a diagram clearly showing *L*, *A* and *C*. Find

a. the distance between *A* and *C*

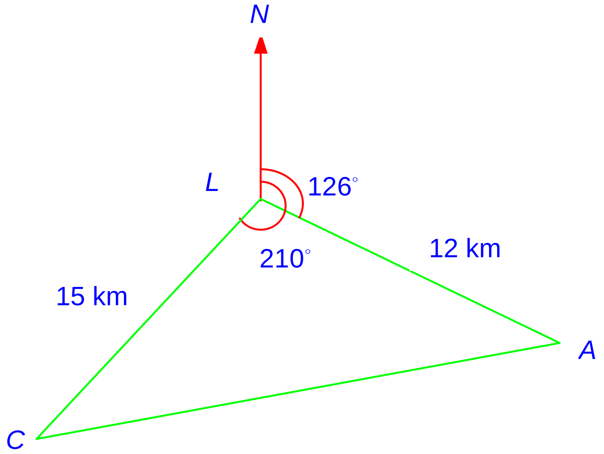
b. the bearing of *A* from *C*

Click here to read the solution to this question

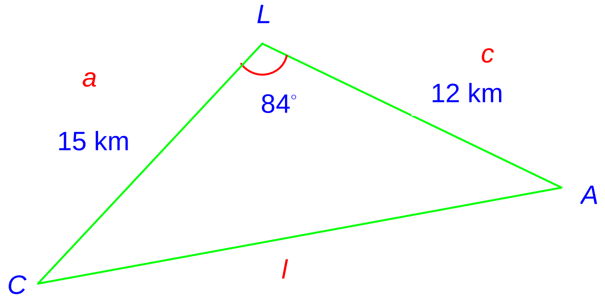
Click here to return to the index

Solution to question 5

First drawing a diagram



Now drawing triangle *LAC*. Note that $\hat{L} = 210^\circ - 126^\circ = 84^\circ$.



a. The distance between the two ships is given by $AC = l$. Using the cosine rule

$$
\begin{aligned}
l^2 &= a^2 + c^2 - 2ac\cos L \\
&= 15^2 + 12^2 - 2(15)(12)\cos 84^\circ \\
&= 331.369\ldots \\
l &= \sqrt{331.369\ldots} \\
&= 18.2\,\text{km}
\end{aligned}
$$

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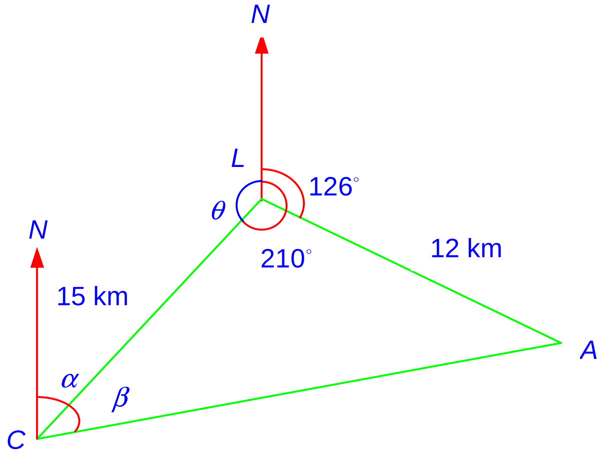
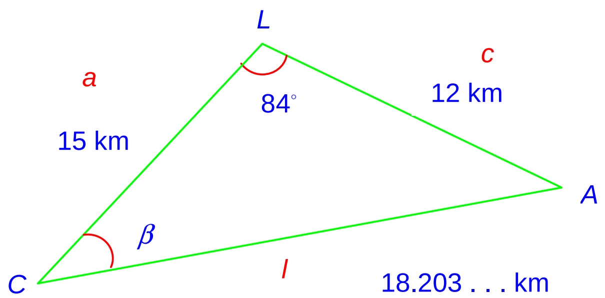
b.



The required bearing is $\alpha+\beta$ as shown in the above diagram.
$\theta = 360^\circ - 210^\circ = 150^\circ \Rightarrow \alpha = 180^\circ - 150^\circ = 30^\circ$ (allied angles).

Considering triangle *CLA* in part **a**



Using the cosine rule $\cos\beta = \cos C = \dfrac{a^2 + l^2 - c^2}{2al}$

$$= \frac{15^2 + 18.205\ldots^2 - 12^2}{2(15)(18.203\ldots)}$$

$$C = \cos^{-1}\left[\frac{15^2 + 18.205\ldots^2 - 12^2}{2(15)(18.203\ldots)}\right]$$

$$= 41.0^\circ$$

The required angle is $\alpha + \beta = 30^\circ + 41^\circ = 71^\circ$
The bearing of A from C is 071°.

Click here to read the question again

Click here to return to the index

Question 6

An airliner flies from Heathwick airport *H*, 400 km on a bearing of 135° to *A* and then 500 km on a bearing of 260° to *B*, and then returns to the airport at *H*.

a. Draw a diagram, showing clearly *H*, *A* and *B*.

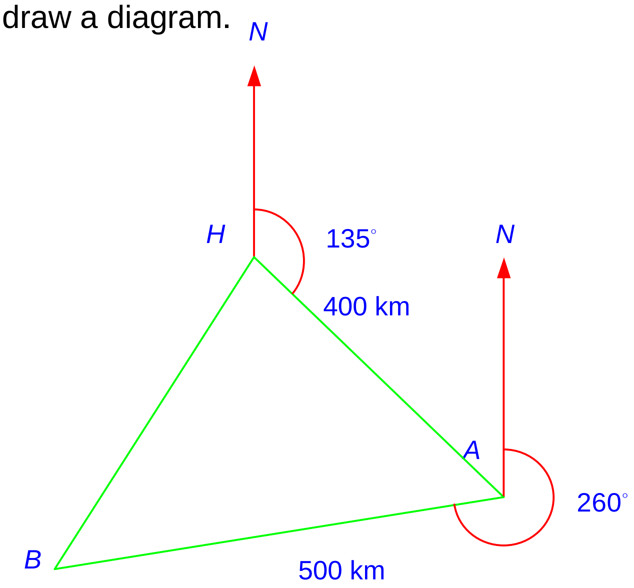
b. Calculate the length and bearing of the return journey from *B* to *H*.

Click here to read the solution to this question

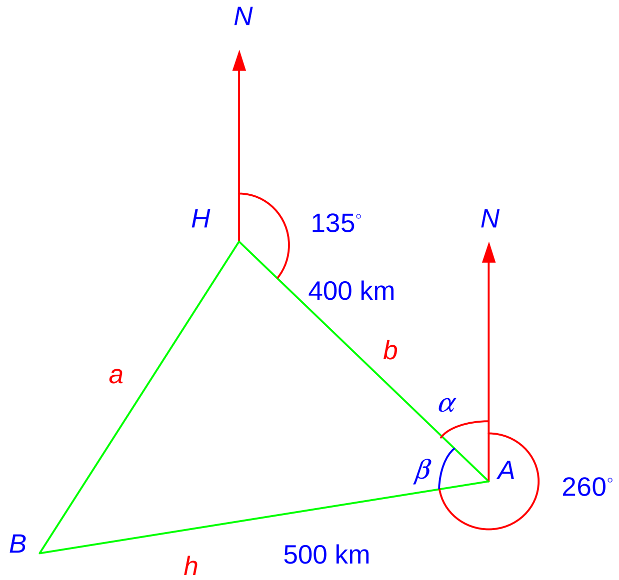
Click here to return to the index

Solution to question 6

a. First draw a diagram.



b.



From the diagram

$\alpha = 180^\circ - 135^\circ = 45^\circ$ therefore $\beta = 360^\circ - 45^\circ - 260^\circ = 55^\circ$

We are required to find $BH = a$

Using the cosine rule we have $a^2 = b^2 + h^2 - 2bh\cos A$

$$= 400^2 + 500^2 - 2(400)(500)\cos 55^\circ$$

$$= 180569.4255$$

$$a = \sqrt{180569.4255}$$

$$= 425\,\text{km}$$

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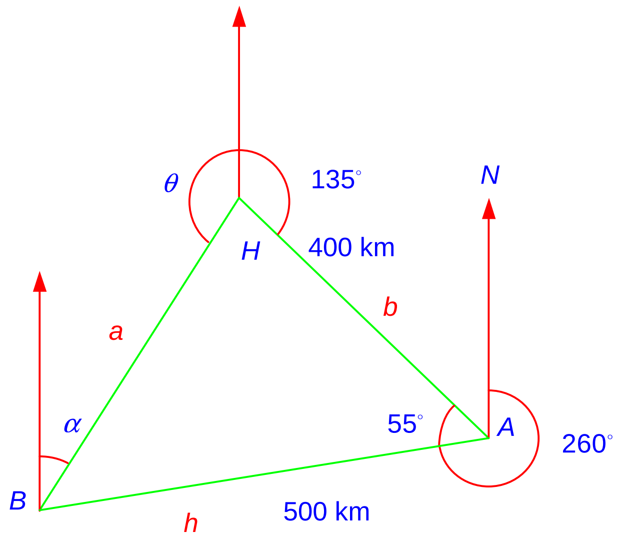

Using cosine rule $\cos H = \dfrac{a^2 + b^2 - h^2}{2ab}$

$$= \frac{424.93\ldots^2 + 400^2 - 500^2}{2(424.93\ldots)(400)}$$

$$H = \cos^{-1}\left[\frac{424.93\ldots^2 + 400^2 - 500^2}{2(424.93\ldots)(400)}\right]$$

$$= 74.5^\circ$$

The bearing of *H* from *B* is given by α

Now $\theta = 360^\circ - 74.5^\circ - 135^\circ = 150.5^\circ$
and hence $\alpha = 180^\circ - 150.5^\circ = 29.5^\circ$

The bearing of *H* from *B* is 029.5°

Click here to read the question again

Click here to return to the index