I.G.C.S.E. Matrices and Transformations

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Question 1

Draw x- and y- axis with values from –10 to 10. Draw the following triangle with vertices $A(3, 2)$, $B(5, 2)$ and $C(5, 4)$. Draw the image of ABC under the following transformations **clearly labelling** the vertices in each case. Write down the coordinates of the vertices in each case.

a. Enlargement scale factor 2, centre (0, 0). Label the image A_1, B_1, C_1.

b. Reflection in the y-axis. Label the image A_2, B_2, C_2.

c. Rotation 270° about (0, 0). Label the image A_3, B_3, C_3.

d. Enlargement scale factor –1, centre (0, 0). Label the image A_4, B_4, C_4.

e. Translation $\begin{pmatrix} -12 \\ -12 \end{pmatrix}$. Label the image A_5, B_5, C_5.

f. Reflection in the line $y = -x$. Label the image A_6, B_6, C_6.

g. Enlargement scale factor –2, centre (0, 3). Label the image A_7, B_7, C_7.

h. Rotation 90° about the origin. Label the image A_8, B_8, C_8.

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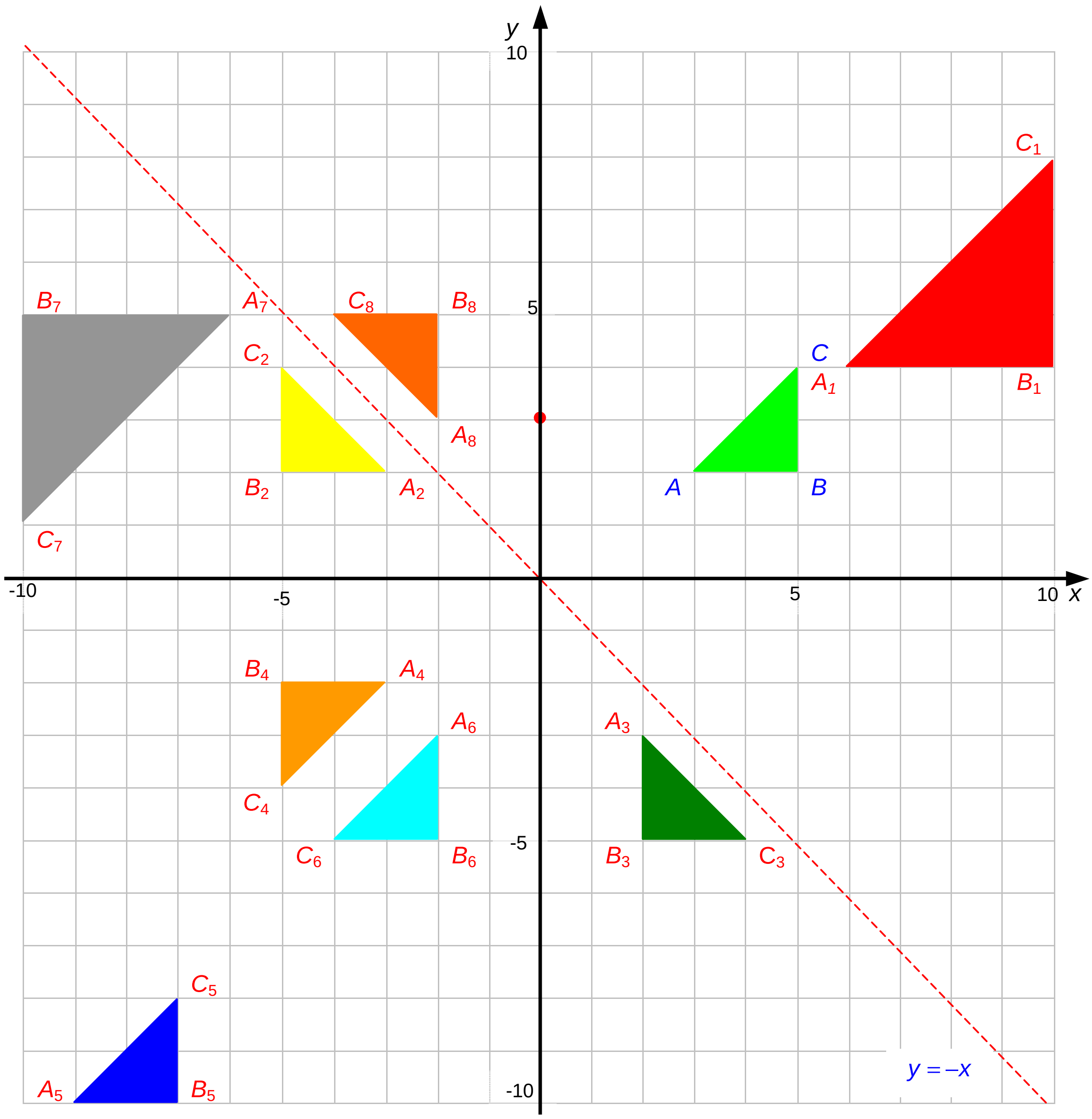
Solution to question 1



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a. A_1, B_1, C_1 is an enlargement scale factor 2, centre (0, 0).
$A_1\ (6, 1), B_1\ (10, 4), C_1\ (10, 8)$.

b. A_2, B_2, C_2 is a reflection in the *y*-axis. $A_2\ (-3, 2), B_2\ (-5, 2), C_2\ (-5, 4)$.

c. A_3, B_3, C_3 is a rotation 270° (anticlockwise) about (0, 0).
$A_3\ (2, -3), B_3\ (2, -5), C_3\ (4, -5)$.

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d. A_4, B_4, C_4 is an enlargement scale factor –1, centre (0, 0).
$A_4\ (-3,-2), B_4\ (-5,-2), C_4\ (-5,-4)$.

e. A_5, B_5, C_5 is a translation $\begin{pmatrix} -12 \\ -12 \end{pmatrix}$. $A_5\ (-9,-10), B_5\ (-7,-10), C_5\ (-7,-8)$.

f. A_6, B_6, C_6 is a reflection in the line $y=-x$.
$A_6\ (-2,-3), B_6\ (-2,-5), C_6\ (-4,-5)$.

g. A_7, B_7, C_7 is an enlargement scale factor –2, centre (0, 3).
$A_7\ (-6,5), B_7\ (-10,5), C_7\ (-10,1)$.

h. A_8, B_8, C_8 is a rotation 90° about the origin.
$A_8\ (-2,3), B_8\ (-2,5), C_8\ (-4,5)$

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Question 2

From the diagram in **question 1** describe the single transformation, which maps

a. A_2, B_2, C_2 to A_3, B_3, C_3

b. A_4, B_4, C_4 to A_6, B_6, C_6

c. A_8, B_8, C_8 to A_2, B_2, C_2

d. A_3, B_3, C_3 to A_4, B_4, C_4

e. A_6, B_6, C_6 to A_3, B_3, C_3

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Solution to question 2

Click here to see the diagram

a. A_2, B_2, C_2 to A_3, B_3, C_3 is a reflection in the line $y = x$.

b. A_4, B_4, C_4 to A_6, B_6, C_6 is a reflection in the line $y = x$.

c. A_8, B_8, C_8 to A_2, B_2, C_2 is a reflection in the line $y = -x$.

d. A_3, B_3, C_3 to A_4, B_4, C_4 is a rotation 270° about (0, 0).

e. A_6, B_6, C_6 to A_3, B_3, C_3 is a reflection in the *y*-axis or $x = 0$.

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Question 3

Draw x- and y- axis with values from –10 to 10. Draw the following triangle with vertices $A(5, 4)$, $B(9, 4)$ and $C(5, 6)$ and its image under a rotation $A_1(-9, -3)$, $B_1(-9, -7)$ and $C_1(-7, -3)$. Show by construction the centre of rotation.

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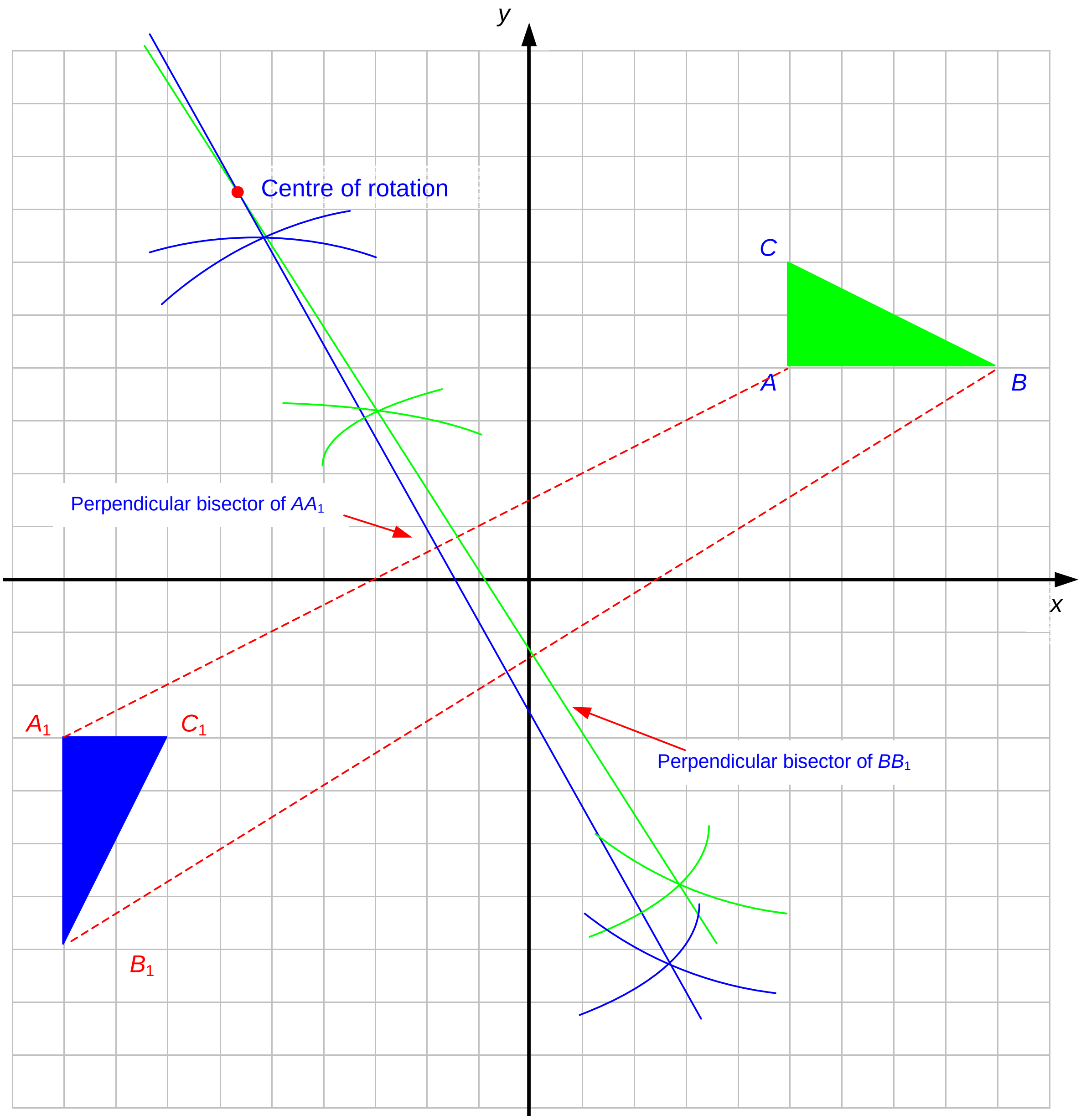
Solution to question 3



The centre of rotation is found by joining two corresponding vertices A to A_1 and B to B_2. Then construct the perpendicular bisectors of both lines using a compass and a ruler. Finally the point of intersection of the two perpendicular bisectors is the centre of rotation.

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Question 4

A is a rotation 270° about (0, 0)
B is a reflection in the line $y = -2$
C is a translation, which maps (-2, 3) to (2, 4)

Find the image of the point (-3, 2) under the following transformations

a. A **b.** A^2 **c.** CB **d.** ABC **e.** $B^{-1}C^{-1}$

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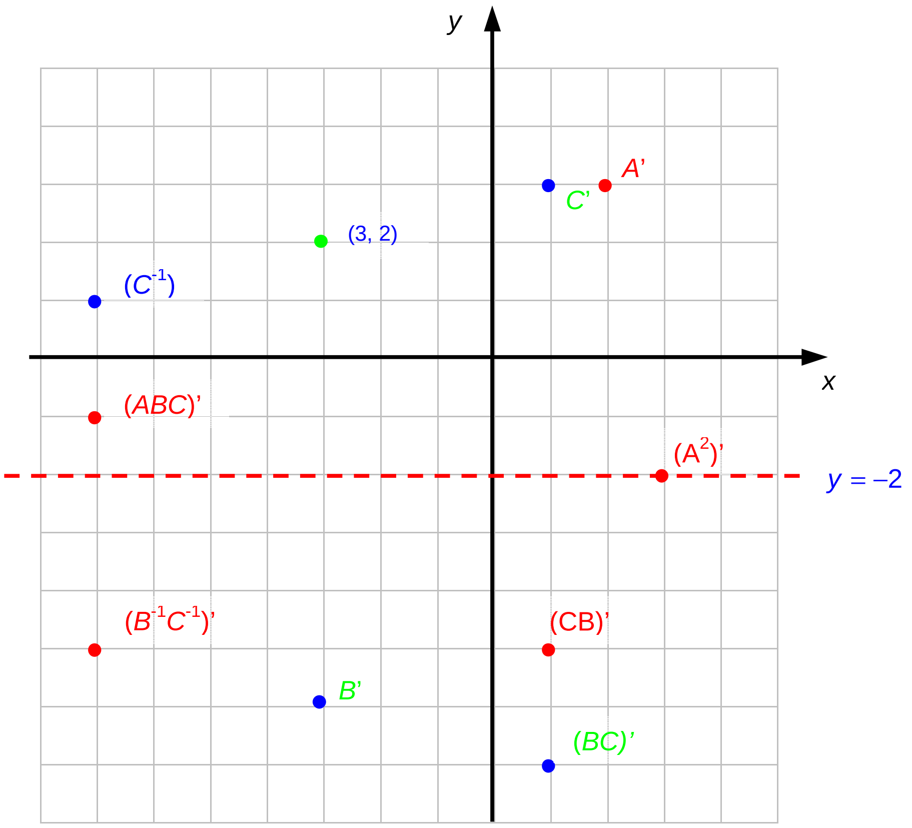
Solution to question 4

A is a rotation 270° about (0, 0)
B is a reflection in the line $y = -2$

C is a translation, which maps (-2, 3) to (2, 4). Note: this is a translation $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$



a. ***A*** (2, 3).

b. ***A***^{2} is the same as a rotation of 180°. (3, -2).

c. ***CB*** is a reflection in the line $y = -2$ followed by a translation $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$. (1, -5)

d. ***ABC*** is a translation $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ followed by a reflection in the line $y = -2$ followed by a rotation of 270°. (-7, -1)

e. ***B***$^{-1}$***C***$^{-1}$ are inverse transformations. This is a translation $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$ followed by a reflection in the line $y = -2$ (self inverse). (-7, -5).

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Question 5

The transformation ***T*** is given by $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ is composed of two transformations.

a. Describe the two transformations.
b. Find the image of the point (2, -1) under the transformation.
c. Find the point, which is mapped by ***T*** onto the point (6, 7).

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Solution to question 5

T is given by $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

a. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ is an enlargement scale factor –2 centre (0, 0), followed by a translation $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

b. The image of (2, -1) is given by

$$\begin{aligned} \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} -4+0 \\ 0+2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ -1 \end{pmatrix} \end{aligned}$$

which is (0, -1).

c. To find the point, which is mapped by ***T*** onto the point (6, 7), we must work with the inverse transformations.

First the point (6, 7) is mapped to (2, 10) by the translation $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

The inverse of an enlargement scale factor –2 is an enlargement scale factor $-\frac{1}{2}$, given by the matrix $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 2 \\ 10 \end{pmatrix} = \begin{pmatrix} -1+0 \\ 0-5 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$, which gives the point (-1, -5).

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Question 6

A is a reflection in the line $y = -x$.
B is a reflection in the x-axis.

Find the matrix, which represents

a. ***A*** **b.** ***B*** **c.** ***AB*** **d.** ***BA***

Describe the single transformations ***AB*** and ***BA***.

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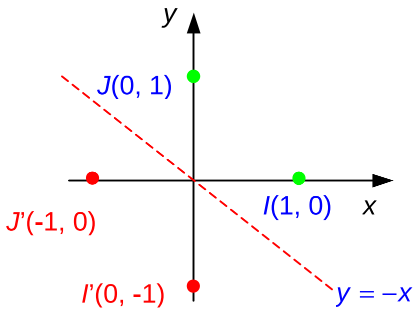
Solution to question 6

A is a reflection in the line $y = -x$.
B is a reflection in the x-axis.

a. ***A***

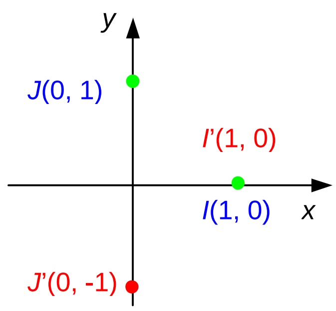


$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}0\\-1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}-1\\0\end{pmatrix}$$

$$\boldsymbol{A} = \begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$$

b. ***B***



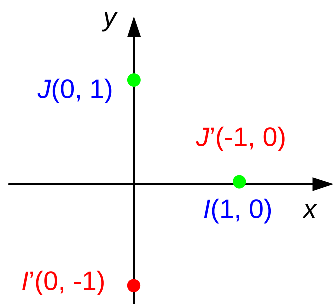
$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}1\\0\end{pmatrix}$$

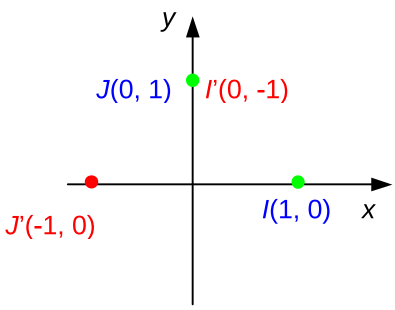
$$\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}0\\-1\end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

c. $\boldsymbol{AB} = \begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$

From the diagram we can see that
AB is a rotation 270° about (0, 0).





d. $\boldsymbol{BA} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix} = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$

From the diagram we can see that
AB is a rotation 90° about (0, 0).

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