IGCSE Matrices

Index:

Please click on the question number you want

You can access the solutions from the end of each question

Question 1

$$A = \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -3 & 2 \\ 7 & 6 \end{pmatrix}.$$

Express as a single matrix the following:

a. $3A$ **b.** $A+B$ **c.** $B-A$ **d.** AB **e.** A^2

Click here to read the solution to this question

Click here to return to the index

Solution to question 1

$$A=\begin{pmatrix}2 & -3\\ 4 & 1\end{pmatrix},\quad B=\begin{pmatrix}-3 & 2\\ 7 & 6\end{pmatrix}.$$

a. $$3A=3\begin{pmatrix}2 & -3\\ 4 & 1\end{pmatrix}=\begin{pmatrix}6 & -9\\ 12 & 3\end{pmatrix}$$

b. $$A+B=\begin{pmatrix}2 & -3\\ 4 & 1\end{pmatrix}+\begin{pmatrix}-3 & 2\\ 7 & 6\end{pmatrix}=\begin{pmatrix}2-3 & -3+2\\ 4+7 & 1+6\end{pmatrix}=\begin{pmatrix}-1 & -1\\ 11 & 7\end{pmatrix}$$

c. $$B-A=\begin{pmatrix}-3 & 2\\ 7 & 6\end{pmatrix}-\begin{pmatrix}2 & -3\\ 4 & 1\end{pmatrix}=\begin{pmatrix}-3-2 & 2-(-3)\\ 7-4 & 6-1\end{pmatrix}=\begin{pmatrix}-5 & 5\\ 3 & 5\end{pmatrix}$$

d. $$AB=\begin{pmatrix}2 & -3\\ 4 & 1\end{pmatrix}\begin{pmatrix}-3 & 2\\ 7 & 6\end{pmatrix}$$

$$=\begin{pmatrix}2\times-3+-3\times7 & 2\times2+-3\times6\\ 4\times-3+1\times7 & 4\times2+1\times6\end{pmatrix}=\begin{pmatrix}-6-21 & 4-18\\ -12+7 & 8+6\end{pmatrix}=\begin{pmatrix}-27 & -14\\ -5 & 14\end{pmatrix}$$

e. $$A^2=\begin{pmatrix}2 & -3\\ 4 & 1\end{pmatrix}\begin{pmatrix}2 & -3\\ 4 & 1\end{pmatrix}$$

$$=\begin{pmatrix}2\times2+-3\times4 & 2\times-3+-3\times1\\ 4\times2+1\times4 & 4\times-3+1\times1\end{pmatrix}=\begin{pmatrix}4-12 & -6-3\\ 8+4 & -12+1\end{pmatrix}=\begin{pmatrix}-8 & -9\\ 12 & -11\end{pmatrix}$$

Click here to read the question again

Click here to return to the index

Question 2

$A = \begin{pmatrix} 6 & 7 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$ $B = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & -3 & 1 \end{pmatrix}$

Find if possible:

a. AB **b.** CA **c.** CB

Click here to read the solution to this question

Click here to return to the index

Solution to question 2

$$A = \begin{pmatrix} 6 & 7 \\ 2 & 1 \\ 1 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \text{ and } C = \begin{pmatrix} 2 & -3 & 1 \end{pmatrix}$$

Find if possible:

a. $$AB = \begin{pmatrix} 6 & 7 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 6\times -4 + 7\times 2 \\ 2\times -4 + 1\times 2 \\ 1\times -4 + 3\times 2 \end{pmatrix} = \begin{pmatrix} -24 + 14 \\ -8 + 2 \\ -4 + 6 \end{pmatrix} = \begin{pmatrix} -10 \\ -6 \\ 2 \end{pmatrix}$$

b. $$CA = \begin{pmatrix} 2 & -3 & 1 \end{pmatrix}\begin{pmatrix} 6 & 7 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 2\times 6 + -3\times 2 + 1\times 1 & 2\times 7 + -3\times 1 + 1\times 3 \end{pmatrix}$$
$$= \begin{pmatrix} 12 - 6 + 1 & 14 - 3 + 3 \end{pmatrix}$$
$$= \begin{pmatrix} 7 & 14 \end{pmatrix}$$

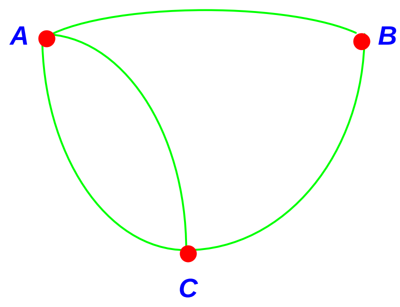
c. *CB* is not possible as the number of rows in the second matrix *B* is 2 and the number of rows in the first matrix *A* is 3.

Click here to read the question again

Click here to return to the index

Question 3

The diagram shows three town *A*, *B* and *C*.



Write down the following:

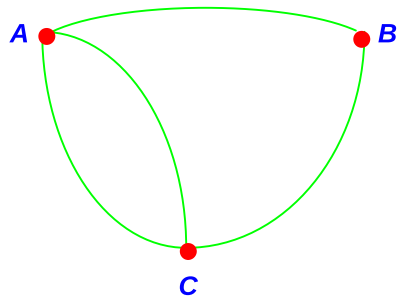
a. the direct route matrix.

b. the two-stage route matrix.

Click here to read the solution to this question

Click here to return to the index

Solution to question 3



a. One stage-matrix is:

$$\text{From}\;\begin{matrix} & \text{To} & & \\ & A & B & C \\ A & 0 & 1 & 2 \\ B & 1 & 0 & 1 \\ C & 2 & 1 & 0 \end{matrix} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

b. The two-stage matrix is found by squaring the one-stage matrix as shown below

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0\times0+1\times1+2\times2 & 0\times1+1\times0+2\times1 & 0\times2+1\times1+2\times0 \\ 1\times0+0\times1+1\times2 & 1\times1+0\times0+1\times1 & 1\times2+0\times1+1\times0 \\ 2\times0+1\times1+0\times2 & 2\times1+1\times0+0\times1 & 2\times2+1\times1+0\times0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+1+4 & 0+0+2 & 0+1+0 \\ 0+0+2 & 1+0+1 & 2+0+0 \\ 0+1+0 & 2+0+0 & 4+1+0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 5 \end{pmatrix}$$

Click here to read the question again

Click here to return to the index

Question 4

The results of three football teams are show below in a matrix, with another matrix showing the allocation of the points.

$$\begin{array}{c} \\ X \\ Y \\ Z \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 6 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 0 & 4 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ W \\ D \\ L \end{array}\begin{array}{c} \text{Points} \\ \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \end{array}$$

Find the number of points for each team.

Click here to read the solution to this question

Click here to return to the index

Solution to question 4

$$\begin{array}{c} \\ X \\ Y \\ Z \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 6 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 0 & 4 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ W \\ D \\ L \end{array}\begin{array}{c} \text{Points} \\ \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \end{array}$$

The number of points for each team is found by multiplying the two matrices together

$$\begin{pmatrix} 6 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 0 & 4 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6\times3+2\times1+1\times0 \\ 3\times3+5\times1+1\times0 \\ 2\times3+0\times1+4\times0 \end{pmatrix} = \begin{pmatrix} 18+2+0 \\ 9+5+0 \\ 6+0+0 \end{pmatrix} = \begin{pmatrix} 20 \\ 14 \\ 6 \end{pmatrix}$$

which gives $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Points} \\ \begin{pmatrix} 20 \\ 14 \\ 6 \end{pmatrix} \end{array}$

Hence team A has 20 points, team B 14 points and team C 6 points.

Click here to read the question again

Click here to return to the index

Question 5

If $M = \begin{pmatrix} 4 & -3 \\ -2 & -5 \end{pmatrix}$. Find the inverse matrix M^{-1}.

Click here to read the solution to this question

Click here to return to the index

Solution to question 5

If $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $M^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

So if $M = \begin{pmatrix} 4 & -3 \\ -2 & -5 \end{pmatrix}$ then

$$\begin{aligned} M^{-1} &= \frac{1}{(4)(-5) - (-3)(-2)}\begin{pmatrix} -5 & 3 \\ 2 & 4 \end{pmatrix} \\ &= \frac{1}{-20 - 6}\begin{pmatrix} -5 & 3 \\ 2 & 4 \end{pmatrix} \\ &= -\frac{1}{26}\begin{pmatrix} -5 & 3 \\ 2 & 4 \end{pmatrix} \\ &= \frac{1}{26}\begin{pmatrix} 5 & -3 \\ -2 & -4 \end{pmatrix} \end{aligned}$$

Click here to read the question again

Click here to return to the index

Question 6

If $AB = I$, where $A = \begin{pmatrix} 2 & 5 \\ 4 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the matrix B.

Click here to read the solution to this question

Click here to return to the index

Solution to question 6

$AB = I$, where $A = \begin{pmatrix} 2 & 5 \\ 4 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{aligned} AB &= I \\ A^{-1}AB &= A^{-1}I \\ IB &= A^{-1} \\ B &= A^{-1} \end{aligned}$$

Now if If $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $M^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Therefore

$$\begin{aligned} A^{-1} &= \frac{1}{(2)(1)-(5)(4)}\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} \\ &= \frac{1}{2-20}\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} \\ &= -\frac{1}{18}\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} \\ &= \frac{1}{18}\begin{pmatrix} -1 & 5 \\ 4 & -2 \end{pmatrix} \end{aligned}$$

Click here to read the question again

Click here to return to the index

Question 7

If $XY = \begin{pmatrix} 2 & 7 \\ -2 & 4 \end{pmatrix}$ and $X = \begin{pmatrix} 3 & 2 \\ 8 & -1 \end{pmatrix}$, find the matrix Y.

Click here to read the solution to this question

Click here to return to the index

Solution to question 7

$XY = \begin{pmatrix} 2 & 7 \\ -2 & 4 \end{pmatrix}$ and $X = \begin{pmatrix} 3 & 2 \\ 8 & -1 \end{pmatrix}$

$$XY = \begin{pmatrix} 2 & 7 \\ -2 & 4 \end{pmatrix}$$

$$X^{-1}XY = X^{-1}\begin{pmatrix} 2 & 7 \\ -2 & 4 \end{pmatrix}$$

$$IY = X^{-1}\begin{pmatrix} 2 & 7 \\ -2 & 4 \end{pmatrix}$$

$$Y = X^{-1}\begin{pmatrix} 2 & 7 \\ -2 & 4 \end{pmatrix}$$

$$X^{-1} = \frac{1}{(3)(-1)-(2)(8)}\begin{pmatrix} -1 & -2 \\ -8 & 3 \end{pmatrix}$$

$$= \frac{1}{-3-16}\begin{pmatrix} -1 & -2 \\ -8 & 3 \end{pmatrix}$$

$$= -\frac{1}{19}\begin{pmatrix} -1 & -2 \\ -8 & 3 \end{pmatrix}$$

$$= \frac{1}{19}\begin{pmatrix} 1 & 2 \\ 8 & -3 \end{pmatrix}$$

$$Y = \frac{1}{19}\begin{pmatrix} 1 & 2 \\ 8 & -3 \end{pmatrix}\begin{pmatrix} 2 & 7 \\ -2 & 4 \end{pmatrix} = \frac{1}{19}\begin{pmatrix} 1\times 2+2\times -2 & 1\times 7+2\times 4 \\ 8\times 2+(-3)\times(-2) & 8\times 7+(-3)\times 4 \end{pmatrix}$$

$$= \frac{1}{19}\begin{pmatrix} 2-4 & 7+8 \\ 16+6 & 56-12 \end{pmatrix} = \frac{1}{19}\begin{pmatrix} -2 & 15 \\ 22 & 44 \end{pmatrix}$$

Click here to read the question again

Click here to return to the index

Question 8

If the matrix $\begin{pmatrix} 2 & -1 \\ 4 & k \end{pmatrix}$ has no inverse find the value of k.

Click here to read the solution to this question

Click here to return to the index

Solution to question 8

For the matrix $\begin{pmatrix} 2 & -1 \\ 4 & k \end{pmatrix}$ to have no inverse then the determinant is equal to zero.

Hence $2k-(-1)(4)=0$

$$2k+4=0$$
$$2k=-4$$
$$k=-2$$

Click here to read the question again

Click here to return to the index