I.G.C.S.E. Linear Programming

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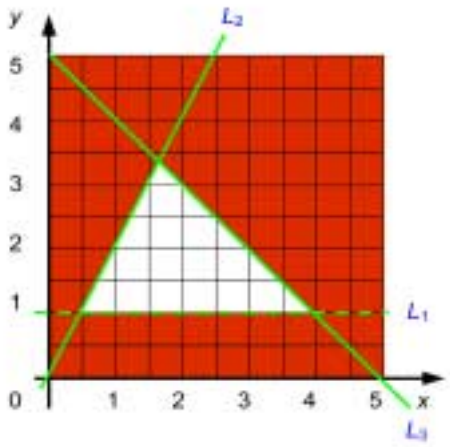
Question 1



a. Find the equations of the lines L_1, L_2 and L_3.

b. The **unshaded region** is defined by three inequalities. Write down these three inequalities.

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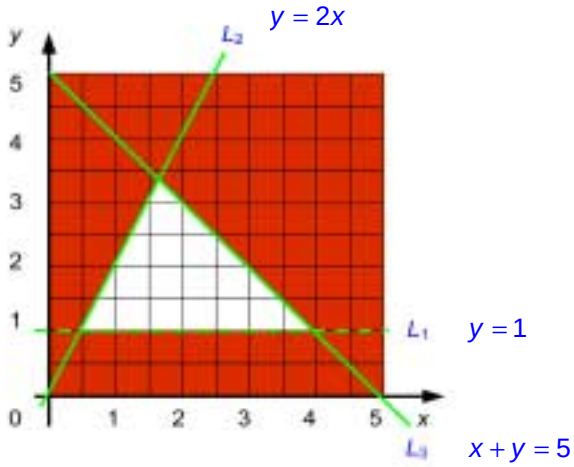
Solution to question 1



a. L_1 is the line $y = 1$ as the y coordinate is always 1 regardless of the x coordinate.
For L_2 if we construct a small table of convenient points, we have

x	0	1	2
y	0	2	4

We can observe that we have the line $y = 2x$.
For L_3 we see that when $y = 0$ we have $x = 5$ the line crosses the x-axis at (5, 0). When $x = 0$ we have $y = 5$, the line crosses the y-axis at (0, 5). Hence we have the line $x + y = 5$.

b. Considering L_1, $y = 1$, the unshaded region is above the line hence $y > 1$. It is not equal to as the line is broken.
Considering L_2 and taking a point not in the line like (2, 2).
At (2, 2) we have $y \le 2x$

$$2 \le 2(2)$$

$2 \le 4$ which is true, ignoring the equal sign.

Considering L_3 and taking a point not on the line like (0, 0).
At (2, 2) we have $x + y \le 5$

$$0 + 0 \le 5$$

$0 \le 5$ which is true, ignoring the equal sign.

Therefore the unshaded region is represented by $y > 0$, $y \le 2x$, $x + y \le 5$.

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Question 2

José and César are tailors. They make x jackets and y suites each week.
José does all the cutting, and César does all the sewing.

To make a jacket takes 5 hours of cutting and 4 hours of sewing.
To make a suit takes 6 hours of cutting and 10 hours of sewing.

Neither tailor works for more than 60 hours a week.

a. For the sewing, show that

$$2x + 5y \leq 30$$

b. Write down another inequality in x and y for the cutting.

c. They make at least 8 jackets each week. Write down another inequality.

d. **i.** Draw axes from 0 to 16, using 1 cm to represent 1 unit on each axes.

ii. On your grid, show the information in parts **a**, **b** and **c**.
Shade the **unwanted** regions.

e. The profit on a jacket is \$30 and on a suit is \$100.
Calculate the maximum profit that José and César can make in a week.

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Solution to question 2

a. From the information in the question that a jacket (x) takes 4 hours of sewing and a suit (y) takes 10 hours of sewing. César, who does the sewing, can only work for 60 hours per week. Hence
$4x+10y \le 60 \Rightarrow 2x+5y \le 30$.

b. From the information in the question that a jacket (x) takes 5 hours of cutting and a suit (y) takes 6 hours of cutting. José, who does the cutting, can only work for 60 hours per week. Hence
$5x+6y \le 60$.

c. They must make at least 8 jackets (x) hence $x \ge 8$.

d. **i.** Drawing the lines, $x=8$ is where the x-coordinate is 8 regardless of what the y-coordinate is.

$2x+5y=30$

x-axis ($y=0$) $2x+5(0)=30 \Rightarrow 2x=30 \Rightarrow x=15$, plot (15, 0)

y-axis ($x=0$) $2(0)+5y=30 \Rightarrow 5y=30 \Rightarrow y=6$, plot (0, 6)

$5x+6y=60$

x-axis ($y=0$) $5x+6(0)=60 \Rightarrow 5x=60 \Rightarrow x=12$, plot (12, 0)

y-axis ($x=0$) $5(0)+6y=60 \Rightarrow 6y=60 \Rightarrow y=10$, plot (0, 10)

(see graph)

ii. Shading.

For $x \ge 8$, shade to the left of the line as we are shading unwanted regions.

Considering $2x+5y \le 30$ and taking a point not in the line like (0, 0).

At (0, 0), $2x+5y \le 30$

$2(0)+5(0) \le 30$

$0 \le 30$ which is true, ignoring the equal sign.

Therefore the point (0, 0) is in the region, we shade the other side of the line.

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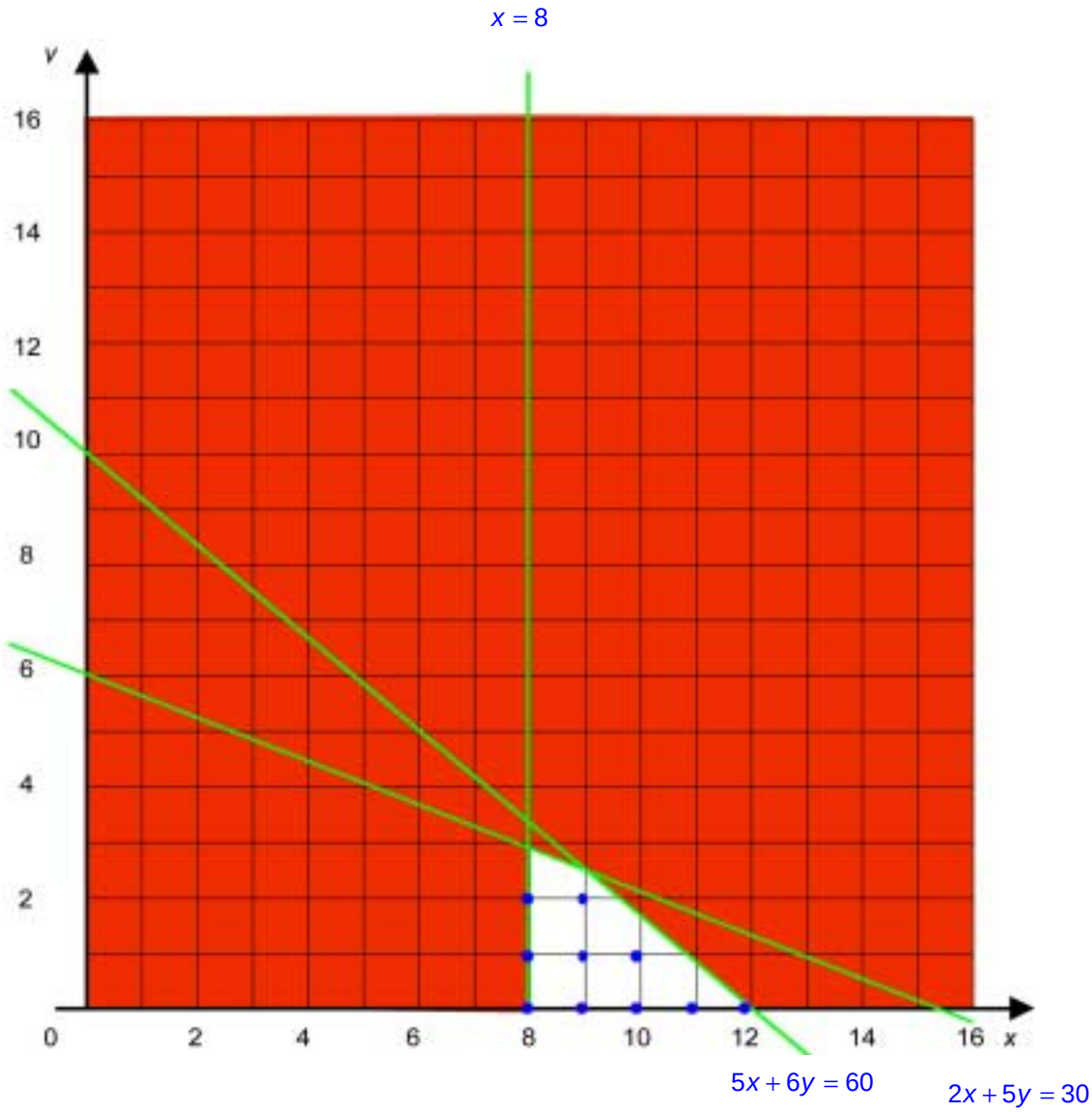
Considering $5x + 6y \leq 60$ and taking a point not on the line like (0, 0).
At (0, 0) we have

$$5x + 6y \leq 60$$

$$5(0) + 6(0) \leq 60$$

$0 \leq 60$ which is true, ignoring the equal sign.

Therefore the point (0, 0) is in the region, we shade the other side of the line.



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e. The blue dots in the unshaded region show the possible combinations of jackets and suits. Making a table and calculating the profit which is $30x + 100y$, for the value that gives the maximum value.

x	8	8	8	9	9	9	10	10	11	12
y	0	1	2	0	1	2	0	1	0	0
$30x$						270				
$100y$						200				
Profit						$470				

Therefore the maximum profit José and César can make is $470.

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