I.G.C.S.E. Geometry 03

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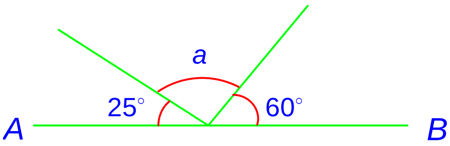
You can access the solutions from the end of each question

Question 1

1. Find the angles with letters, showing clearly all working out. (*AB* is always a straight line).

a.



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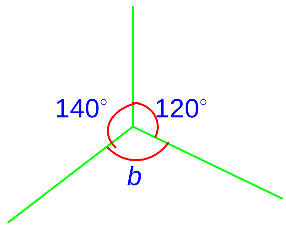

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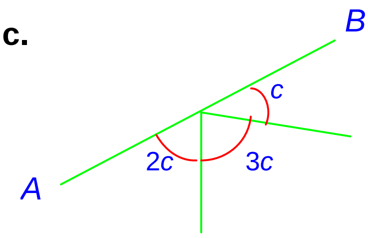

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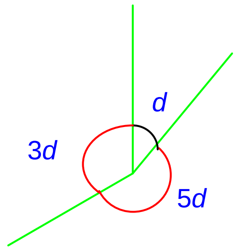

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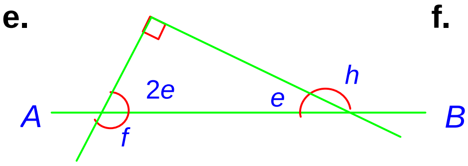
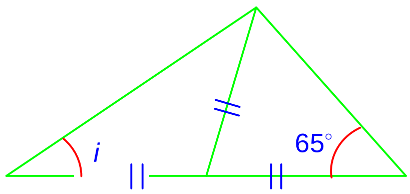


f.



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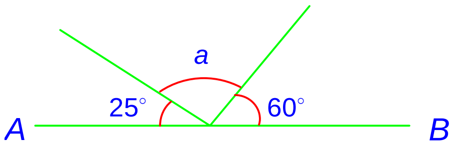
Solution to question 1

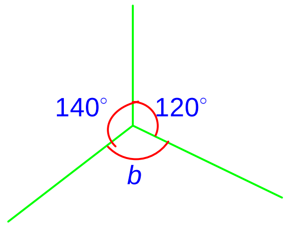
a.



Angles on a straight line add up to 180°

$$25^\circ + a + 60^\circ = 180^\circ$$
$$a = 180^\circ - 25^\circ - 60^\circ$$
$$= 95^\circ$$

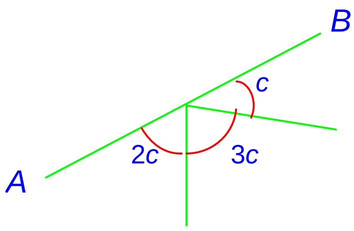
b.



Angles at a point add up to 360°

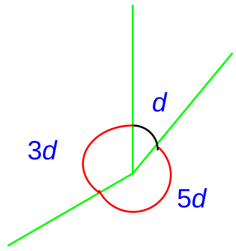
$$140^\circ + 120^\circ + b = 360^\circ$$
$$b = 360^\circ - 140^\circ - 120^\circ$$
$$= 100^\circ$$

c.



$$2c + 3c + c = 180^\circ$$
$$6c = 180^\circ$$
$$c = 30^\circ$$

d.



$$3d + d + 5d = 360^\circ$$
$$9d = 360^\circ$$
$$d = 40^\circ$$

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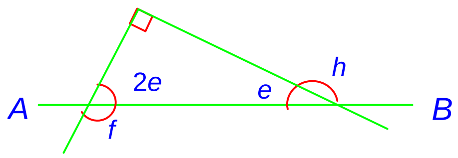
e.



Angles in a triangle add up to 180°.

$$90^\circ + 2e + e = 180^\circ$$
$$3e = 90^\circ$$
$$e = 30^\circ$$
$$f + 2e = 180^\circ$$
$$f + 2(30^\circ) = 180^\circ$$
$$f + 60^\circ = 180^\circ$$
$$f = 120^\circ$$

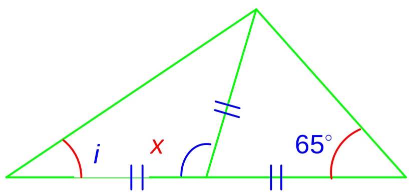
The opposite exterior angle is equal to the two opposite interior angles.

$$h = 2e + 90^\circ$$
$$= 2(30^\circ) + 90^\circ$$
$$= 150^\circ$$

or

$$e + h = 180^\circ$$
$$30^\circ + h = 180^\circ$$
$$h = 150^\circ$$

f.



Add an angle x to the diagram

The triangles are isosceles so

$x = 2(65^\circ)$ The opposite exterior angle is equal to the two opposite interior angles.

Therefore

$$i = \frac{180^\circ - 130^\circ}{2}$$
$$= 25^\circ$$

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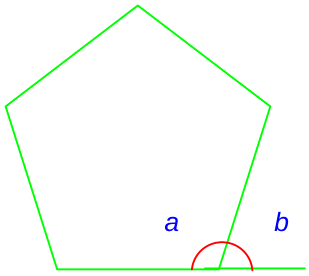
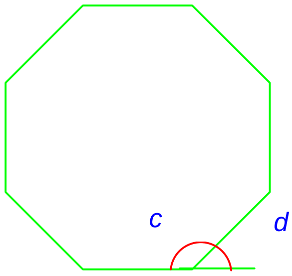
Question 2

Find angles marked for the following regular polygons.

a.



b.



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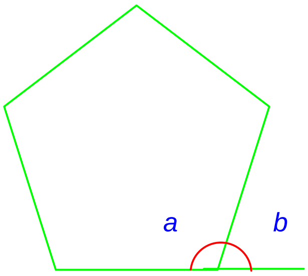
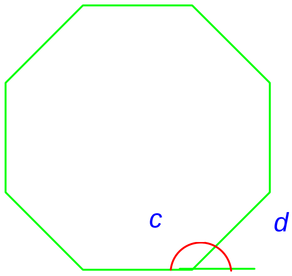
Solution to question 2

a.



b.



The sum of the exterior angles of any polygon is 360°

The sum of the exterior and interior angle is 180°

$$b = \frac{360^{\circ}}{5}$$
$$= 72^{\circ}$$
$$a + b = 180^{\circ}$$
$$a + 72^{\circ} = 180^{\circ}$$
$$a = 108^{\circ}$$

$$d = \frac{360^{\circ}}{8}$$
$$= 45^{\circ}$$
$$c + d = 180^{\circ}$$
$$c + 45^{\circ} = 180^{\circ}$$
$$c = 135^{\circ}$$

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Question 3

Calculate the number of sides of a regular polygon whose interior angles are each 150°.

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Solution to question 3

The sum of the exterior and interior angle is 180°

$$\begin{aligned} i + e &= 180^\circ \\ 150^\circ + e &= 180^\circ \\ e &= 30^\circ \end{aligned}$$

The sum of the exterior angles of any polygon is 360°

The number of sides is $\frac{360^\circ}{30^\circ} =$ 12 sides

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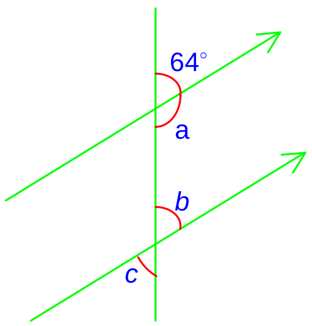
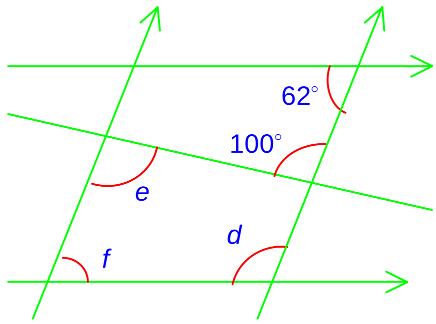
Question 4

In the following questions find the angles marked with letters, giving reasons for your answers.

a.



b.



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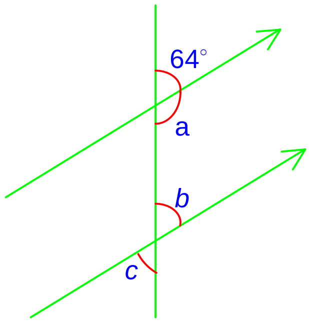
Solution to question 4

a.



$a + 64^\circ = 180^\circ$

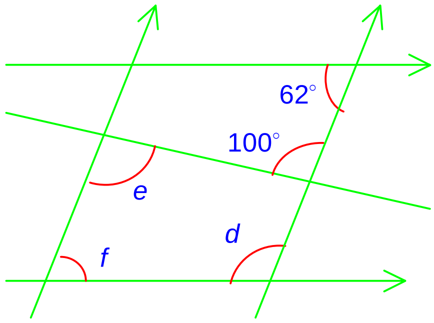
$a = 180^\circ - 64^\circ$

$a = 116^\circ$

$b = 64^\circ$ (corresponding angles)

$c = 64^\circ$ (vertically opposite angles)

b.



$e = 100^\circ$ (alternate angles)

$d + 62^\circ = 180^\circ$ (allied angles)

$d = 118^\circ$

$f + d = 180^\circ$ (allied angles)

$f + 118^\circ = 180^\circ$

$f = 62^\circ$

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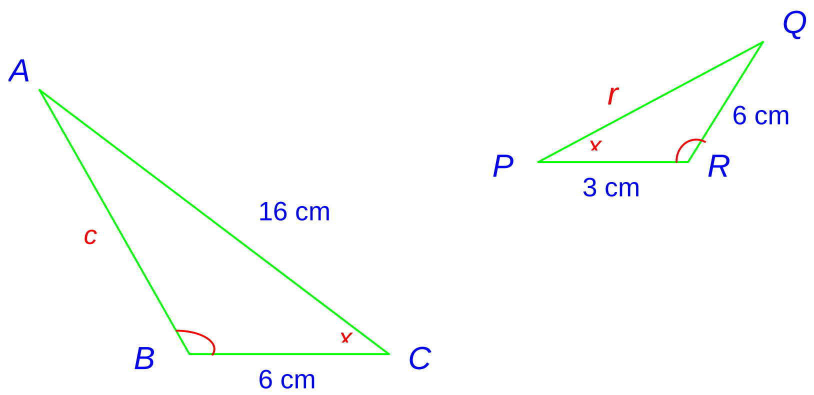
Question 5

Show that triangles ABC and PQR are similar and find the sides marked with letters.



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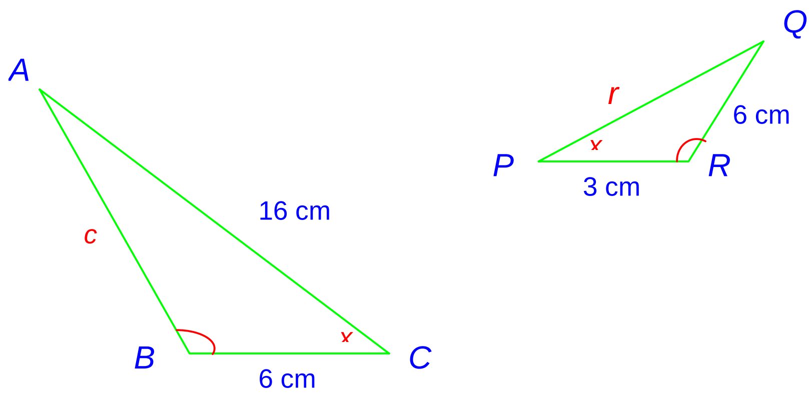
Solution to question 5

We have to show that the two angles are equal in both triangles.



$$\hat{B} = \hat{R}$$
$$\hat{C} = \hat{P}$$
$$\Rightarrow \hat{A} = \hat{Q}$$

Hence ΔABC is similar to ΔPQR

$$\text{Now } \frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$
$$\frac{c}{6} = \frac{6}{3} = \frac{16}{r}$$

Hence $\frac{c}{6} = \frac{6}{3} \Rightarrow c = \frac{6 \times 6}{3} = 18\,\text{cm}$

and $\frac{6}{3} = \frac{16}{r} \Rightarrow \frac{r}{16} = \frac{3}{6} \Rightarrow r = \frac{3 \times 16}{6} = 8\,\text{cm}$

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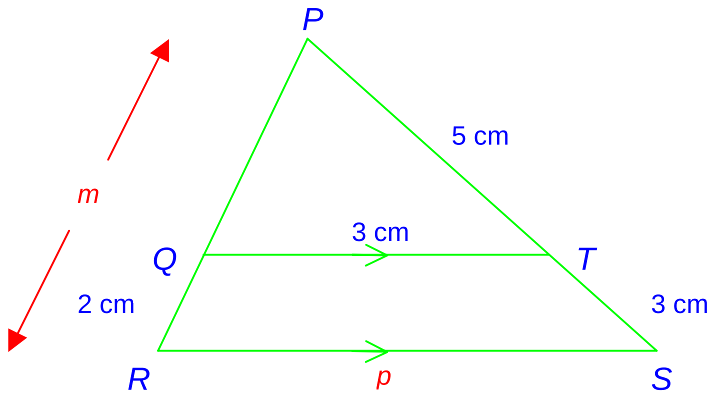
Question 6

Show that triangles PQT and PRS are similar and find the sides marked with letters.



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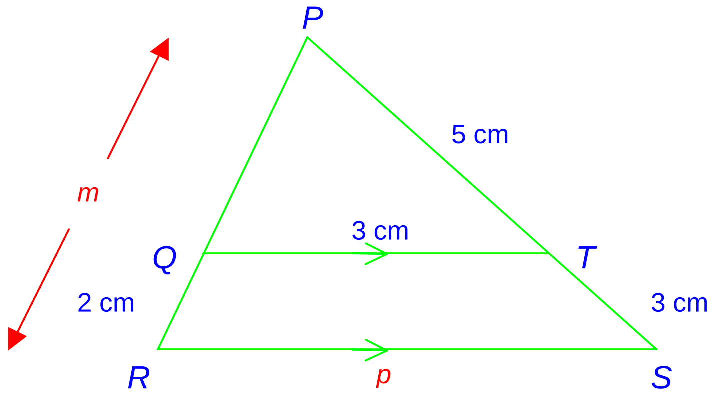
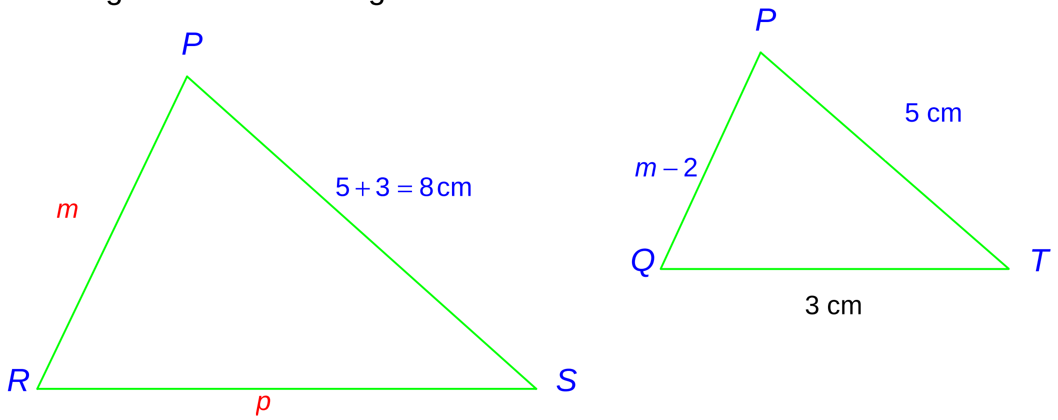
Solution to question 6



Split the diagram into two triangles



Now considering ΔPRS and ΔPQT
we have $\hat{P} = \hat{P}$ (same angle in both triangles), $\hat{R} = \hat{Q}$ (corresponding angles) and therefore $\hat{S} = \hat{T}$. Therefore ΔPRS and ΔPQT are similar.

Now $\dfrac{PR}{PQ} = \dfrac{RS}{QT} = \dfrac{SP}{TP} \Rightarrow \dfrac{m}{m-2} = \dfrac{p}{3} = \dfrac{8}{5}$

Hence $\dfrac{m}{m-2} = \dfrac{8}{5} \Rightarrow 5m = 8(m-2) \Rightarrow 5m = 8m - 16 \Rightarrow 3m = 16 \Rightarrow m = \dfrac{16}{3}$ cm

and $\dfrac{p}{3} = \dfrac{8}{5} \Rightarrow p = \dfrac{8 \times 3}{5} = \dfrac{24}{5}$ cm

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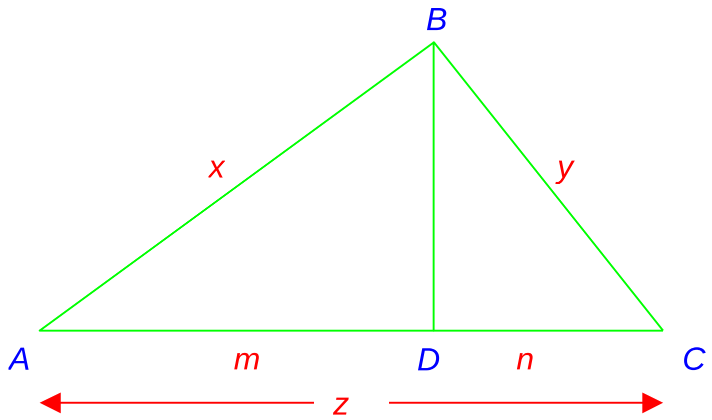
Question 7

In the diagram $A\hat{B}C = A\hat{D}B = 90^\circ$, $AD = m$ and $DC = n$.

a. Use similar triangles to show that $x^2 = mz$

b. Find a similar expression for y^2

c. Add the expressions for x^2 and y^2 and hence prove Pythagoras' theorem.



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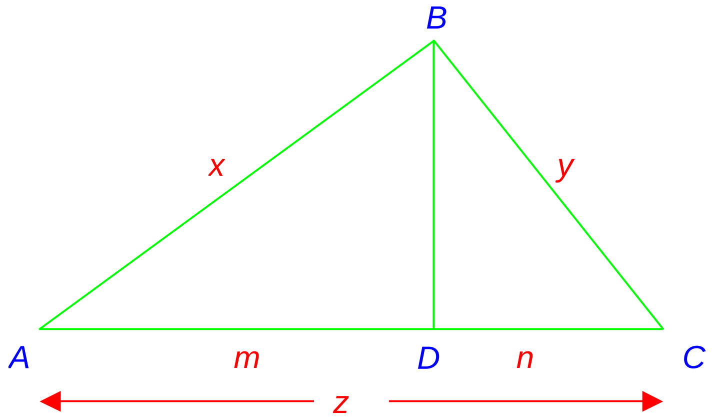
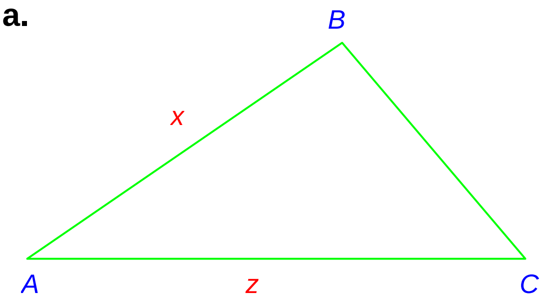
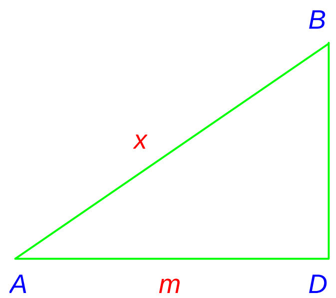
Solution to question 7



a.

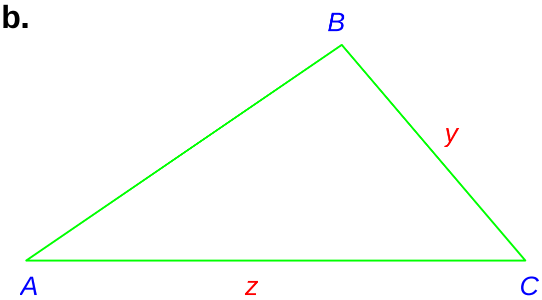


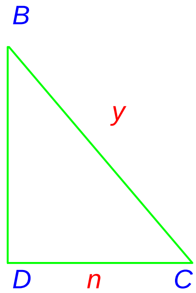


In ΔABC and ΔADB
$\hat{A} = \hat{A}$ (the same angle in both triangles), $\hat{B} = \hat{D} = 90^\circ$ hence $\hat{C} = \hat{B}$.
Therefore ΔABC and ΔADB are similar.

$$\frac{AB}{AD} = \frac{BC}{DB} = \frac{CA}{BA} \Rightarrow \frac{x}{m} = \frac{z}{x} \Rightarrow x^2 = mz$$

b.





In ΔABC and ΔBCD
$\hat{C} = \hat{C}$ (the same angle in both triangles), $\hat{B} = \hat{D} = 90^\circ$ hence $\hat{A} = \hat{B}$.
Therefore ΔABC and ΔBCD are similar.

$$\frac{CB}{CD} = \frac{BA}{DB} = \frac{AC}{BC} \Rightarrow \frac{y}{n} = \frac{z}{y} \Rightarrow y^2 = nz$$

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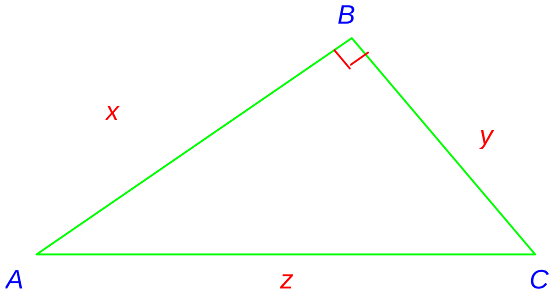
c. Now $x^2 + y^2 = mz + nz$

$$= (m+n)z$$

$$= z^2$$



Hence $z^2 = x^2 + y^2$, which is Pythagoras' theorem for ΔABC.

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Question 8

The volumes of two similar objects are 64 cm^3 and 343 cm^3 respectively.

a. Find the length ratio.

b. Find the ratio of the surface areas.

c. Given that the larger object has a surface area of 105 cm^2, find the surface area of the smaller object.

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Solution to question 8

a. The ratio of the volumes is $\frac{64}{343}$

Therefore the length ratio is $\sqrt[3]{\frac{64}{343}} = \frac{4}{7}$

b. The area ratio is $\left(\frac{4}{7}\right)^2 = \frac{16}{49}$

c. The surface area of the smaller object is

$$\frac{16}{\cancel{49}^{7}} \times \cancel{105}^{15} = \frac{240}{7} = 34\frac{2}{7}\text{cm}^2$$

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