I.G.C.S.E. Functions & Vectors

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Question 1

The following functions f , g and h are defined as follows:

$$f : x \to 4x - 1, \quad g : x \to \frac{x^2 - 2}{3}, \quad h : x \to \frac{4}{x}.$$

Find

a. $f(7)$, $f(-7)$, $f\left(\tfrac{1}{7}\right)$.

b. $g(4)$, $g(-3)$, $g\left(\tfrac{1}{2}\right)$.

c. $h(10)$, $h(-4)$, $h\left(-\tfrac{1}{8}\right)$

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Solution to question 1

a. $f : x \to 4x - 1$

$f\left(7\right) = 4\left(7\right) - 1 = 28 - 1 = 27$

$f\left(-7\right) = 4\left(-7\right) - 1 = -28 - 1 = -29$

$f\left(\tfrac{1}{7}\right) = 4\left(\tfrac{1}{7}\right) - 1 = \tfrac{4}{7} - 1 = -\tfrac{3}{7}$

b. $g : x \to \dfrac{x^2 - 2}{3}$

$g(4) = \dfrac{\left(4\right)^2 - 2}{3} = \dfrac{16 - 2}{3} = \dfrac{14}{3} = 4\dfrac{2}{3}$

$g(-3) = \dfrac{\left(-3\right)^2 - 2}{3} = \dfrac{9 - 2}{3} = \dfrac{7}{3} = 2\dfrac{1}{3}$

$g\left(\tfrac{1}{2}\right) . g(\tfrac{1}{2}) = \dfrac{\left(\tfrac{1}{2}\right)^2 - 2}{3} = \dfrac{\tfrac{1}{4} - 2}{3} = \dfrac{-\tfrac{7}{4}}{3} = -\dfrac{7}{12}$

c. $h : x \to \dfrac{4}{x}$

$h(10) = \dfrac{4}{\left(10\right)} = \dfrac{2}{5}$

$h(-4) = \dfrac{4}{\left(-4\right)} = -1$

$h\left(-\tfrac{1}{8}\right)\, h(-\tfrac{1}{8}) = \dfrac{4}{\left(-\tfrac{1}{8}\right)} = -32$

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Question 2

For the following functions $f : x \to 2 - 3x$ and $g : x \to x^2 - 2x$, find

a. the value of x such that $f(x) = -3$

b. the values of x such that $g(x) = 3$

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Solution to question 2

a. $f : x \to 2-3x$

$$f(x) = -3 \Rightarrow 2-3x = -3$$
$$-3x = -5$$
$$x = \frac{5}{3}$$

b. $g : x \to x^2 - 2x$

$$g(x) = 3 \Rightarrow \quad x^2 - 2x = 3$$
$$x^2 - 2x - 3 = 0$$

product = -3
sum = -2
factors -3, 1

$$x^2 - 3x + x - 3 = 0$$
$$x(x-3) + (x-3) = 0$$
$$(x-3)(x+1) = 0$$
$$\Rightarrow x = 3 \text{ or } x = -1$$

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Question 3

Given that the function f is defined as $f : x \rightarrow ax + b$, where *a* and *b* are constants. If $f\left(2\right) = 3$ and $f\left(-4\right) = 9$.

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Solution to question 3

$f : x \to ax + b$

$f(2) = 3 \Rightarrow 2a + b = 3 \ldots 1.$

$f(-4) = 9 \Rightarrow -4a + b = 9 \ldots 2.$

Solving equations 1 and 2 simultaneously we have

$$\begin{aligned} 1-2 \qquad 2a + b &= 3 \ldots 1. \\ -4a + b &= 9 \ldots 2. \\ 6a \qquad &= -6 \end{aligned}$$

$\Rightarrow a = -1$

Substitute $a = -1$ into equation 1 we have

$$\begin{aligned} 2(-1) + b &= 3 \\ -2 + b &= 3 \\ b &= 5 \end{aligned}$$

Check in equation 2 we have $-4(-1) + (5) = 4 + 5 = 9$

Hence $a = -1$ and $b = 5$.

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Question 4

Given the functions $l(x) = 3x - 2$, $m(x) = 2x^2 + 4$ and $n(x) = \frac{4}{x}$ find

a. $lm(x)$ **b.** $ml(x)$ **c.** $lmn(x)$ **d.** $lmn(-2)$.

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Solution to question 4

$l(x)=3x-2$, $m(x)=2x^2+4$ and $n(x)=\dfrac{4}{x}$

a. $lm(x)=l\left(2x^2+4\right)=3\left(2x^2+4\right)-2=6x^2+12-2=6x^2+10=2\left(3x^2+5\right)$

b. $ml(x)=m(3x-2)=2(3x-2)^2+4=2\left(9x^2-12x+4\right)+4$

$=18x^2-24x+8+4=18x^2-24x+12=6\left(3x^2-4x+2\right)$

c. $lmn(x)=lm\left(\dfrac{4}{x}\right)=l\left[2\left(\dfrac{4}{x}\right)^2+4\right]=l\left(\dfrac{32}{x}+4\right)=3\left(\dfrac{32}{x}+4\right)-2=\dfrac{96}{x}+12-2$

$=\dfrac{96}{x}+10=\dfrac{96+10x}{x}=\dfrac{2(44+5x)}{x}$

d. $lmn(-2)=\dfrac{2\left[44+5(-2)\right]}{(-2)}=-34$

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Question 5

Given the functions $f : x \rightarrow 7x - 3$ and $g : x \rightarrow \frac{3x-5}{4}$ find the inverse functions $f^{-1}(x)$ and $g^{-1}(x)$.

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Solution to question 5

$f : x \to 7x - 3$

Let $\quad y = 7x - 3$

$y + 3 = 7x$

$\dfrac{y+3}{7} = x$

Interchange the x and y to give $f^{-1}(x) = \dfrac{x+3}{7}$

$g : x \to \dfrac{3x-5}{4}$

Let $\quad y = \dfrac{3x-5}{4}$

$4y = 3x - 5$

$4y + 5 = 3x$

$\dfrac{4y+5}{3} = x$

Interchange the x and y to give $g^{-1}(x) = \dfrac{4x+5}{3}$

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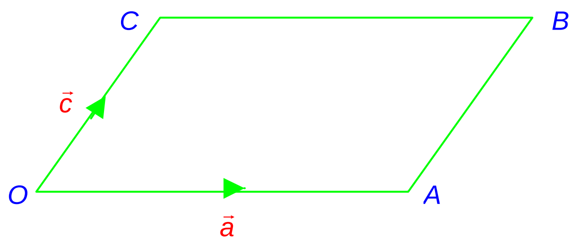
Question 6

OABC is a parallelogram where $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$.



Find in terms of $\vec{a}$ and $\vec{c}$ only

a. $\overrightarrow{AB}$ **b.** $\overrightarrow{BC}$ **c.** $\overrightarrow{CA}$ **d.** $\overrightarrow{BO}$.

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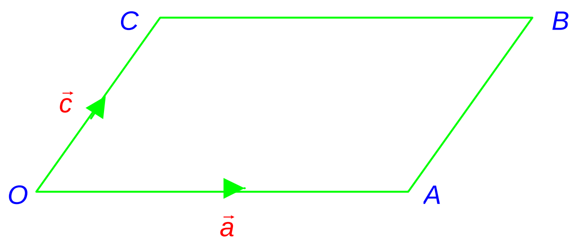
Solution to question 6

OABC is a parallelogram where $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$.



a. $\overrightarrow{AB} = \overrightarrow{OC} = \vec{c}$

b. $\overrightarrow{BC} = -\overrightarrow{OA} = -\vec{a}$

c. $\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA} = -\vec{c} + \vec{a} = \vec{a} - \vec{c}$

d. $\overrightarrow{BO} = \overrightarrow{BC} + \overrightarrow{CO} = -\vec{a} - \vec{c}$

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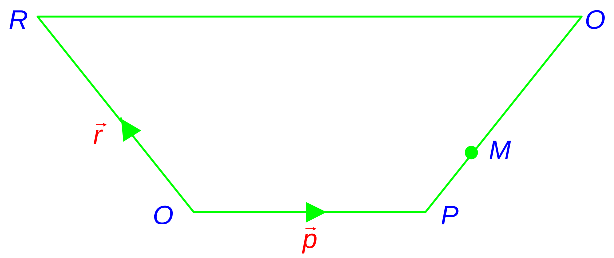
Question 7

$OPQR$ is a trapezium where $RQ = 3OP$ and M is the point on PQ such that $PM = 2MQ$.



Given that $\overrightarrow{OP} = \vec{p}$ and $\overrightarrow{OR} = \vec{r}$ find in terms of $\vec{p}$ and $\vec{r}$ only

a. $\overrightarrow{PR}$ **b.** $\overrightarrow{RQ}$ **c.** $\overrightarrow{PQ}$ **d.** $\overrightarrow{PM}$ **e.** $\overrightarrow{MR}$

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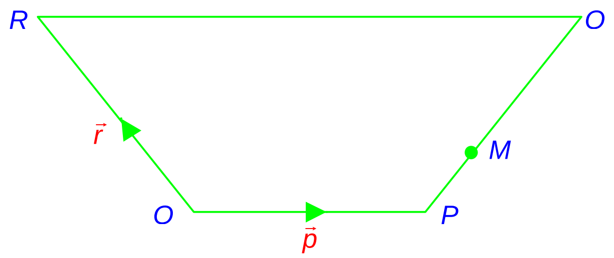
Solution to question 7

OPQR is a trapezium where $RQ = 3OP$ and M is the point on *PQ* such that $PM = 2MQ$.



$\overrightarrow{OP} = \vec{p}$ and $\overrightarrow{OR} = \vec{r}$

a. $\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -\vec{p} + \vec{r} = \vec{r} - \vec{p}$

b. $\overrightarrow{RQ} = 3\overrightarrow{OP} = 3\vec{p}$

c. $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OR} + \overrightarrow{RQ} = -\vec{p} + \vec{r} + 3\vec{p} = 2\vec{p} + \vec{r}$

d. $\overrightarrow{PM} = \frac{1}{3}\overrightarrow{PQ} = \frac{1}{3}(2\vec{p} + \vec{r}) = \frac{2}{3}\vec{p} + \frac{1}{3}\vec{r}$

e. $\overrightarrow{MR} = \overrightarrow{MQ} + \overrightarrow{QR} = \frac{2}{3}\overrightarrow{PQ} + \overrightarrow{QR} = \frac{2}{3}(2\vec{p} + \vec{r}) - 3\vec{p} = \frac{4}{3}\vec{p} + \frac{2}{3}\vec{r} - 3\vec{p} = \frac{2}{3}\vec{r} - \frac{5}{3}\vec{p}$

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Question 8

Given the following vectors $\vec{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$. Find the following giving your answers exactly, wherever appropriate.

a. $\vec{a} + \vec{b}$ **b.** $\vec{a} - \vec{b}$ **c.** $3\vec{a} - 2\vec{b}$ **d.** $\left|\vec{b}\right|$ **e.** $\left|3\vec{a} - 2\vec{b}\right|$

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Solution to question 8

$\vec{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$.

a. $\vec{a} + \vec{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

b. $\vec{a} - \vec{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

c. $3\vec{a} - 2\vec{b} = 3\begin{pmatrix} -2 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -6 \\ 9 \end{pmatrix} - \begin{pmatrix} 8 \\ -10 \end{pmatrix} = \begin{pmatrix} -14 \\ 19 \end{pmatrix}$

d. $\left|\vec{b}\right| = \sqrt{4^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41}$

e. $\left|3\vec{a} - 2\vec{b}\right| = \sqrt{(-14)^2 + 19^2} = \sqrt{196 + 361} = \sqrt{557}$

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