

## $O A B C$ is a field.

$A$ is 88 metres due North of $O$.
$B$ is 146 metres from $O$ on a bearing of $040^{\circ}$.
$C$ is equidistant from $A$ and from $B$. The bearing of $C$ from $O$ is $098^{\circ}$.
(a) Using a scale of 1 centimetre to represent 10 metres, make an accurate scale drawing of the field $O A B C$, by
(i) constructing the triangle $O A B$,
(ii) drawing the locus of points equidistant from $A$ and from $B$,
(iii) completing the scale diagram of $O A B C$.
(b) Use your scale drawing to write down
(i) the distance $O C$ correct to the nearest metre,
(ii) the size of angle $O A B$ correct to the nearest degree.
(c) Find the bearing of $A$ from $B$.
(d) A donkey in the field is not more than 40 metres from $C$ and is closer to $B$ than to $A$. Shade the area where the donkey could be and label it $D$.
(e) A horse in the field is not more than 20 metres from the side $A B$ and is closer to $A$ than to $B$. Shade the area where the horse could be and label it $H$.


A sphere, centre $C$, rests on horizontal ground at $A$ and touches a vertical wall at $D$.
A straight plank of wood, $G B W$, touches the sphere at $B$, rests on the ground at $G$ and against the wall at $W$. The wall and the ground meet at $X$.
Angle $W G X=42^{\circ}$.
(a) Find the values of $a, b, c, d$ and $e$ marked on the diagram.
(b) Write down one word which completes the following sentence.
'Angle $C G A$ is $21^{\circ}$ because triangle GBC and triangle GAC are $\ldots \ldots \ldots \ldots \ldots \ldots .$. .
(c) The radius of the sphere is 54 cm .
(i) Calculate the distance $G A$. Show all your working.
(ii) Show that $G X=195 \mathrm{~cm}$ correct to the nearest centimetre.
(iii) Calculate the length of the plank $G W$.
(iv) Find the distance $B W$.


In quadrilateral $A B C D, A B=77 \mathrm{~m}, B C=120 \mathrm{~m}, C D=60 \mathrm{~m}$ and diagonal $A C=55 \mathrm{~m}$. Angle $C A D=45^{\circ}$, angle $B A C=x^{\circ}$ and angle $A D C=y^{\circ}$.
(a) Calculate the value of $x$.
(b) Calculate the value of $y$.
(c) The bearing of $D$ from $A$ is $090^{\circ}$.

Find the bearing of
(i) $A$ from $C$,
(ii) $B$ from $A$.

NOT TO SCALE

$A B C D$ is a cyclic quadrilateral.
$A B=9.5 \mathrm{~cm}, B C=11.1 \mathrm{~cm}$, angle $A B C=70^{\circ}$ and angle $C A D=37^{\circ}$.
(a) Calculate the length of $A C$.
(b) Explain why angle $A D C=110^{\circ}$.
(c) Calculate the length of $A D$.
(d) A point $E$ lies on the circle such that triangle $A C E$ is isosceles, with $E A=E C$.
(i) Write down the size of angle $A E C$.
(ii) Calculate the area of triangle $A C E$.


NOT TO
SCALE

The diagram shows a trapezium $A B C D$.
$A B=12 \mathrm{~cm}, D C=9 \mathrm{~cm}$ and the perpendicular distance between these parallel sides is 7 cm . $A D=B C$.
(a) Approximately halfway down your page, draw a line $A B$ of length 12 cm .
(b) Using a straight edge and compasses only, construct the perpendicular bisector of $A B$.
(c) Complete an accurate drawing of the trapezium $A B C D$.
(d) Measure angle $A B C$, giving your answer correct to the nearest degree.
(e) Use trigonometry to calculate angle $A B C$.

Show all your working and give your answer correct to 1 decimal place.
(f) On your diagram,
(i) draw the locus of points inside the trapezium which are 5 cm from $D$,
(ii) using a straight edge and compasses only, construct the locus of points equidistant from $D A$ and from $D C$,
(iii) shade the region inside the trapezium containing points which are less than 5 cm from $D$ and nearer to $D A$ than to $D C$.


NOT TO
$A, B, C$ and $D$ lie on a circle.
$A C$ and $B D$ intersect at $X$.
Angle $A B X=55^{\circ}$ and angle $A X B=92^{\circ}$.
$B X=26.8 \mathrm{~cm}, A X=40.3 \mathrm{~cm}$ and $X C=20.1 \mathrm{~cm}$.
(i) Calculate the area of triangle $A X B$.

You must show your working.
(ii) Calculate the length of $A B$.

You must show your working.
(iii) Write down the size of angle $A C D$. Give a reason for your answer.
(iv) Find the size of angle $B D C$.
(v) Write down the geometrical word which completes the statement
"Triangle $A X B$ is to triangle $D X C$."
(vi) Calculate the length of $X D$. You must show your working.
(b)


NOT TO
SCALE

In the diagram $P Q$ is parallel to $R S$.
$P S$ and $Q R$ intersect at $X$.
$P X=y \mathrm{~cm}, Q X=(y+2) \mathrm{cm}, R X=(2 y-1) \mathrm{cm}$ and $S X=(y+1) \mathrm{cm}$.
(i) Show that $y^{2}-4 y-2=0$.
(ii) Solve the equation $y^{2}-4 y-2=0$.

Show all your working and give your answers correct to two decimal places.
(iii) Write down the length of $R X$.


A circle, centre $O$, touches all the sides of the regular octagon $A B C D E F G H$ shaded in the diagram.
The sides of the octagon are of length 12 cm .
$B A$ and $G H$ are extended to meet at $P . H G$ and $E F$ are extended to meet at $Q$.
(a) (i) Show that angle $B A H$ is $135^{\circ}$.
(ii) Show that angle $A P H$ is $90^{\circ}$.
(b) Calculate
(i) the length of PH ,
(ii) the length of $P Q$,
(iii) the area of triangle $A P H$,
(iv) the area of the octagon.
(c) Calculate
(i) the radius of the circle,
(ii) the area of the circle as a percentage of the area of the octagon.


The diagram shows three straight horizontal roads in a town, connecting points $P, A$ and $B$. $P B=250 \mathrm{~m}$, angle $A P B=23^{\circ}$ and angle $B A P=126^{\circ}$.
(a) Calculate the length of the $\operatorname{road} A B$.

$$
\text { Answer(a) } A B=
$$

(b) The bearing of $A$ from $P$ is $303^{\circ}$.

Find the bearing of
(i) $B$ from $P$,
(ii) $A$ from $B$.

$A B C D$ is a quadrilateral and $B D$ is a diagonal.
$A B=26 \mathrm{~cm}, B D=24 \mathrm{~cm}$, angle $A B D=40^{\circ}$, angle $C B D=40^{\circ}$ and angle $C D B=30^{\circ}$.
(a) Calculate the area of triangle $A B D$.

$$
\text { Answer(a) ................................ } \mathrm{cm}^{2}
$$

(b) Calculate the length of $A D$.

> Answer(b)
cm [4]
(c) Calculate the length of $B C$.

> Answer(c)
$\qquad$ cm
(d) Calculate the shortest distance from the point $C$ to the line $B D$.


NOT TO
SCALE

The diagram shows some straight line distances between Auckland $(A)$, Hamilton $(H)$, Tauranga $(T)$ and Rotorua $(R)$. $A T=180 \mathrm{~km}, A H=115 \mathrm{~km}$ and $H T=90 \mathrm{~km}$.
(a) Calculate angle $H A T$.

Show that this rounds to $25.0^{\circ}$, correct to 3 significant figures.

Answer(a)
(b) The bearing of $H$ from $A$ is $150^{\circ}$.

Find the bearing of
(i) $T$ from $A$,
Answer(b)(i)
(ii) $A$ from $T$.
(c) Calculate how far $T$ is east of $A$.
Answer(c)

km
(d) Angle $T H R=30^{\circ}$ and angle $H R T=70^{\circ}$.

Calculate the distance $T R$.
(e) On a map the distance representing $H T$ is 4.5 cm .

The scale of the map is $1: n$.

Calculate the value of $n$.
$9 \quad$ (a)


The lines $A B$ and $C D E$ are parallel.
$A D$ and $C B$ intersect at $X$.
$A B=9 \mathrm{~cm}, C D=6 \mathrm{~cm}$ and $D X=3 \mathrm{~cm}$.
(i) Complete the following statement.

Triangle $A B X$ is $\qquad$ to triangle $D C X$.
(ii) Calculate the length of $A X$.

$$
\text { Answer(a)(ii) } A X=
$$

(iii) The area of triangle $D C X$ is $6 \mathrm{~cm}^{2}$.

Calculate the area of triangle $A B X$.
Answer(a)(iii)

$$
\mathrm{cm}^{2}
$$

(iv) Angle $B A X=x^{\circ}$ and angle $A B X=y^{\circ}$.

Find angle $A X B$ and angle $X D E$ in terms of $x$ and/or $y$.

$$
\begin{aligned}
\text { Answer(a)(iv) Angle } A X B & =\text {........................ } \\
\text { Angle } X D E & =\text {......................... }
\end{aligned}
$$

(b)

$P, Q, R$ and $S$ lie on a circle, centre $O$.
Angle $O P S=42^{\circ}$ and angle $P R Q=35^{\circ}$.

## Calculate

(i) angle $P O S$,

Answer(b)(i) Angle $P O S=$
(ii) angle $P R S$,

Answer(b)(ii) Angle $P R S=$
(iii) angle $S P Q$,

Answer(b)(iii) Angle $S P Q=$
(iv) angle $P S Q$.

Answer(b)(iv) Angle $P S Q=$
(c) The interior angle of a regular polygon is 8 times as large as the exterior angle.

Calculate the number of sides of the polygon.


NOT TO SCALE

In the quadrilateral $A B C D, A B=3 \mathrm{~cm}, A D=11 \mathrm{~cm}$ and $D C=8 \mathrm{~cm}$.
The diagonal $A C=5 \mathrm{~cm}$ and angle $B A C=90^{\circ}$.
Calculate
(a) the length of $B C$,
(b) angle $A C D$,
(c) the area of the quadrilateral $A B C D$.


NOT TO
SCALE

The diagram shows two triangles $A C B$ and $A P Q$.
Angle $P A Q=$ angle $B A C$ and angle $A Q P=$ angle $A B C$.
$A B=4 \mathrm{~cm}, B C=3.6 \mathrm{~cm}$ and $A Q=3 \mathrm{~cm}$.
(i) Complete the following statement.

Triangle $A C B$ is to triangle $A P Q$.
(ii) Calculate the length of $P Q$.
(iii) The area of triangle $A C B$ is $5.6 \mathrm{~cm}^{2}$.

Calculate the area of triangle $A P Q$.
(b)

$R, H, S, T$ and $U$ lie on a circle, centre $O$.
$H T$ is a diameter and $M N$ is a tangent to the circle at $T$.
Angle $R T M=61^{\circ}$.
Find
(i) angle RTH ,

$$
\text { Answer(b)(i) Angle } R T H=
$$

(ii) angle $R H T$,

$$
\text { Answer(b)(ii) Angle } R H T=
$$

(iii) angle $R S T$,
Answer(b)(iii) Angle RST =
(iv) angle RUT.

Answer(b)(iv) Angle $R U T=$ $\qquad$
(c) $A B C D E F$ is a hexagon.

The interior angle $B$ is $4^{\circ}$ greater than interior angle $A$.
The interior angle $C$ is $4^{\circ}$ greater than interior angle $B$, and so on, with each of the next interior angles $4^{\circ}$ greater than the previous one.
(i) By how many degrees is interior angle $F$ greater than interior angle $A$ ?
Answer(c)(i)
(ii) Calculate interior angle $A$.


The diagram shows a sketch of the net of a solid tetrahedron (triangular prism).
The right-angled triangle $A B C$ is its base.
$A C=8 \mathrm{~cm}, B C=6 \mathrm{~cm}$ and $A B=10 \mathrm{~cm} . F C=C E=5 \mathrm{~cm}$.
(a) (i) Show that $B E=\sqrt{ } 61 \mathrm{~cm}$.
(ii) Write down the length of $D B$.
(iii) Explain why $D A=\sqrt{ } 89 \mathrm{~cm}$.
(b) Calculate the size of angle DBA.
(c) Calculate the area of triangle DBA.
(d) Find the total surface area of the solid.
(e) Calculate the volume of the solid.
[The volume of a tetrahedron is $\frac{1}{3}$ (area of the base) $\times$ perpendicular height.]


Felipe $(F)$ stands 17 metres from a bridge $(B)$ and 32 metres from a tree $(T)$.
The points $F, B$ and $T$ are on level ground and angle $B F T=40^{\circ}$.
(a) Calculate
(i) the distance $B T$,
(ii) the angle $B T F$.
(b) The bearing of $B$ from $F$ is $085^{\circ}$. Find the bearing of
(i) $T$ from $F$,
(ii) $F$ from $T$,
(iii) $B$ from $T$.
(c) The top of the tree is 30 metres vertically above $T$.

Calculate the angle of elevation of the top of the tree from $F$.

(a) During a soccer match a player runs from $A$ to $B$ and then from $B$ to $C$ as shown in the diagram. $A B=40 \mathrm{~m}, B C=45 \mathrm{~m}$ and $A C=70 \mathrm{~m}$.
(i) Show by calculation that angle $B A C=37^{\circ}$, correct to the nearest degree.
(ii) The bearing of $C$ from $A$ is $051^{\circ}$. Find the bearing of $B$ from $A$.
(iii) Calculate the area of triangle $A B C$.
(b) $x$ - and $y$-axes are shown in the diagram.
$\overrightarrow{A C}=\binom{p}{q}$, where $p$ and $q$ are measured in metres.
(i) Show that $p=54.4$.
(ii) Find the value of $q$.
(c) Another player is standing at $D$.
$B C=45 \mathrm{~m}$, angle $B C D=54^{\circ}$ and angle $D B C=32^{\circ}$.
Calculate the length of $B D$.

NOT TO
SCALE


The quadrilateral $P Q R S$ shows the boundary of a forest.
A straight 15 kilometre road goes due East from $P$ to $R$.
(a) The bearing of $S$ from $P$ is $030^{\circ}$ and $P S=7 \mathrm{~km}$.
(i) Write down the size of angle $S P R$.
(ii) Calculate the length of $R S$.
(b) Angle $R P Q=55^{\circ}$ and $Q R=14 \mathrm{~km}$.
(i) Write down the bearing of $Q$ from $P$.
(ii) Calculate the acute angle $P Q R$.
(iii) Calculate the length of $P Q$.
(c) Calculate the area of the forest, correct to the nearest square kilometre.


NOT TO SCALE

The diagram shows the positions of four cities in Africa, Windhoek $(W)$, Johannesburg $(J)$, Harari $(H)$ and Lusaka ( $L$ ).
$W L=1400 \mathrm{~km}$ and $W H=1600 \mathrm{~km}$.
Angle $L W H=13^{\circ}$, angle $H W J=36^{\circ}$ and angle $W J H=95^{\circ}$.
(a) Calculate the distance $L H$.
(b) Calculate the distance $W J$.
(c) Calculate the area of quadrilateral $W J H L$.
(d) The bearing of Lusaka from Windhoek is $060^{\circ}$.

Calculate the bearing of
(i) Harari from Windhoek,
(ii) Windhoek from Johannesburg.
(e) On a map the distance between Windhoek and Harari is 8 cm . Calculate the scale of the map in the form $1: n$.


The diagram shows the plan of a garden.
The garden is a trapezium with $A B=26$ metres, $D C=18$ metres and angle $D A B=80^{\circ}$.
A straight path from $B$ to $D$ has a length of 30 metres.
(a) (i) Using a scale of 1:200, draw an accurate plan of the garden.
(ii) Measure and write down the size of angle $A D B$ and the size of angle $D C B$.
(iii) A second path is such that all points on it are equidistant from $A B$ and from $A D$.

Using a straight edge and compasses only, construct this path on your plan.
(iv) A third path is such that all points on it are equidistant from $A$ and from $D$.

Using a straight edge and compasses only, construct this path on your plan.
(v) In the garden, vegetables are grown in the region which is nearer to $A B$ than to $A D$ and nearer to $A$ than to $D$.

Shade this region on your plan.
(b) Use trigonometry, showing all your working, to calculate
(i) angle $A D B$,
(ii) the length of $B C$,
(iii) the area of the garden.


To avoid an island, a ship travels 40 kilometres from $A$ to $B$ and then 60 kilometres from $B$ to $C$.
The bearing of $B$ from $A$ is $080^{\circ}$ and angle $A B C$ is $115^{\circ}$.
(a) The ship leaves $A$ at 1155 .

It travels at an average speed of $35 \mathrm{~km} / \mathrm{h}$.
Calculate, to the nearest minute, the time it arrives at $C$.
(b) Find the bearing of
(i) $A$ from $B$,
(ii) $C$ from $B$.
(c) Calculate the straight line distance $A C$.
(d) Calculate angle $B A C$.
(e) Calculate how far $C$ is east of $A$.

$A B C D E$ is a pentagon.
A circle, centre $O$, passes through the points $A, C, D$ and $E$.
Angle $E A C=36^{\circ}$, angle $C A B=78^{\circ}$ and $A B$ is parallel to $D C$.
(a) Find the values of $x, y$ and $z$, giving a reason for each.
(b) Explain why $E D$ is not parallel to $A C$.
(c) Find the value of angle EOC.
(d) $A B=A C$.

Find the value of angle $A B C$.


The diagram shows a toy boat.
$A C=16.5 \mathrm{~cm}, A B=19.5 \mathrm{~cm}$ and $P R=11 \mathrm{~cm}$.
Triangles $A B C$ and $P Q R$ are similar.
(i) Calculate $P Q$.
(ii) Calculate $B C$.

$$
\text { Answer(a)(ii) } B C=
$$

cm [3]
(iii) Calculate angle $A B C$.
(iv) The toy boat is mathematically similar to a real boat.

The length of the real boat is 32 times the length of the toy boat. The fuel tank in the toy boat holds 0.02 litres of diesel.

Calculate how many litres of diesel the fuel tank of the real boat holds.

Answer(a)(iv) ...................................... litres
(b)


The diagram shows a field $D E F G$, in the shape of a quadrilateral, with a footpath along the diagonal $D F$.
$D F=105 \mathrm{~m}$ and $F G=67 \mathrm{~m}$.
Angle $E D F=70^{\circ}$, angle $E F D=32^{\circ}$ and angle $D F G=143^{\circ}$.
(i) Calculate $D G$.
(ii) Calculate $E F$.
(a)


## NOT TO

 SCALE$A, B, C$ and $D$ are points on the circumference of a circle centre $O$. $A C$ is a diameter.
$B D=B C$ and angle $D B C=62^{\circ}$.
Work out the values of $w, x, y$ and $z$.
Give a reason for each of your answers.
$w=$............................... because ..... [2]
$x=$ because ..... [2]
$y=$ because ..... [2]
$z=$ because ..... [2]
(b)


[^0]

The diagram shows the positions of London $(L)$, Dubai $(D)$ and Colombo $(C)$.
(a) (i) Show that $L C$ is 8710 km correct to the nearest kilometre.

Answer(a)(i)
(ii) Calculate the angle $C L D$.


The diagram shows five straight roads.
$P Q=4.5 \mathrm{~km}, Q R=4 \mathrm{~km}$ and $P R=7 \mathrm{~km}$.
Angle $R P S=40^{\circ}$ and angle $P S R=85^{\circ}$.
(a) Calculate angle $P Q R$ and show that it rounds to $110.7^{\circ}$.

Answer(a)
(b) Calculate the length of the road $R S$ and show that it rounds to 4.52 km .

Answer(b)
(c) Calculate the area of the quadrilateral $P Q R S$.
[Use the value of $110.7^{\circ}$ for angle $P Q R$ and the value of 4.52 km for $R S$.]


[^0]:    (i) Write down $\overrightarrow{A B}$ as a column vector.

