4 Answer the whole of this question on a sheet of graph paper.

x	−4	−3	−2	−1	0	1	2	3	4
f(x)	−8	4.5	8	5.5	0	−5.5	−8	−4.5	8

(a) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 4 units on the y-axis, draw axes for $-4 \leqslant x \leqslant 4$ and $-8 \leqslant y \leqslant 8$.
Draw the curve $y = \mathrm{f}(x)$ using the table of values given above. [5]

(b) Use your graph to solve the equation $\mathrm{f}(x) = 0$. [2]

(c) On the same grid, draw $y = \mathrm{g}(x)$ for $-4 \leqslant x \leqslant 4$, where $\mathrm{g}(x) = x + 1$. [2]

(d) Write down the value of

(i) $\mathrm{g}(1)$,

(ii) $\mathrm{fg}(1)$,

(iii) $\mathrm{g}^{-1}(4)$,

(iv) the **positive** solution of $\mathrm{f}(x) = \mathrm{g}(x)$. [4]

(e) Draw the tangent to $y = \mathrm{f}(x)$ at $x = 3$. Use it to calculate an estimate of the gradient of the curve at this point. [3]

---

2 Answer all of this question on a sheet of graph paper.

(a) $\mathrm{f}(x) = x^2 - x - 3$.

x	−3	−2	−1	0	1	2	3	4
f(x)	p	3	−1	-3	q	−1	3	r

(i) Find the values of p, q and r. [3]

(ii) Draw the graph of $y = \mathrm{f}(x)$ for $-3 \leqslant x \leqslant 4$.
Use a scale of 1 cm to represent 1 unit on each axis. [4]

(iii) By drawing a suitable line, estimate the gradient of the graph at the point where $x = -1$. [3]

(b) $\mathrm{g}(x) = 6 - \dfrac{x^3}{3}$.

x	−2	−1	0	1	2	3
g(x)	8.67	u	v	5.67	3.33	−3

(i) Find the values of u and v. [2]

(ii) On the same grid as **part (a) (ii)** draw the graph of $y = \mathrm{g}(x)$ for $-2 \leqslant x \leqslant 3$. [4]

(c) (i) Show that the equation $\mathrm{f}(x) = \mathrm{g}(x)$ simplifies to $x^3 + 3x^2 - 3x - 27 = 0$. [1]

(ii) Use your graph to write down a solution of the equation $x^3 + 3x^2 - 3x - 27 = 0$. [1]

---

4 Answer the whole of this question on a sheet of graph paper.

The table gives values of $f(x) = 2^x$, for $-2 \leqslant x \leqslant 4$.

x	-2	-1	0	1	2	3	4
$f(x)$	p	0.5	q	2	4	r	16

(a) Find the values of p, q and r. [3]

(b) Using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis, draw the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 4$. [5]

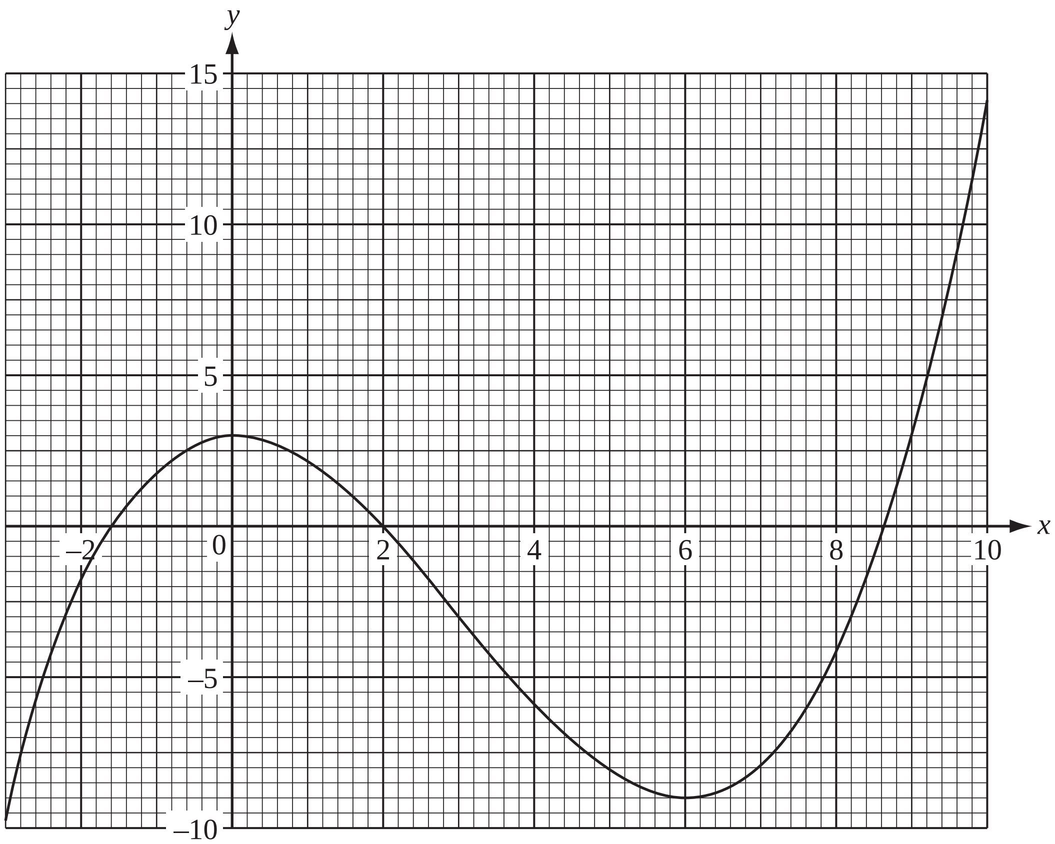
(c) Use your graph to solve the equation $2^x = 7$. [1]

(d) What value does $f(x)$ approach as x decreases? [1]

(e) By drawing a tangent, estimate the gradient of the graph of $y = f(x)$ when $x = 1.5$. [3]

(f) On the same grid draw the graph of $y = 2x + 1$ for $0 \leqslant x \leqslant 4$. [2]

(g) Use your graph to find the non-integer solution of $2^x = 2x + 1$. [2]



The diagram shows the accurate graph of $y = f(x)$.

(a) Use the graph to find

(i) f(0), [1]

(ii) f(8). [1]

(b) Use the graph to solve

(i) $f(x) = 0$, [2]

(ii) $f(x) = 5$. [1]

(c) k is an integer for which the equation $f(x) = k$ has exactly two solutions.

Use the graph to find the two values of k. [2]

(d) Write down the range of values of x for which the graph of $y = f(x)$ has a negative gradient. [2]

(e) The equation $f(x) + x - 1 = 0$ can be solved by drawing a line on the grid.

(i) Write down the equation of this line. [1]

(ii) How many solutions are there for $f(x) + x - 1 = 0$? [1]

8 Answer the whole of this question on a sheet of graph paper.
Use one side for your working and one side for your graphs.

Alaric invests \$100 at 4% per year **compound interest.**

(a) How many dollars will Alaric have after 2 years? [2]

(b) After x years, Alaric will have y dollars.
He knows a formula to calculate y.
The formula is $y = 100 \times 1.04^x$

x (Years)	0	10	20	30	40
y (Dollars)	100	p	219	q	480

Use this formula to calculate the values of p and q in the table. [2]

(c) Using a scale of 2 cm to represent 5 years on the x-axis and 2 cm to represent \$50 on the y-axis, draw an x-axis for $0 \leqslant x \leqslant 40$ and a y-axis for $0 \leqslant y \leqslant 500$.

Plot the five points in the table and draw a smooth curve through them. [5]

(d) Use your graph to estimate

(i) how many dollars Alaric will have after 25 years, [1]

(ii) how many years, to the nearest year, it takes for Alaric to have \$200. [1]

(e) Beatrice invests \$100 at 7% per year **simple interest**.

(i) Show that after 20 years Beatrice has \$240. [2]

(ii) How many dollars will Beatrice have after 40 years? [1]

(iii) On the **same grid**, draw a graph to show how the \$100 which Beatrice invests will increase during the 40 years. [2]

(f) Alaric first has more than Beatrice after n years.
Use your graphs to find the value of n. [1]

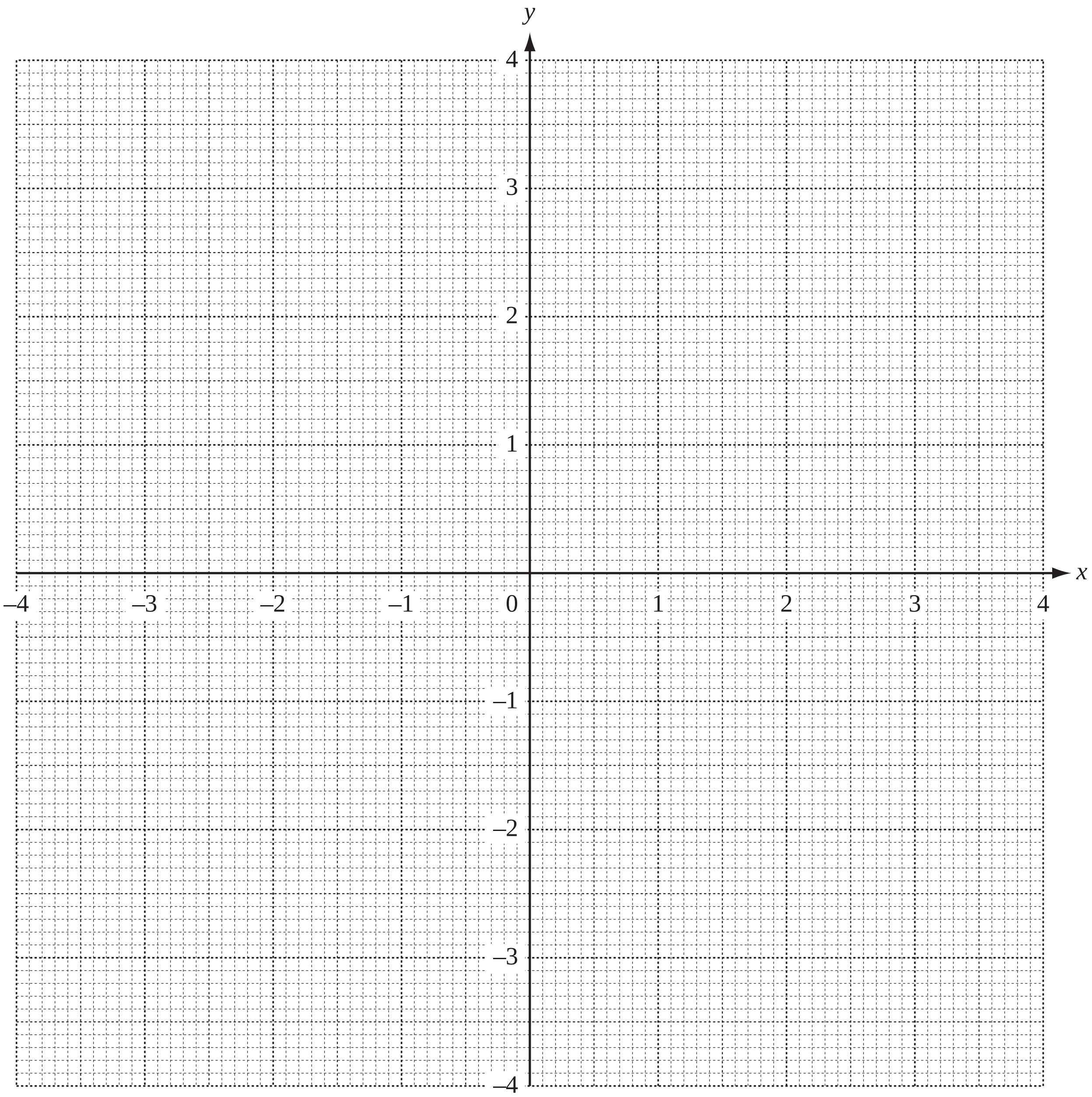
For Examiner's Use

5 **(a)** The table shows some values for the equation $y = \frac{x}{2} - \frac{2}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

x	–4	–3	–2	–1.5	–1	–0.5	0.5	1	1.5	2	3	4
y	–1.5	–0.83	0	0.58			–3.75		–0.58	0	0.83	1.5

(i) Write the missing values of y in the empty spaces. [3]

(ii) On the grid, draw the graph of $y = \frac{x}{2} - \frac{2}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.



[5]

(b) Use your graph to solve the equation $\frac{x}{2}-\frac{2}{x}=1$.

Answer(b) $x =$ .... or $x =$ .... [2]

(c) **(i)** By drawing a tangent, work out the gradient of the graph where $x = 2$.

Answer(c)(i) .... [3]

(ii) Write down the gradient of the graph where $x = -2$.

Answer(c)(ii) .... [1]

(d) **(i)** On the grid, draw the line $y = -x$ for $-4 \leqslant x \leqslant 4$. [1]

(ii) Use your graphs to solve the equation $\frac{x}{2}-\frac{2}{x}=-x$.

Answer(d)(ii) $x =$ .... or $x =$ .... [2]

(e) Write down the equation of a straight line which passes through the origin and does **not** intersect the graph of $y=\frac{x}{2}-\frac{2}{x}$.

Answer(e) .... [2]

10 $f(x) = 2x - 1$ $g(x) = x^2 + 1$ $h(x) = 2^x$

For Examiner's Use

(a) Find the value of

(i) $f\left(-\frac{1}{2}\right)$,

Answer(a)(i) .... [1]

(ii) $g(-5)$,

Answer(a)(ii) .... [1]

(iii) $h(-3)$.

Answer(a)(iii) .... [1]

(b) Find the inverse function $f^{-1}(x)$.

Answer(b) $f^{-1}(x) =$ .... [2]

(c) $g(x) = z$.
Find x in terms of z.

Answer(c) $x =$ .... [2]

(d) Find $gf(x)$, in its simplest form.

Answer(d) $gf(x) =$ .... [2]

(e) h(x) = 512.
Find the value of x.

Answer(e) x = ... [1]

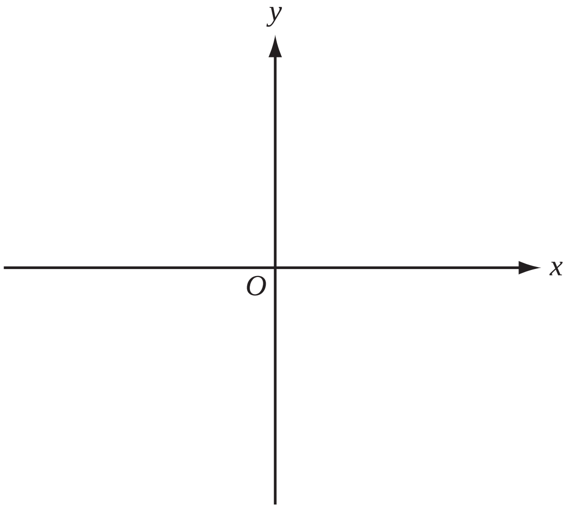
(f) Solve the equation $2\mathrm{f}(x) + \mathrm{g}(x) = 0$, giving your answers correct to 2 decimal places.

Answer(f) x = ... or x = ... [5]

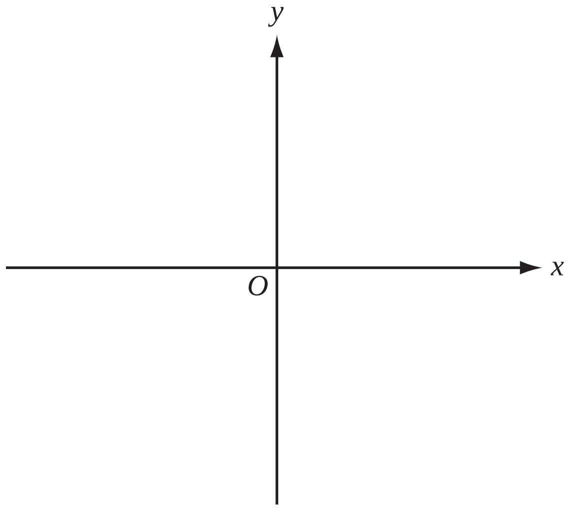
(g) Sketch the graph of

(i) $y = \mathrm{f}(x)$,

(ii) $y = \mathrm{g}(x)$.



(i) $y = \mathrm{f}(x)$



(ii) $y = \mathrm{g}(x)$

[3]

For Examiner's Use

For Examiner's Use

8 **(a)** $f(x) = 2^x$

Complete the table.

x	–2	–1	0	1	2	3	4
$y = f(x)$		0.5	1	2	4		

[3]

(b) $g(x) = x(4 - x)$

Complete the table.

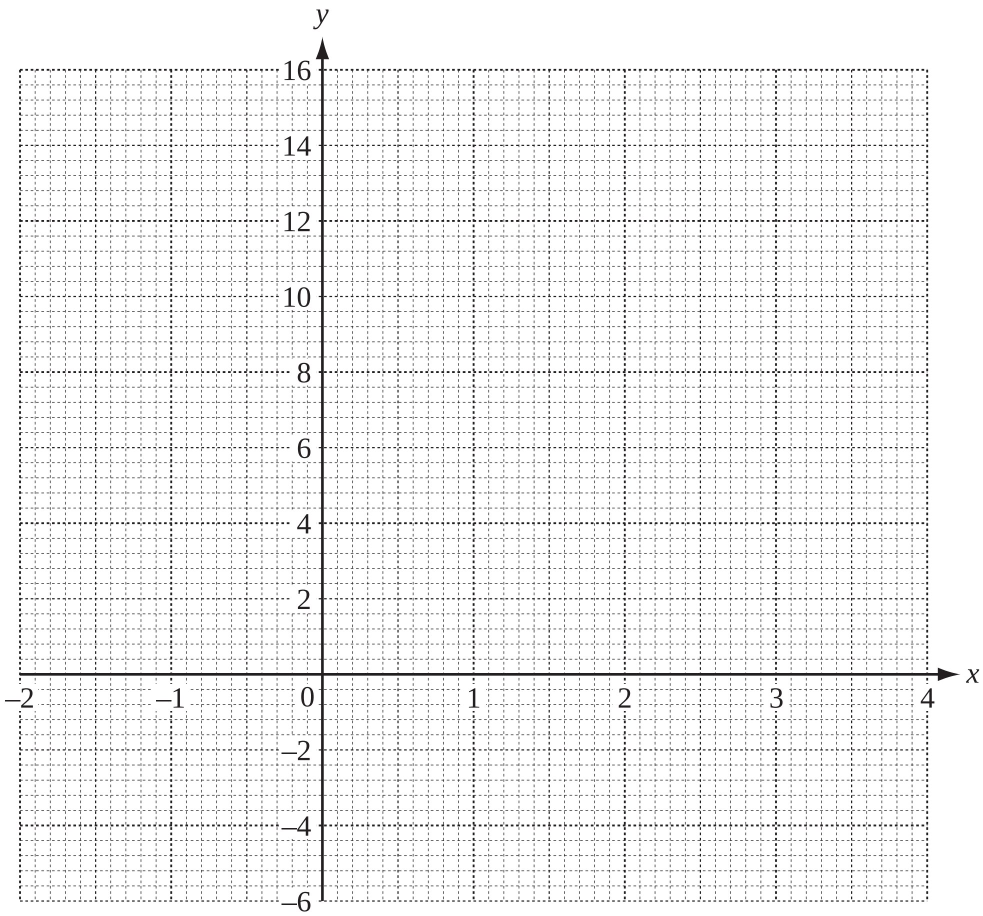
x	–1	0	1	2	3	4
$y = g(x)$		0	3		3	0

[2]

(c) On the grid, draw the graphs of

(i) $y = f(x)$ for $-2 \leqslant x \leqslant 4$, [3]

(ii) $y = g(x)$ for $-1 \leqslant x \leqslant 4$. [3]



(d) Use your graphs to solve the following equations.

(i) $f(x) = 10$

Answer(d)(i) $x =$ .... [1]

(ii) $f(x) = g(x)$

Answer(d)(ii) $x =$ .... or $x =$ .... [2]

(iii) $f^{-1}(x) = 1.7$

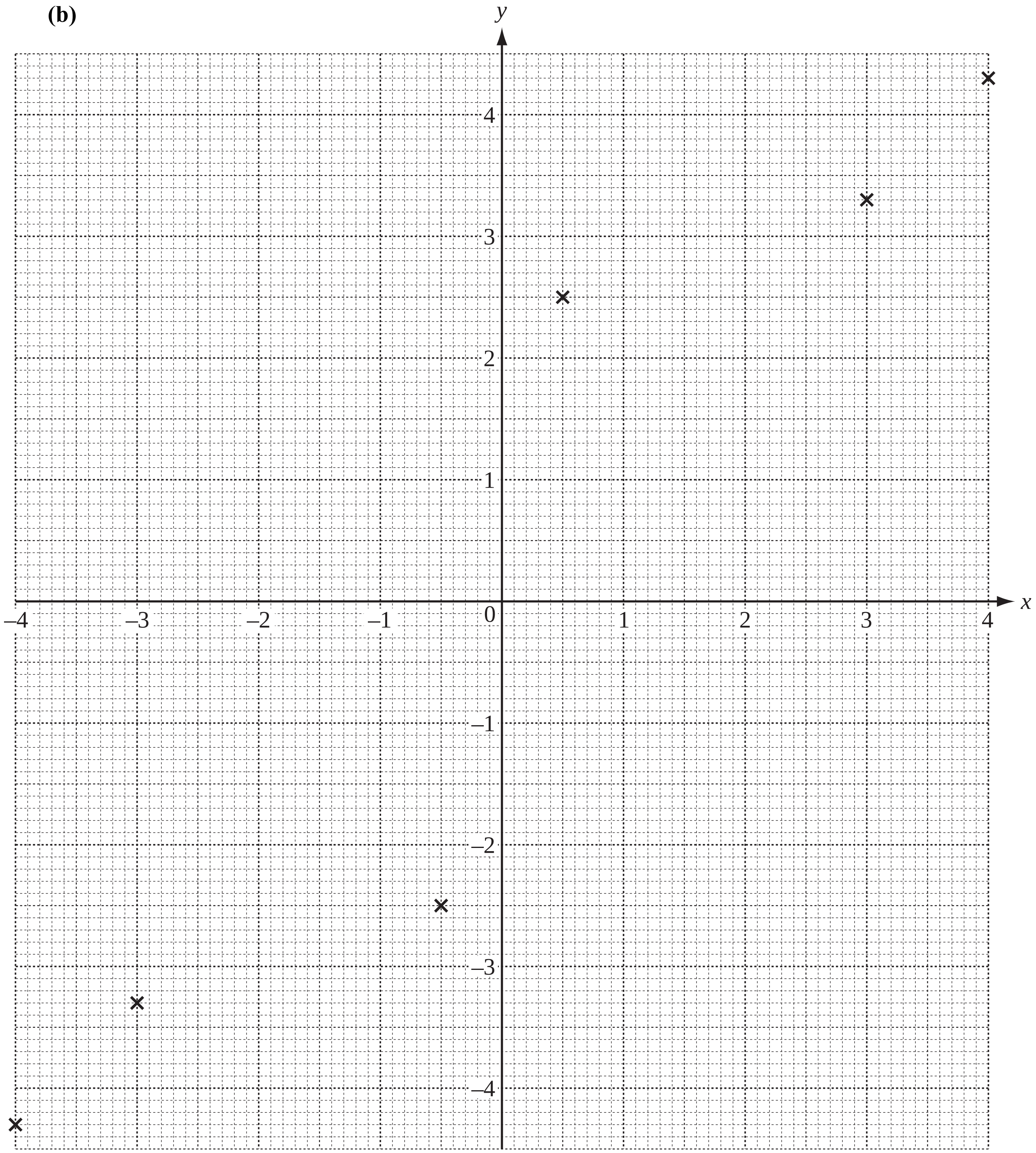
Answer(d)(iii) $x =$ .... [1]

6 **(a)** Complete the table of values for $y = x + \frac{1}{x}$.

x	–4	–3	–2	–1	–0.5	0.5	1	2	3	4
y	–4.3	–3.3			–2.5	2.5			3.3	4.3

[2]

(b)



On the grid, draw the graph of $y = x + \frac{1}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

Six of the ten points have been plotted for you. [3]

For Examiner's Use

(c) There are three integer values of k for which the equation $x + \frac{1}{x} = k$ has **no** solutions.

Write down these three values of k.

Answer(c) $k =$ .... or $k =$ .... or $k =$ .... [2]

(d) Write down the ranges of x for which the gradient of the graph of $y = x + \frac{1}{x}$ is positive.

Answer(d) .... [2]

(e) To solve the equation $x + \frac{1}{x} = 2x + 1$, a straight line can be drawn on the grid.

(i) Draw this line on the grid for $-2.5 \leqslant x \leqslant 1.5$. [2]

(ii) On the grid, show how you would find the solutions. [1]

(iii) Show how the equation $x + \frac{1}{x} = 2x + 1$ can be rearranged into the form $x^2 + bx + c = 0$ and find the values of b and c.

Answer(e)(iii) $b =$ ....

$c =$ .... [3]

5 **Answer the whole of this question on a sheet of graph paper.**

(a) The table gives values of $f(x) = \frac{24}{x^2} + x^2$ for $0.8 \leqslant x \leqslant 6$.

x	0.8	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$	38.1	25	12.9	10	10.1	11.7	l	m	n	26	31	36.7

Calculate, correct to 1 decimal place, the values of l, m and n. [3]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 5 units on the y-axis, draw an x-axis for $0 \leqslant x \leqslant 6$ and a y-axis for $0 \leqslant y \leqslant 40$.

Draw the graph of $y = f(x)$ for $0.8 \leqslant x \leqslant 6$. [6]

(c) Draw the tangent to your graph at $x = 1.5$ and use it to calculate an estimate of the gradient of the curve at this point. [4]

(d) **(i)** Draw a straight line joining the points (0, 20) and (6, 32). [1]

(ii) Write down the equation of this line in the form $y = mx + c$. [2]

(iii) Use your graph to write down the x-values of the points of intersection of this line and the curve $y = f(x)$. [2]

(iv) Draw the tangent to the curve which has the same gradient as your line in **part d(i)**. [1]

(v) Write down the equation for the tangent in **part d(iv)**. [2]

4 Answer the whole of this question on a sheet of graph paper.

t	0	1	2	3	4	5	6	7
$f(t)$	0	25	37.5	43.8	46.9	48.4	49.2	49.6

(a) Using a scale of 2 cm to represent 1 unit on the horizontal t-axis and 2 cm to represent 10 units on the y-axis, draw axes for $0 \leqslant t \leqslant 7$ and $0 \leqslant y \leqslant 60$.
Draw the graph of the curve $y = f(t)$ using the table of values above. [5]

(b) $f(t) = 50(1 - 2^{-t})$.

(i) Calculate the value of f(8) and the value of f(9). [2]

(ii) Estimate the value of $f(t)$ when t is large. [1]

(c) **(i)** Draw the tangent to $y = f(t)$ at $t = 2$ and use it to calculate an estimate of the gradient of the curve at this point. [3]

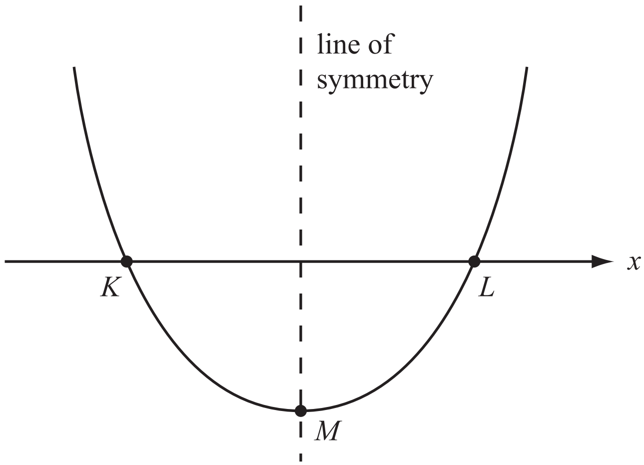
(ii) The function $f(t)$ represents the speed of a particle at time t.
Write down what quantity the gradient gives. [1]

(d) **(i)** On the same grid, draw $y = g(t)$ where $g(t) = 6t + 10$, for $0 \leqslant t \leqslant 7$. [2]

(ii) Write down the range of values for t where $f(t) > g(t)$. [2]

(iii) The function $g(t)$ represents the speed of a second particle at time t.
State whether the first or second particle travels the greater distance for $0 \leqslant t \leqslant 7$.
You **must** give a reason for your answer. [2]

7 A sketch of the graph of the quadratic function $y = px^2 + qx + r$ is shown in the diagram.



The graph cuts the x-axis at K and L.
The point M lies on the graph and on the line of symmetry.

(a) When $p = 1,\quad q = -2,\quad r = -3$, find

(i) the y-coordinate of the point where $x = 4$, [1]

(ii) the coordinates of K and L, [3]

(iii) the coordinates of M. [2]

(b) Describe how the above sketch of the graph would change in each of the following cases.

(i) p is negative. [1]

(ii) $p = 1,\quad q = r = 0$. [1]

(c) Another quadratic function is $y = ax^2 + bx + c$.

(i) Its graph passes through the origin.
Write down the value of c. [1]

(ii) The graph also passes through the points (3, 0) and (4, 8).
Find the values of a and b. [4]

5 **Answer the whole of this question on one sheet of graph paper.**

$$f(x) = 1 - \frac{1}{x^2}, \quad x \neq 0.$$

(a)

x	-3	-2	-1	-0.5	-0.4	-0.3		0.3	0.4	0.5	1	2	3
$f(x)$	p	0.75	0	-3	-5.25	q		q	-5.25	-3	0	0.75	p

Find the values of p and q. [2]

(b) **(i)** Draw an x-axis for $-3 \leqslant x \leqslant 3$ using 2 cm to represent 1 unit and a y-axis for $-11 \leqslant y \leqslant 2$ using 1 cm to represent 1 unit. [1]

(ii) Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and for $0.3 \leqslant x \leqslant 3$. [5]

(c) Write down an integer k such that $f(x) = k$ has no solutions. [1]

(d) **On the same grid**, draw the graph of $y = 2x - 5$ for $-3 \leqslant x \leqslant 3$. [2]

(e) **(i)** Use your graphs to find solutions of the equation $1 - \frac{1}{x^2} = 2x - 5$. [3]

(ii) Rearrange $1 - \frac{1}{x^2} = 2x - 5$ into the form $ax^3 + bx^2 + c = 0$, where a, b and c are integers. [2]

(f) **(i)** Draw a tangent to the graph of $y = f(x)$ which is parallel to the line $y = 2x - 5$. [1]

(ii) Write down the equation of this tangent. [2]

4 **Answer the whole of this question on a sheet of graph paper.**

$$f(x) = 3x - \frac{1}{x^2} + 3, \ x \neq 0.$$

(a) The table shows some values of f(x).

x	−3	−2.5	−2	−1.5	−1	−0.5	−0.4	−0.3	0.3	0.4	0.5	1	1.5	2	2.5	3
f(x)	p	−4.7	−3.3	−1.9	−1	−2.5	−4.5	−9.0	−7.2	−2.1	0.5	q	7.1	8.8	10.3	r

Find the values of p, q and r. [3]

(b) Draw axes using a scale of 1 cm to represent 0.5 units for $-3 \leqslant x \leqslant 3$ and 1 cm to represent 2 units for $-10 \leqslant y \leqslant 12$. [1]

(c) On your grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$. [5]

(d) Use your graph to solve the equations

(i) $3x - \frac{1}{x^2} + 3 = 0$, [1]

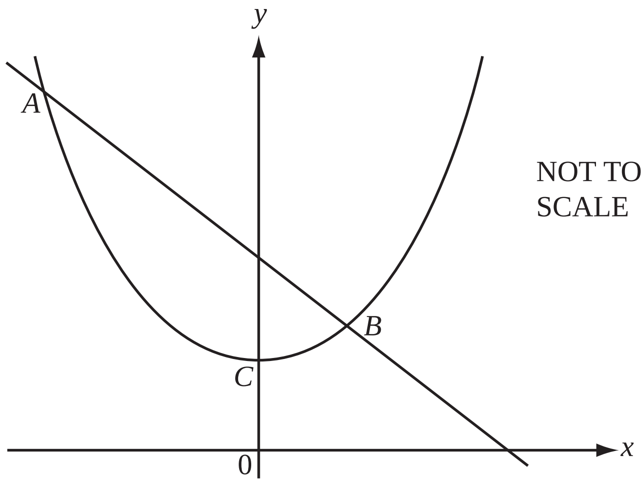
(ii) $3x - \frac{1}{x^2} + 7 = 0$. [3]

(e) $g(x) = 3x + 3$.
On the same grid, draw the graph of $y = g(x)$ for $-3 \leqslant x \leqslant 3$. [2]

(f) **(i)** Describe briefly what happens to the graphs of $y = f(x)$ and $y = g(x)$ for large positive or negative values of x. [1]

(ii) Estimate the gradient of $y = f(x)$ when $x = 100$. [1]

3



The diagram shows a sketch of $y = x^2 + 1$ and $y = 4 - x$.

(a) Write down the co-ordinates of

(i) the point C, [1]

(ii) the points of intersection of $y = 4 - x$ with each axis. [2]

(b) Write down the gradient of the line $y = 4 - x$. [1]

(c) Write down the range of values of x for which the gradient of the graph of $y = x^2 + 1$ is negative. [1]

(d) The two graphs intersect at A and B.

Show that the x co-ordinates of A and B satisfy the equation $x^2 + x - 3 = 0$. [1]

(e) Solve the equation $x^2 + x - 3 = 0$, giving your answers correct to 2 decimal places. [4]

(f) Find the co-ordinates of the mid-point of the straight line AB. [2]

3 Answer the whole of this question on a sheet of graph paper.

The table shows some of the values of the function $f(x) = x^2 - \frac{1}{x}$, $x \neq 0$.

x	−3	−2	−1	−0.5	−0.2	0.2	0.5	1	2	3
y	9.3	4.5	2.0	2.3	p	−5.0	−1.8	q	3.5	r

(a) Find the values of p, q and r, correct to 1 decimal place. [3]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw an x-axis for $-3 \leqslant x \leqslant 3$ and a y-axis for $-6 \leqslant y \leqslant 10$.

Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$. [6]

(c) (i) By drawing a suitable straight line, find the three values of x where $f(x) = -3x$. [3]

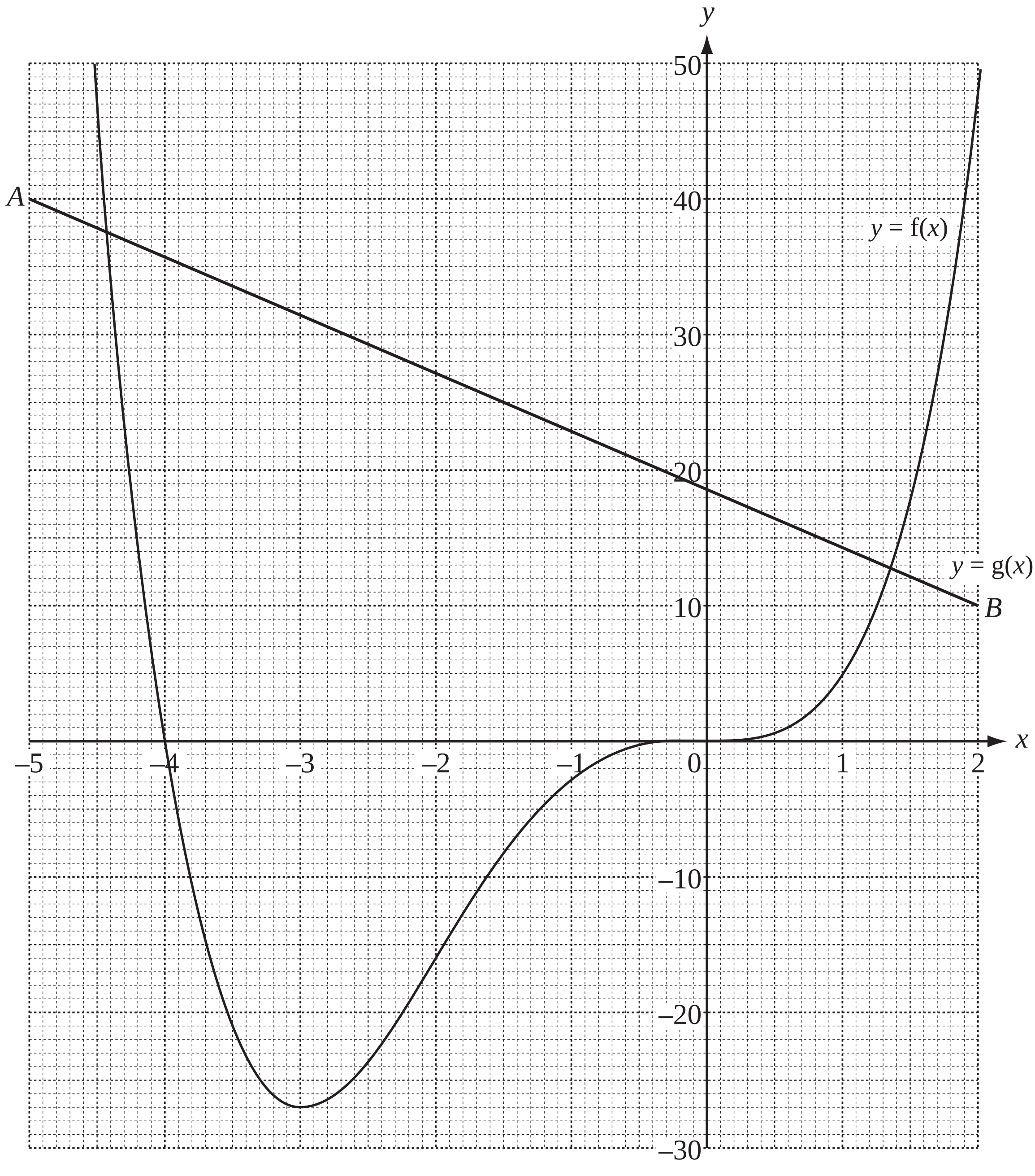
(ii) $x^2 - \frac{1}{x} = -3x$ can be written as $x^3 + ax^2 + b = 0$.

Find the values of a and b. [2]

(d) Draw a tangent to the graph of $y = f(x)$ at the point where $x = -2$.

Use it to estimate the gradient of $y = f(x)$ when $x = -2$. [3]

6



The graphs of $y = f(x)$ and $y = g(x)$ are shown above.

(a) Find the value of

(i) f(−2),

Answer(a)(i) .... [1]

g(0).

Answer(a)(ii) .... [1]

(b) Use the graphs to solve

(i) the equation f(x) = 20,

Answer(b)(i) x = ... or x = ... [2]

(ii) the equation f(x) = g(x),

Answer(b)(ii) x = ... or x = ... [2]

(iii) the inequality f(x) < g(x).

Answer(b)(iii) .... [1]

(c) Use the points A and B to find the gradient of y = g(x) as an exact fraction.

Answer(c) .... [2]

(d) On the grid, draw the graph of y = g(x) – 10. [2]

(e) **(i)** Draw the tangent to the graph of y = f(x) at (–3, –27). [1]

(ii) Write down the equation of this tangent.

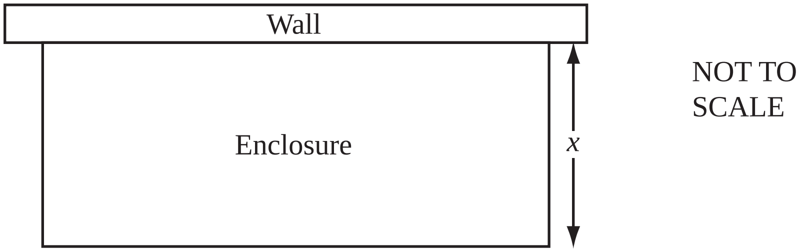
Answer(e)(ii) .... [1]

(f) A region, R, contains points whose co-ordinates satisfy the inequalities

$$-3 \leqslant x \leqslant -2, \qquad y \leqslant 40 \qquad \text{and} \qquad y \geqslant \text{g}(x).$$

On the grid, draw suitable lines and label this region R. [2]

3



A farmer makes a rectangular enclosure for his animals.
He uses a wall for one side and a total of 72 metres of fencing for the other three sides.

The enclosure has width x metres and area A square metres.

(a) Show that $A = 72x - 2x^2$.

Answer (a)

[2]

(b) Factorise completely $72x - 2x^2$.

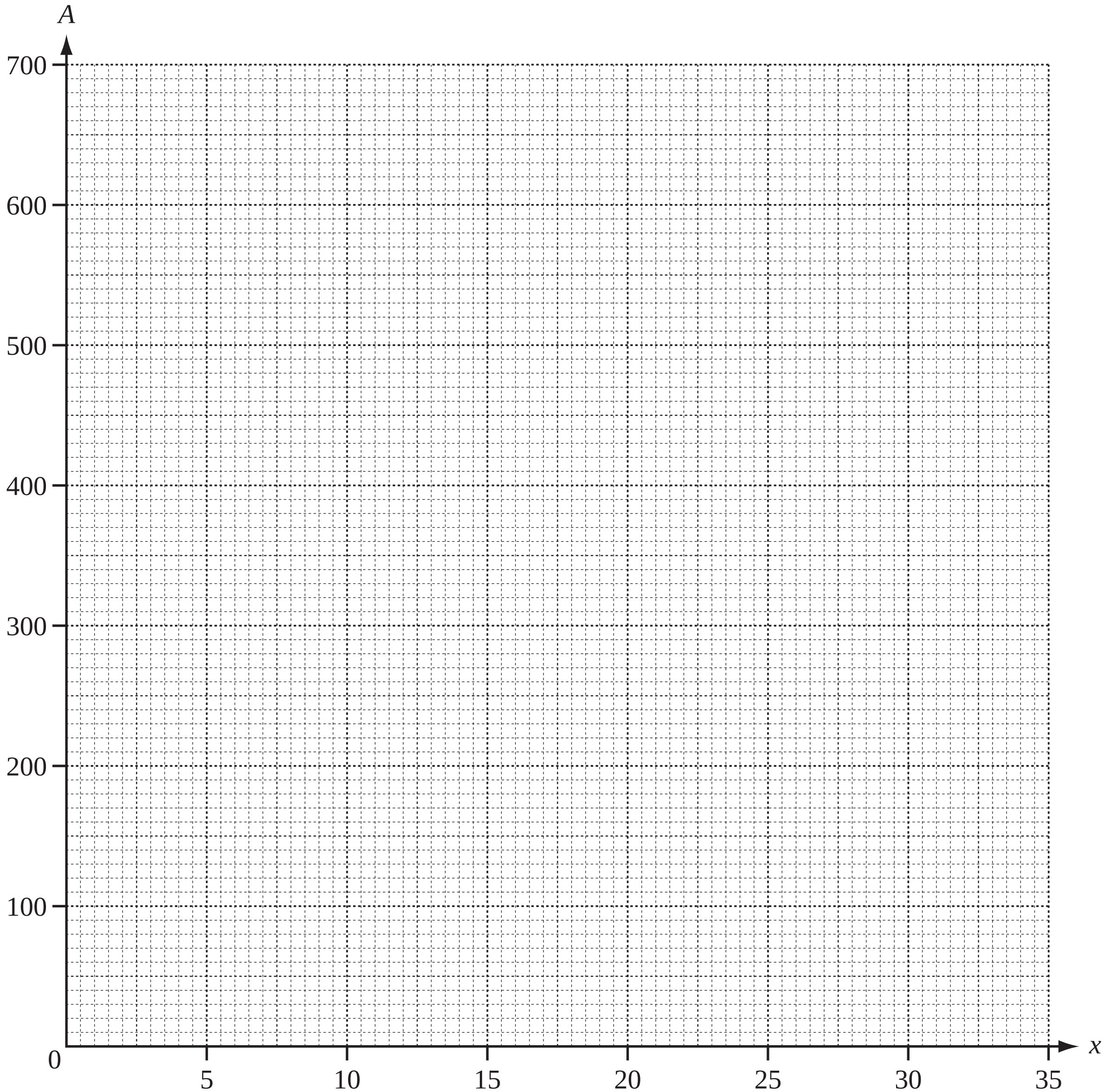
Answer(b) .... [2]

(c) Complete the table for $A = 72x - 2x^2$.

x	0	5	10	15	20	25	30	35
A	0	310	520			550	360	

[3]

(d) Draw the graph of $A = 72x - 2x^2$ for $0 \leqslant x \leqslant 35$ on the grid opposite.



[4]

(e) Use your graph to find

(i) the values of x when $A = 450$,

Answer(e)(i) $x =$ .... or $x =$ .... [2]

(ii) the maximum area of the enclosure.

Answer(e)(ii) .... m^2 [1]

(f) Each animal must have at least 12 m^2 for grazing.

Calculate the greatest number of animals that the farmer can keep in an enclosure which has an area of 500 m^2.

8 **(a)** $f(x) = 2x - 1$ $g(x) = x^2$

For Examiner's Use

Work out

(i) f(2),

Answer(a)(i) .... [1]

(ii) g(– 2),

Answer(a)(ii) .... [1]

(iii) ff(x) in its simplest form,

Answer(a)(iii) ff(x) = ... [2]

(iv) $f^{-1}(x)$, the inverse of f(x),

Answer(a)(iv) $f^{-1}(x)$ = ... [2]

(v) x when gf(x) = 4.

Answer(a)(v) x = ... or x = ... [4]

(b) y is **inversely** proportional to x and $y = 8$ when $x = 2$.

Find,

(i) an equation connecting y and x,

Answer(b)(i) .... [2]

(ii) y when $x = \frac{1}{2}$.

2 $f(x) = 6 + x^2$ $g(x) = 4x - 1$

(a) Find

(i) g(3),

Answer(a)(i) .... [1]

(ii) f (–4).

Answer(a)(ii) .... [1]

(b) Find the inverse function $g^{-1}(x)$.

Answer(b) $g^{-1}(x) =$ .... [2]

(c) Find fg(x) in its simplest form.

Answer(c) fg(x) = ... [3]

(d) Solve the equation gg(x) = 3.

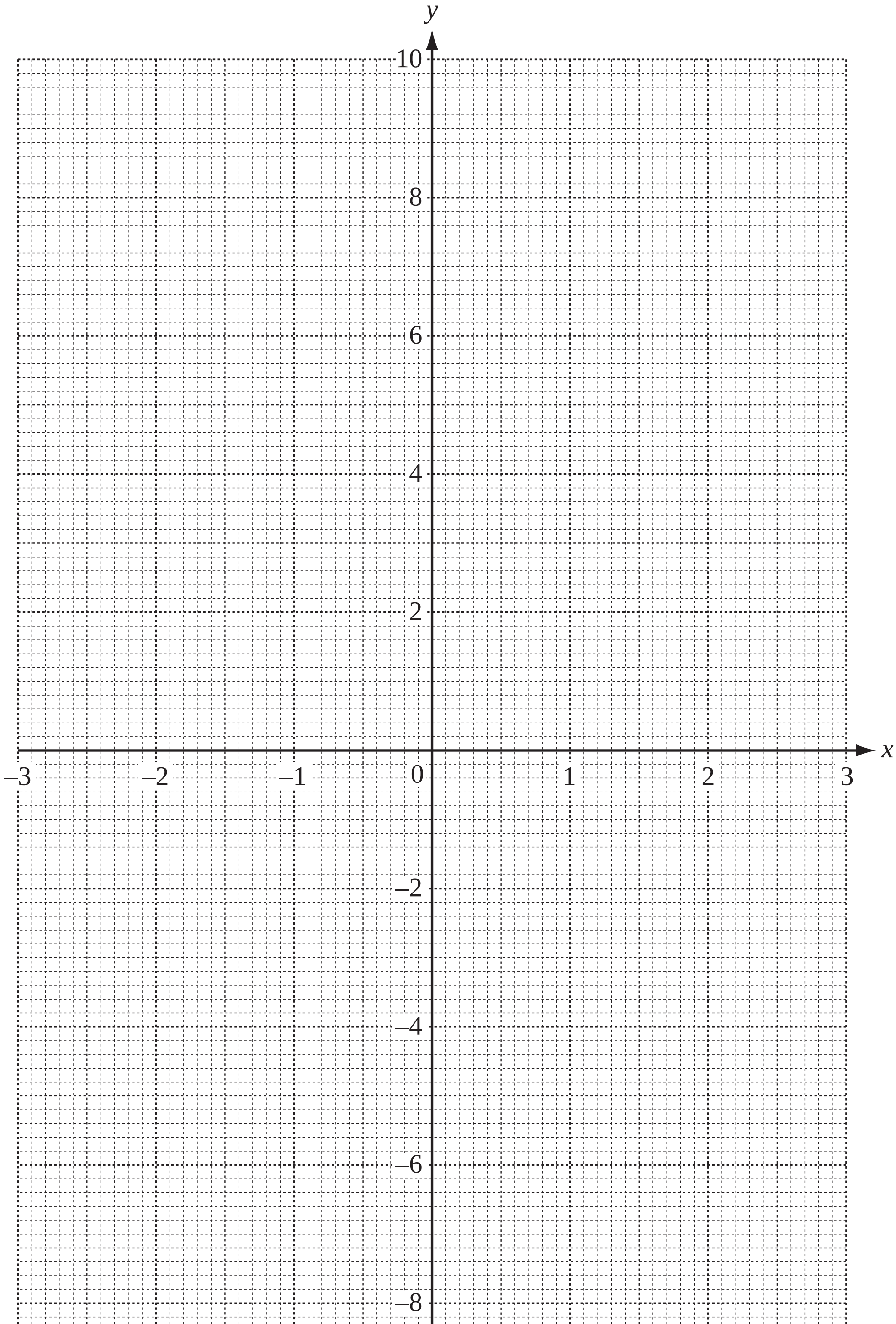
For Examiner's Use

7 **(a)** Complete the table for the function $f(x) = \frac{2}{x} - x^2$.

x	–3	–2	–1	–0.5	–0.2		0.2	0.5	1	2	3
$f(x)$	–9.7	–5			–10.0		10.0	3.75	1		–8.3

[3]

(b) On the grid draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.



(c) Use your graph to

(i) solve f(x) = 2,

Answer(c)(i) x = ... [1]

(ii) find a value for k so that f(x) = k has 3 solutions.

Answer(c)(ii) k = ... [1]

(d) Draw a suitable line on the grid and use your graphs to solve the equation $\frac{2}{x} - x^2 = 5x$.

Answer(d) x = ... or x = ... [3]

(e) Draw the tangent to the graph of y = f(x) at the point where x = –2.

Use it to calculate an estimate of the gradient of y = f(x) when x = –2.

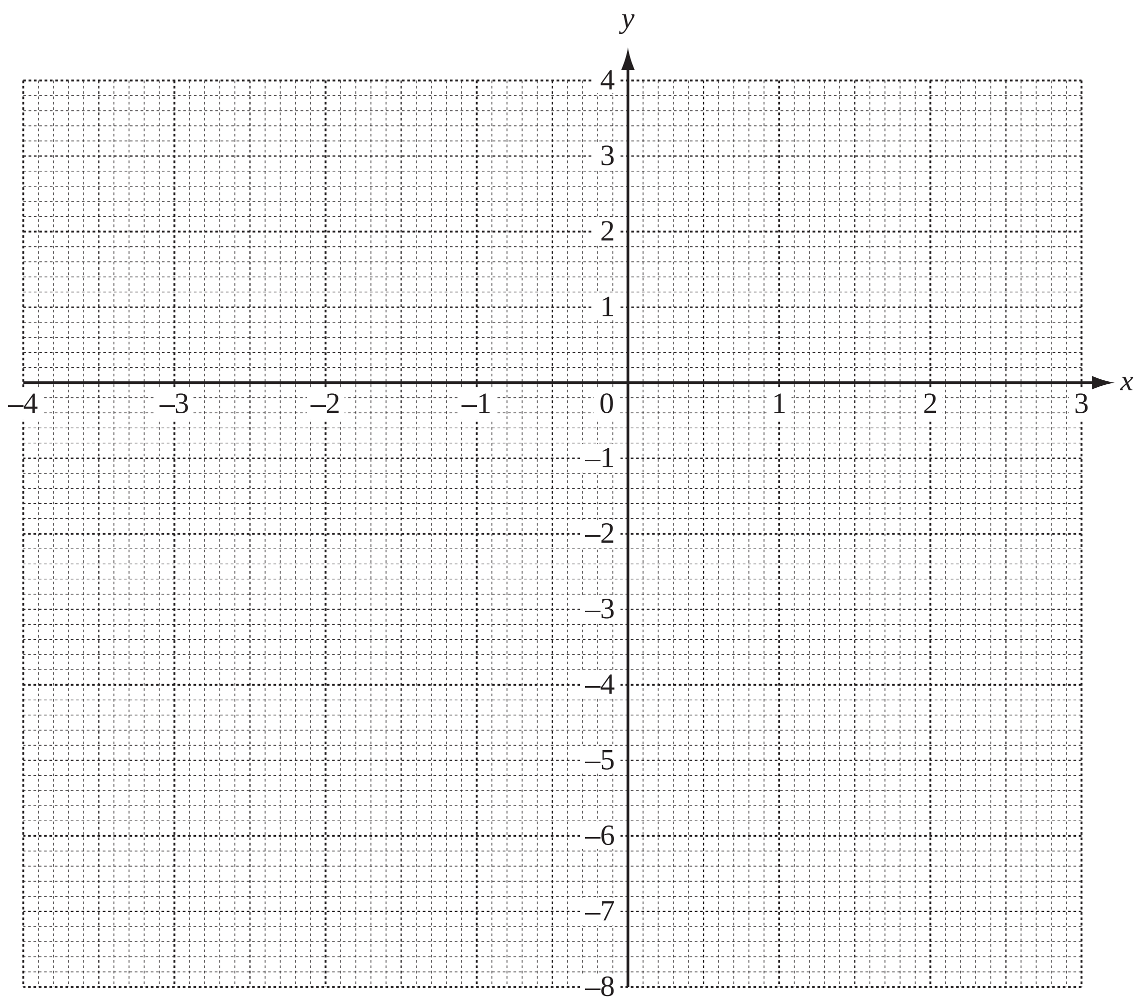
Answer(e) .... [3]

7 **(a)** Complete the table for the function $f(x) = \frac{x^3}{10} + 1$.

x	–4	–3	–2	–1	0	1	2	3
f(x)		–1.7	0.2	0.9	1	1.1	1.8	

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 3$.



[4]

(c) Complete the table for the function $g(x) = \frac{4}{x}$, $x \neq 0$.

x	–4	–3	–2	–1	1	2	3
g(x)	–1	–1.3				2	1.3

[2]

For Examiner's Use

(d) On the grid, draw the graph of $y = g(x)$ for $-4 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 3$. [3]

(e) **(i)** Use your graphs to solve the equation $\dfrac{x^3}{10} + 1 = \dfrac{4}{x}$.

Answer(e)(i) $x =$ .... or $x =$ .... [2]

(ii) The equation $\dfrac{x^3}{10} + 1 = \dfrac{4}{x}$ can be written as $x^4 + ax + b = 0$.

Find the values of a and b.

Answer(e)(ii) $a =$ ....

$b =$ .... [2]