Linear Equations in Two Variables

2

In this chapter we cover topics related to graphing and the applications of graphing. Graphs appear in magazines and newspapers and in other aspects of day-to-day life. Furthermore, in many fields of study such as the sciences and business, graphs are used to display data (information).

The following words are key terms used in this chapter. Search for them in the puzzle and in the text throughout the chapter. By the end of this chapter, you should be familiar with all of these terms.

Key Terms

coordinate	midpoint
origin	*x*-intercept
x-axis	*y*-intercept
y-axis	horizontal
quadrant	vertical
slope	

W	M	V	E	P	O	L	S	W	W	M	M	R	E	N	E	F
V	D	P	D	I	D	F	K	O	O	R	Z	J	J	P	P	Q
Z	E	R	G	N	V	R	P	W	L	X	J	C	K	A	W	J
O	H	R	I	F	F	X	P	P	E	B	P	F	P	Y	X	S
J	M	V	T	I	M	T	P	E	C	R	E	T	N	I	X	C
U	R	P	N	I	W	Y	I	N	T	E	R	C	E	P	T	R
E	W	K	N	L	C	J	V	D	Y	Z	R	N	G	X	D	I
T	T	D	I	V	Y	A	Z	L	X	N	R	T	K	A	K	V
A	N	P	G	T	E	L	L	A	Z	T	Q	E	H	X	V	E
N	I	R	I	N	Y	D	K	T	T	K	N	B	T	I	Z	Z
I	O	W	R	A	E	H	A	N	Y	L	K	N	A	S	Y	X
D	P	Q	O	R	C	E	S	O	T	X	Q	T	S	X	K	Q
R	D	A	V	D	L	T	O	Z	C	U	N	F	R	Y	O	J
O	I	E	F	A	O	U	G	I	T	D	A	I	D	A	P	X
O	M	C	H	U	L	T	K	R	R	Z	C	E	K	X	T	T
C	Y	K	I	Q	R	U	R	O	D	W	L	N	B	I	S	U
Z	W	J	U	R	R	P	S	H	E	C	X	M	N	S	N	N

Section 2.1 The Rectangular Coordinate System and Midpoint Formula

Concepts

1. The Rectangular Coordinate System
2. Plotting Points
3. The Midpoint Formula

1. The Rectangular Coordinate System

One application of algebra is the graphical representation of numerical information (or data). For example, Table 2-1 shows the percentage of individuals who participate in leisure sports activities according to the age of the individual.

Table 2-1

Age (years)	Percentage of Individuals Participating in Leisure Sports Activities
20	59%
30	52%
40	44%
50	34%
60	21%
70	18%

Source: U.S. National Endowment for the Arts.

Information in table form is difficult to picture and interpret. However, when the data are presented in a graph, there appears to be a downward trend in the participation in leisure sports activities for older age groups (Figure 2-1). In this example, two variables are related: age and the percentage of individuals who participate in leisure sports activities.

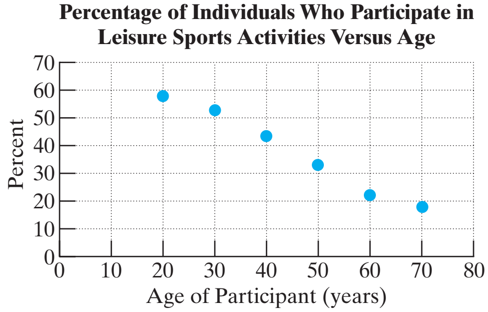


Figure 2-1

To picture two variables simultaneously, we use a graph with two number lines drawn at right angles to each other (Figure 2-2). This forms a **rectangular coordinate system.** The horizontal line is called the ***x*-axis,** and the vertical line is called the ***y*-axis.** The point where the lines intersect is called the **origin.** On the *x*-axis, the numbers to the right of the origin are positive,

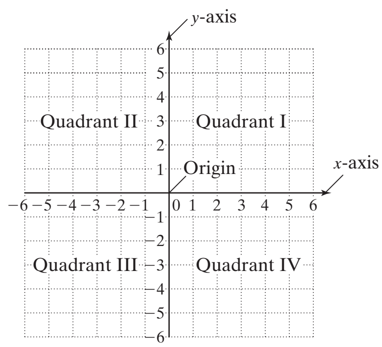


Figure 2-2

and the numbers to the left are negative. On the y-axis, the numbers above the origin are positive, and the numbers below are negative. The x- and y-axes divide the graphing area into four regions called **quadrants.**

2. Plotting Points

Points graphed in a rectangular coordinate system are defined by two numbers as an **ordered pair** (x, y). The first number (called the first coordinate or abscissa) is the horizontal position from the origin. The second number (called the second coordinate or ordinate) is the vertical position from the origin. Example 1 shows how points are plotted in a rectangular coordinate system.

Example 1 Plotting Points

Plot each point and state the quadrant or axis where it is located.

a. $(4, 1)$ **b.** $(-3, 4)$ **c.** $(4, -3)$

d. $(-\frac{5}{2}, -2)$ **e.** $(0, 3)$ **f.** $(-4, 0)$

Solution:

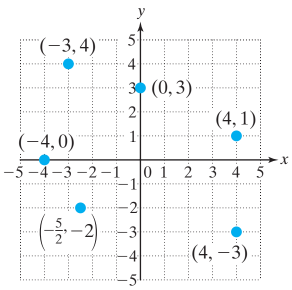


Figure 2-3

a. The point $(4, 1)$ is in quadrant I.

b. The point $(-3, 4)$ is in quadrant II.

c. The point $(4, -3)$ is in quadrant IV.

d. The point $(-\frac{5}{2}, -2)$ can also be written as $(-2.5, -2)$. This point is in quadrant III.

e. The point $(0, 3)$ is on the y-axis.

f. The point $(-4, 0)$ is located on the x-axis.

TIP: Notice that the points $(-3, 4)$ and $(4, -3)$ are in different quadrants. Changing the order of the coordinates changes the location of the point. That is why points are represented by *ordered* pairs (Figure 2-3).

Skill Practice Plot the point and state the quadrant or axis where it is located.

1a. $(3, 5)$ **b.** $(-4, 0)$ **c.** $(2, -1)$

d. $(0, 3)$ **e.** $(-2, -2)$ **f.** $(-5, 2)$

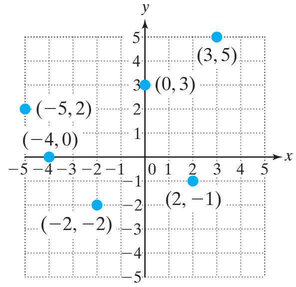
The effective use of graphs for mathematical models requires skill in identifying points and interpreting graphs.

Example 2 Interpreting a Graph

Kristine started a savings plan at the beginning of the year and plotted the amount of money she deposited in her savings account each month. The graph of her savings is shown in Figure 2-4. The values on the x-axis represent the first 6 months of the year, and the values on the y-axis represent the amount of money in dollars that she saved. Refer to Figure 2-4 to answer the questions. Let $x = 1$ represent January on the horizontal axis.

Skill Practice Answers

1a. $(3, 5)$; quadrant I
b. $(-4, 0)$; x-axis
c. $(2, -1)$; quadrant IV
d. $(0, 3)$; y-axis
e. $(-2, -2)$; quadrant III
f. $(-5, 2)$; quadrant II



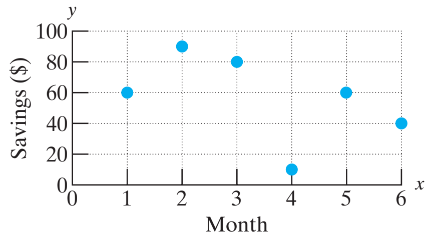


Figure 2-4

TIP: The scale on the x- and y-axes may be different. This often happens in applications. See Figure 2-4.

a. What is the y-coordinate when the x-coordinate is 6? Interpret the meaning of the ordered pair in the context of this problem.

b. In which month did she save the most? How much did she save?

c. In which month did she save the least? How much did she save?

d. How much did she save in March?

e. In which two months did she save the same amount? How much did she save in these months?

Solution:

a. When x is 6, the y-coordinate is 40. This means that in June, Kristine saved \$40.

b. The point with the greatest y-coordinate occurs when x is 2. She saved the most money, \$90, in February.

c. The point with the lowest y-coordinate occurs when x is 4. She saved the least amount, \$10, in April.

d. In March, the x-coordinate is 3 and the y-coordinate is 80. She saved \$80 in March.

e. The two points with the same y-coordinate occur when $x = 1$ and when $x = 5$. She saved \$60 in both January and May.

Skill Practice Refer to Figure 2-4.

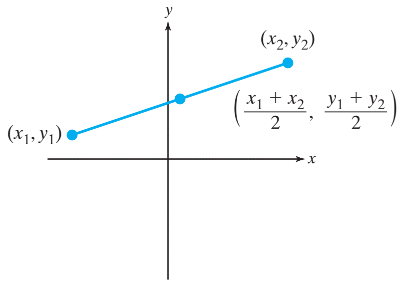
2a. In which month(s) did Kristine save \$80?

b. How much did Kristine save in June?

c. What was the total amount saved during these 6 months?

3. The Midpoint Formula

Consider two points in the coordinate plane and the line segment determined by the points. It is sometimes necessary to determine the point that is halfway between the endpoints of the segment. This point is called the **midpoint.** If the coordinates of the endpoints are represented by (x_1, y_1) and (x_2, y_2), then the midpoint of the segment is given by the following formula.



Midpoint formula: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Skill Practice Answers

2a. March
b. \$40
c. \$340

TIP: The midpoint of a line segment is found by taking the *average* of the x-coordinates and the *average* of the y-coordinates of the endpoints.

Example 3 Finding the Midpoint of a Segment

Find the midpoint of the line segment with the given endpoints.

a. $(-4, 6)$ and $(8, 1)$ **b.** $(-1.2, -3.1)$ and $(-6.6, 1.2)$

Solution:

a. $(-4, 6)$ and $(8, 1)$

$\left(\dfrac{-4+8}{2}, \dfrac{6+1}{2}\right)$ Apply the midpoint formula.

$\left(2, \dfrac{7}{2}\right)$ Simplify.

The midpoint of the segment is $(2, \frac{7}{2})$.

b. $(-1.2, -3.1)$ and $(-6.6, 1.2)$

$\left(\dfrac{-1.2 + -6.6}{2}, \dfrac{-3.1 + 1.2}{2}\right)$ Apply the midpoint formula.

$(-3.9, -0.95)$ Simplify.

Skill Practice Find the midpoint of the line segment with the given endpoints.

3. $(5, 6)$ and $(-10, 4)$ **4.** $(-2.6, -6.3)$ and $(1.2, 4.1)$

Example 4 Applying the Midpoint Formula

A map of a national park is created so that the ranger station is at the origin of a rectangular grid. Two hikers are located at positions $(2, 3)$ and $(-5, -2)$ with respect to the ranger station, where all units are in miles. The hikers would like to meet at a point halfway between them (Figure 2-5), but they are too far apart to communicate their positions to each other via radio. However, the hikers are both within radio range of the ranger station. If the ranger station relays each hiker's position to the other, at what point on the map should the hikers meet?

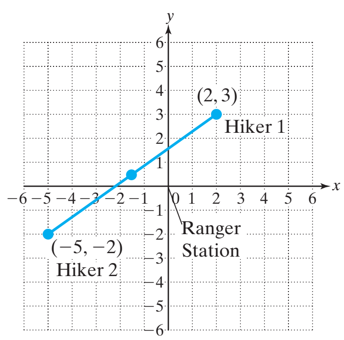


Figure 2-5

Skill Practice Answers

3. $\left(-\dfrac{5}{2}, 5\right)$ **4.** $(-0.7, -1.1)$

Solution:

To find the halfway point on the line segment between the two hikers, apply the midpoint formula:

$$(2, 3) \quad \text{and} \quad (-5, -2)$$
$$(x_1, y_1) \quad \text{and} \quad (x_2, y_2)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

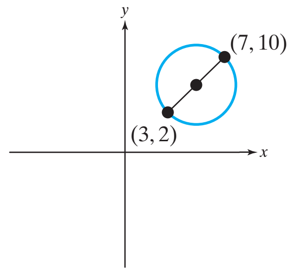
$$\left(\frac{2 + (-5)}{2}, \frac{3 + (-2)}{2}\right) \qquad \text{Apply the midpoint formula.}$$

$$\left(\frac{-3}{2}, \frac{1}{2}\right) \qquad \text{Simplify.}$$

The halfway point between the hikers is located at $(-\frac{3}{2}, \frac{1}{2})$ or $(-1.5, 0.5)$.

Skill Practice

5. Find the center of the circle in the figure, given that the endpoints of a diameter are (3, 2) and (7, 10).



Skill Practice Answers

5. (5, 6)

Section 2.1 Practice Exercises

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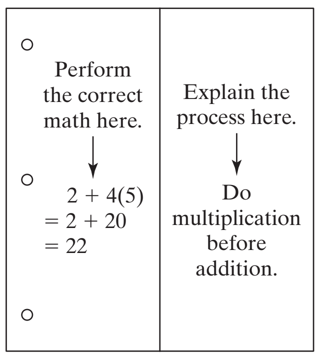
Study Skills Exercises

1. After getting a test back, it is a good idea to correct the test so that you do not make the same errors again. One recommended approach is to use a clean sheet of paper, and divide the paper down the middle vertically, as shown. For each problem that you missed on the test, rework the problem correctly on the left-hand side of the paper. Then give a written explanation on the right-hand side of the paper. To reinforce the correct procedure, return to the section of text from which the problem was taken and do several more problems.

Take the time this week to make corrections from your last test.



2. Define the key terms.

a. Rectangular coordinate system **b.** ***x*-Axis** **c.** ***y*-Axis**

d. Origin **e. Quadrant** **f. Ordered pair**

g. Midpoint

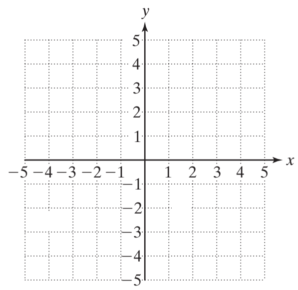
Concept 1: The Rectangular Coordinate System

3. Given the coordinates of a point, explain how to determine which quadrant the point is in.

4. What is meant by the word *ordered* in the term *ordered pair*?

Concept 2: Plotting Points

5. Plot the points on a rectangular coordinate system.

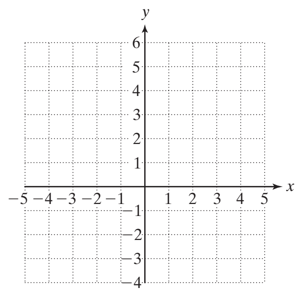


a. $(-2, 1)$ **b.** $(0, 4)$

c. $(0, 0)$ **d.** $(-3, 0)$

e. $\left(\frac{3}{2}, -\frac{7}{3}\right)$ **f.** $(-4.1, -2.7)$

6. Plot the points on a rectangular coordinate system.



a. $(-2, 5)$ **b.** $\left(\frac{5}{2}, 0\right)$

c. $(4, -3)$ **d.** $(0, -2)$

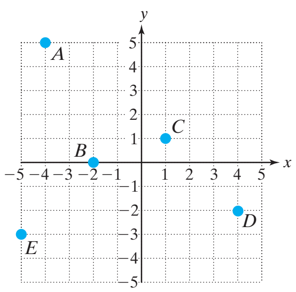
e. $(2, 2)$ **f.** $(-3, -3)$

7. A point on the x-axis will have what y-coordinate?

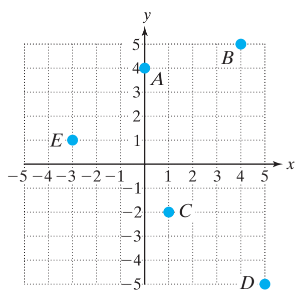
8. A point on the y-axis will have what x-coordinate?

For Exercises 9–12, give the coordinates of the labeled points, and state the quadrant or axis where the point is located.

9.



10.



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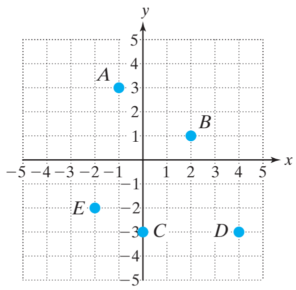

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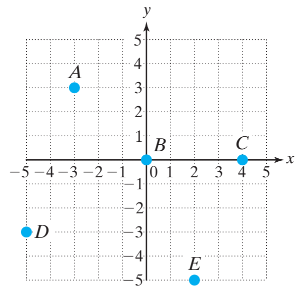
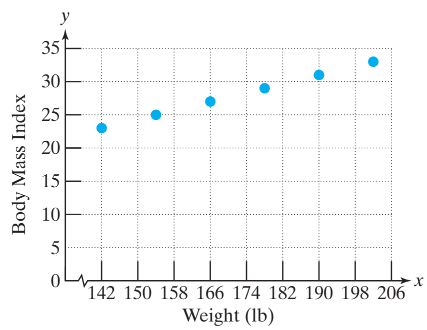


For Exercises 13–14, refer to the graphs to answer the questions.

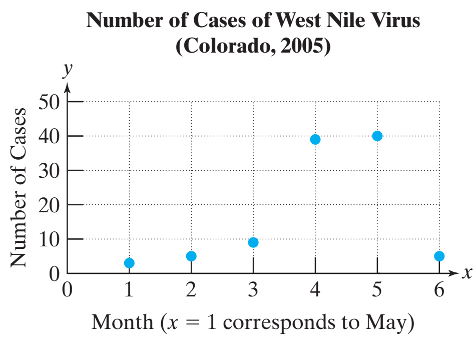
13. The fact that obesity is increasing in both children and adults is of great concern to health care providers. One way to measure obesity is by using the body mass index. Body mass is calculated based on the height and weight of an individual. The graph shows the relationship between body mass index and weight for a person who is 5′6″ tall.



a. What is the body mass index for a 5′6″ person who weighs 154 lb?

b. What is the weight of a 5′6″ person whose body mass index is 29?

14. The graph shows the number of cases of West Nile virus reported in Colorado during the months of May through October 2005. The month of May is represented by $x = 1$ on the x-axis. (Source: Centers for Disease Control.)



a. Which month had the greatest number of cases reported? Approximately how many cases were reported?

b. Which month had the fewest cases reported? Approximately how many cases were reported?

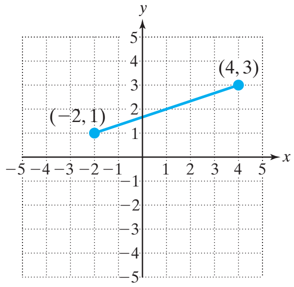
c. Which months had fewer than 10 cases of the virus reported?

d. Approximately how many cases of the virus were reported in August?

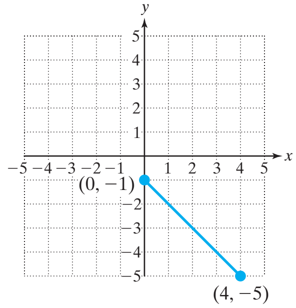
Concept 3: The Midpoint Formula

For Exercises 15–18, find the midpoint of the line segment. Check your answers by plotting the midpoint on the graph.

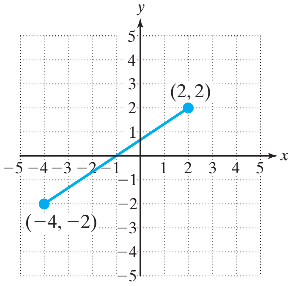
15.



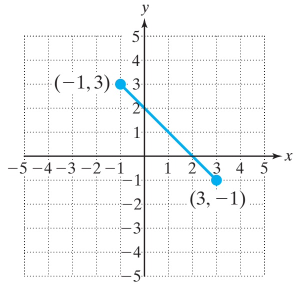
16.



17.



18.



For Exercises 19–26, find the midpoint of the line segment between the two given points.

19. $(4, 0)$ and $(-6, 12)$

20. $(-7, 2)$ and $(-3, -2)$

21. $(-3, 8)$ and $(3, -2)$

22. $(0, 5)$ and $(4, -5)$

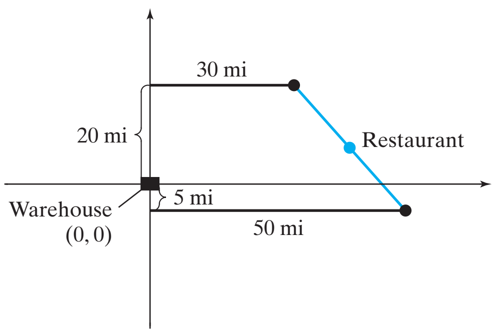
23. $(5, 2)$ and $(-6, 1)$

24. $(-9, 3)$ and $(0, -4)$

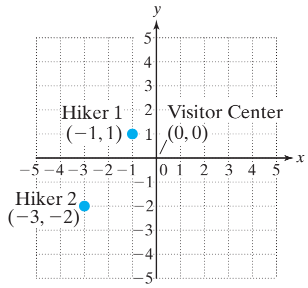
25. $(-2.4, -3.1)$ and $(1.6, 1.1)$

26. $(0.8, 5.3)$ and $(-4.2, 7.1)$

27. Two courier trucks leave the warehouse to make deliveries. One travels 20 mi north and 30 mi east. The other truck travels 5 mi south and 50 mi east. If the two drivers want to meet for lunch at a restaurant at a point halfway between them, where should they meet relative to the warehouse? (*Hint:* Label the warehouse as the origin, and find the coordinates of the restaurant. See the figure.)



28. A map of a hiking area is drawn so that the Visitor Center is at the origin of a rectangular grid. Two hikers are located at positions $(-1, 1)$ and $(-3, -2)$ with respect to the Visitor Center where all units are in miles. A campground is located exactly halfway between the hikers. What are the coordinates of the campground?



29. Find the center of a circle if a diameter of the circle has endpoints $(-1, 2)$ and $(3, 4)$.

30. Find the center of a circle if a diameter of the circle has endpoints $(-3, 3)$ and $(7, -1)$.

Section 2.2 Linear Equations in Two Variables

Concepts

1. Linear Equations in Two Variables
2. Graphing Linear Equations in Two Variables
3. x-Intercepts and y-Intercepts
4. Horizontal and Vertical Lines

1. Linear Equations in Two Variables

Recall from Section 1.4 that an equation in the form $ax + b = 0$ is called a linear equation in one variable. In this section we will study linear equations in *two* variables.

Linear Equation in Two Variables

Let A, B, and C be real numbers such that A and B are not both zero. A **linear equation in two variables** is an equation that can be written in the form

$Ax + By = C$ This form is called *standard form.*

A *solution* to a linear equation in two variables is an ordered pair (x, y) that makes the equation a true statement.

Example 1 Determining Solutions to a Linear Equation

For the linear equation $-2x + 3y = 8$, determine whether the order pair is a solution.

a. $(-4, 0)$ **b.** $(2, -4)$ **c.** $\left(1, \dfrac{10}{3}\right)$

Solution:

a. $-2x + 3y = 8$ — The ordered pair $(-4, 0)$ indicates that $x = -4$ and $y = 0$.

$-2(-4) + 3(0) \stackrel{?}{=} 8$ — Substitute $x = -4$ and $y = 0$ into the equation.

$8 + 0 = 8$ ✔ (true) — The ordered pair $(-4, 0)$ makes the equation a true statement. The ordered pair is a solution to the equation.

b. $-2x + 3y = 8$ — Test the point $(2, -4)$.

$-2(2) + 3(-4) \stackrel{?}{=} 8$ — Substitute $x = 2$ and $y = -4$ into the equation.

$-4 + (-12) \stackrel{?}{=} 8$

$-16 \stackrel{?}{=} 8$ (false) — The ordered pair $(2, -4)$ does not make the equation a true statement. The ordered pair is *not* a solution to the equation.

c. $-2x + 3y = 8$ — Test the point $(1, \frac{10}{3})$.

$-2(1) + 3\left(\frac{10}{3}\right) \stackrel{?}{=} 8$ — Substitute $x = 1$ and $y = \frac{10}{3}$.

$-2 + 10 = 8$ ✔ (true) — The ordered pair $(1, \frac{10}{3})$ is a solution to the equation.

Skill Practice Determine whether each ordered pair is a solution for the equation $x + 4y = -8$.

1a. $(-2, -1)$ **b.** $(4, -3)$ **c.** $(-14, 1.5)$

2. Graphing Linear Equations in Two Variables

Consider the linear equation $x - y = 3$. The solutions to the equation are ordered pairs such that the difference of x and y is 3. Several solutions are given in the following list:

Solution	Check
(x, y)	$x - y = 3$
$(3, 0)$	$3 - 0 = 3$ ✔
$(4, 1)$	$4 - 1 = 3$ ✔
$(0, -3)$	$0 - (-3) = 3$ ✔
$(-1, -4)$	$-1 - (-4) = 3$ ✔
$(2, -1)$	$2 - (-1) = 3$ ✔

Skill Practice Answers

1a. Not a solution
b. Solution
c. Solution

By graphing these ordered pairs, we see that the solution points line up (see Figure 2-6). There are actually an infinite number of solutions to the equation $x - y = 3$. The graph of all solutions to a linear equation forms a line in the xy-plane. Conversely, each ordered pair on the line is a solution to the equation.

To graph a linear equation, it is sufficient to find two solution points and draw the line between them. We will find three solution points and use the third point as a check point. This is demonstrated in Example 2.

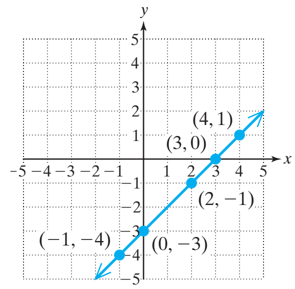


Figure 2-6

Example 2 Graphing a Linear Equation in Two Variables

Graph the equation $3x + 5y = 15$.

Solution:

We will find three ordered pairs that are solutions to the equation. In the table, we have selected arbitrary values for x or y and must complete the ordered pairs.

x	y	
0		$\longrightarrow$ (0,)
	2	$\longrightarrow$ (, 2)
5		$\longrightarrow$ (5,)

From the first row, substitute $x = 0$.

$$\begin{aligned} 3x + 5y &= 15 \\ 3(0) + 5y &= 15 \\ 5y &= 15 \\ y &= 3 \end{aligned}$$

From the second row, substitute $y = 2$.

$$\begin{aligned} 3x + 5y &= 15 \\ 3x + 5(2) &= 15 \\ 3x + 10 &= 15 \\ 3x &= 5 \\ x &= \frac{5}{3} \end{aligned}$$

From the third row, substitute $x = 5$.

$$\begin{aligned} 3x + 5y &= 15 \\ 3(5) + 5y &= 15 \\ 15 + 5y &= 15 \\ 5y &= 0 \\ y &= 0 \end{aligned}$$

The completed list of ordered pairs is shown as follows. To graph the equation, plot the three solutions and draw the line through the points (Figure 2-7). Arrows on the ends of the line indicate that points on the line extend infinitely in both directions.

x	y	
0	3	$\longrightarrow$ (0, 3)
$\frac{5}{3}$	2	$\longrightarrow$ $\left(\frac{5}{3}, 2\right)$
5	0	$\longrightarrow$ (5, 0)

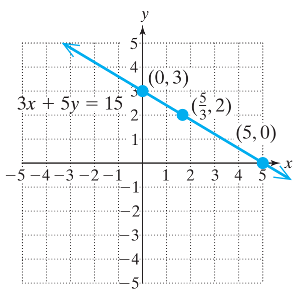


Figure 2-7

Skill Practice

2. Given $2x - y = 1$, complete the table and graph the line through the points.

x	y
0	
	5
1	

Example 3 Graphing a Linear Equation in Two Variables

Graph the equation $y = \frac{1}{2}x - 2$.

Solution:

Because the y-variable is isolated in the equation, it is easy to substitute a value for x and simplify the right-hand side to find y. Since any number for x can be used, choose numbers that are multiples of 2 that will simplify easily when multiplied by $\frac{1}{2}$.

x	y
0	
2	
4	

Substitute $x = 0$.

$y = \frac{1}{2}(0) - 2$

$y = 0 - 2$

$y = -2$

Substitute $x = 2$.

$y = \frac{1}{2}(2) - 2$

$y = 1 - 2$

$y = -1$

Substitute $x = 4$.

$y = \frac{1}{2}(4) - 2$

$y = 2 - 2$

$y = 0$

The completed list of ordered pairs is as follows. To graph the equation, plot the three solutions and draw the line through the points (Figure 2-8).

x	y	
0	-2	$\longrightarrow (0, -2)$
2	-1	$\longrightarrow (2, -1)$
4	0	$\longrightarrow (4, 0)$

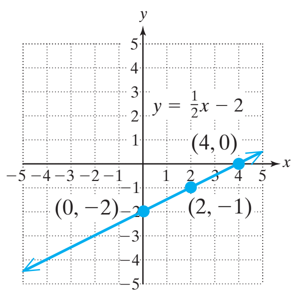


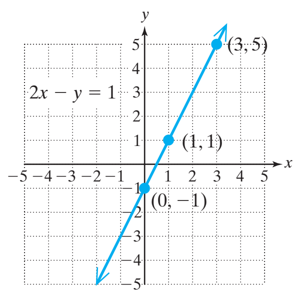
Figure 2-8

Skill Practice

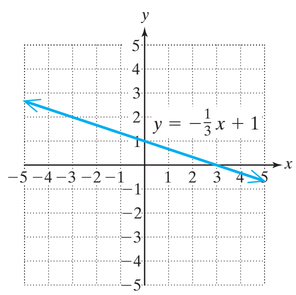
3. Graph the equation $y = -\frac{1}{3}x + 1$. *Hint:* Select values of x that are multiples of 3.

Skill Practice Answers

2.



3.



3. x-Intercepts and y-Intercepts

For many applications of graphing, it is advantageous to know the points where a graph intersects the x- or y-axis. These points are called the x- and y-intercepts.

In Figure 2-7, the x-intercept is $(5, 0)$. In Figure 2-8, the x-intercept is $(4, 0)$. In general, a point on the x-axis must have a y-coordinate of zero. In Figure 2-7, the y-intercept is $(0, 3)$. In Figure 2-8, the y-intercept is $(0, -2)$. In general, a point on the y-axis must have an x-coordinate of zero.

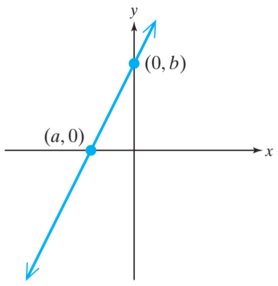


Figure 2-9

Definition of x- and y-Intercepts

An **x-intercept*** is a point $(a, 0)$ where a graph intersects the x-axis. (see Figure 2-9.)

A **y-intercept** is a point $(0, b)$ where a graph intersects the y-axis. (see Figure 2-9.)

*In some applications, an x-intercept is defined as the *x-coordinate* of a point of intersection that a graph makes with the x-axis. For example, if an x-intercept is at the point $(3, 0)$, it is sometimes stated simply as 3 (the y-coordinate is understood to be zero). Similarly, a y-intercept is sometimes defined as the *y-coordinate* of a point of intersection that a graph makes with the y-axis. For example, if a y-intercept is at the point $(0, 7)$, it may be stated simply as 7 (the x-coordinate is understood to be zero).

To find the x- and y-intercepts from an equation in x and y, follow these steps:

Steps to Find the x- and y-Intercepts from an Equation

Given an equation in x and y,

1. Find the x-intercept(s) by substituting $y = 0$ into the equation and solving for x.
2. Find the y-intercept(s) by substituting $x = 0$ into the equation and solving for y.

Example 4 Finding the x- and y-Intercepts of a Line

Find the x- and y-intercepts of the line $2x + 4y = 8$. Then graph the line.

Solution:

To find the x-intercept, substitute $y = 0$.

$$2x + 4y = 8$$
$$2x + 4(0) = 8$$
$$2x = 8$$
$$x = 4$$

The x-intercept is $(4, 0)$.

To find the y-intercept, substitute $x = 0$.

$$2x + 4y = 8$$
$$2(0) + 4y = 8$$
$$4y = 8$$
$$y = 2$$

The y-intercept is $(0, 2)$.

In this case, the intercepts are two distinct points and may be used to graph the line. A third point can be found to verify that the points all fall on the same line (points that lie on the same line are said to be *collinear*). Choose a different value for either x or y, such as $y = 4$.

$$2x + 4y = 8$$

$$2x + 4(4) = 8 \qquad \text{Substitute } y = 4.$$

$$2x + 16 = 8 \qquad \text{Solve for } x.$$

$$2x = -8$$

$$x = -4 \qquad \text{The point } (-4, 4) \text{ lines up with the other two points (Figure 2-10).}$$

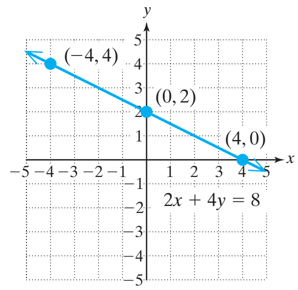


Figure 2-10

Skill Practice

4. Given $y = 2x - 4$, find the x- and y-intercepts. Then graph the line.

Example 5 Finding the *x*- and *y*-Intercepts of a Line

Find the x- and y-intercepts of the line $y = \frac{1}{4}x$. Then graph the line.

Solution:

To find the x-intercept, substitute $y = 0$.

$$y = \frac{1}{4}x$$

$$(0) = \frac{1}{4}x$$

$$0 = x$$

The x-intercept is $(0, 0)$.

To find the y-intercept, substitute $x = 0$.

$$y = \frac{1}{4}x$$

$$y = \frac{1}{4}(0)$$

$$y = 0$$

The y-intercept is $(0, 0)$.

Notice that the x- and y-intercepts are both located at the origin $(0, 0)$. In this case, the intercepts do not yield two distinct points. Therefore, another point is necessary to draw the line. We may pick any value for either x or y. However, for this equation, it would be particularly convenient to pick a value for x that is a multiple of 4 such as $x = 4$.

$$y = \frac{1}{4}x$$

$$y = \frac{1}{4}(4) \qquad \text{Substitute } x = 4.$$

$$y = 1$$

The point $(4, 1)$ is a solution to the equation (Figure 2-11).

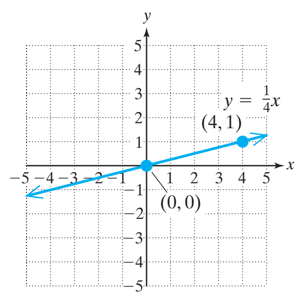


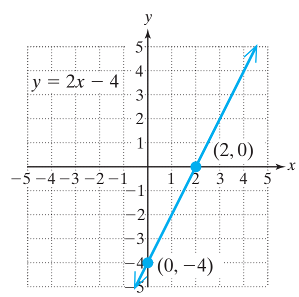
Figure 2-11

Skill Practice

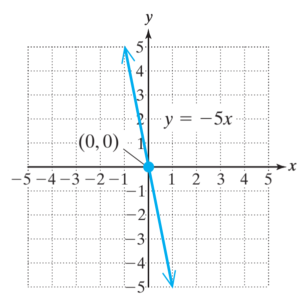
5. Given $y = -5x$, find the x- and y-intercepts. Then graph the line.

Skill Practice Answers

4.



5.



Example 6 Interpreting the *x*- and *y*-Intercepts of a Line

Companies and corporations are permitted to depreciate assets that have a known useful life span. This accounting practice is called *straight-line depreciation*. In this procedure the useful life span of the asset is determined, and then the asset is depreciated by an equal amount each year until the taxable value of the asset is equal to zero.

The J. M. Gus trucking company purchases a new truck for \$65,000. The truck will be depreciated at \$13,000 per year. The equation that describes the depreciation line is

$$y = 65{,}000 - 13{,}000x$$

where y represents the value of the truck in dollars and x is the age of the truck in years.

a. Find the x- and y-intercepts. Plot the intercepts on a rectangular coordinate system, and draw the line that represents the straight-line depreciation.

b. What does the x-intercept represent in the context of this problem?

c. What does the y-intercept represent in the context of this problem?

Solution:

a. To find the x-intercept, substitute $y = 0$.

$$0 = 65{,}000 - 13{,}000x$$

$$13{,}000x = 65{,}000$$

$$x = 5$$

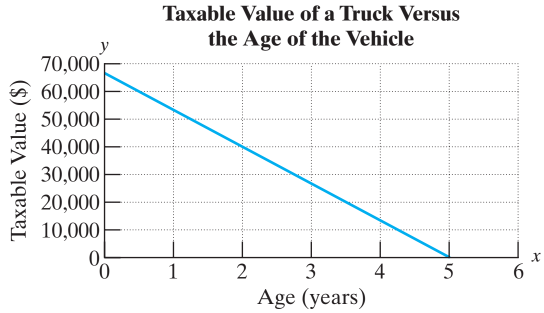
The x-intercept is $(5, 0)$.

To find the y-intercept, substitute $x = 0$.

$$y = 65{,}000 - 13{,}000(0)$$

$$y = 65{,}000$$

The y-intercept is $(0, 65{,}000)$.



b. The x-intercept $(5, 0)$ indicates that when the truck is 5 years old, the taxable value of the truck will be \$0.

c. The y-intercept $(0, 65{,}000)$ indicates that when the truck was new (0 years old), its taxable value was \$65,000.

Skill Practice

6. Acme motor company tests the engines of its trucks by running the engines in a laboratory. The engines burn 4 gal of fuel per hour. The engines begin the test with 30 gal of fuel. The equation $y = 30 - 4x$ represents the amount of fuel y left in the engine after x hours.

a. Find the x- and y-intercepts.

b. Interpret the y-intercept in the context of this problem.

c. Interpret the x-intercept in the context of this problem.

Skill Practice Answers

6a. x-intercept: $(7.5, 0)$; y-intercept: $(0, 30)$

b. The y-intercept $(0, 30)$ represents the amount of fuel in the truck initially (after 0 hr). After 0 hr, the tank contains 30 gal of fuel.

c. The x-intercept $(7.5, 0)$ represents the amount of fuel in the truck after 7.5 hr. After 7.5 hr the tank contains 0 gal. It is empty.

4. Horizontal and Vertical Lines

Recall that a linear equation can be written in the form $Ax + By = C$, where A and B are not both zero. If either A or B is 0, then the resulting line is horizontal or vertical, respectively.

Definitions of Vertical and Horizontal Lines

1. A **vertical line** is a line that can be written in the form $x = k$, where k is a constant.
2. A **horizontal line** is a line that can be written in the form $y = k$, where k is a constant.

Example 7 Graphing a Vertical Line

Graph the line $x = 6$.

Solution:

Because this equation is in the form $x = k$, the line is vertical and must cross the x-axis at $x = 6$. We can also construct a table of solutions to the equation $x = 6$. The choice for the x-coordinate must be 6, but y can be any real number (Figure 2-12).

x	y
6	−8
6	1
6	4
6	8

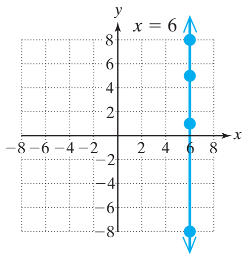


Figure 2-12

Skill Practice

7. Graph the line $x = -4$.

Example 8 Graphing a Horizontal Line

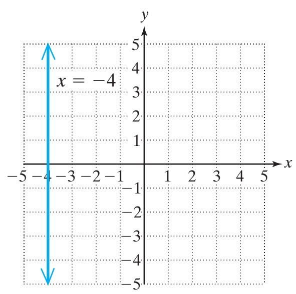
Graph the line $4y = -7$.

Solution:

The equation $4y = -7$ is equivalent to $y = -\frac{7}{4}$. Because the line is in the form $y = k$, the line must be horizontal and must pass through the y-axis at $y = -\frac{7}{4}$ (Figure 2-13).

Skill Practice Answers

7.



We can also construct a table of solutions to the equation $4y = -7$. The choice for the y-coordinate must be $-\frac{7}{4}$, but x can be any real number.

x	y
0	$-\frac{7}{4}$
-3	$-\frac{7}{4}$
2	$-\frac{7}{4}$

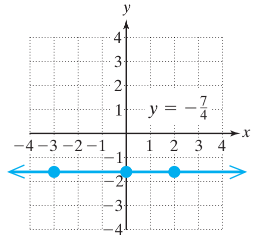


Figure 2-13

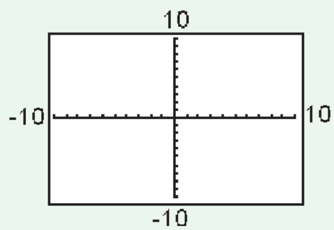
Skill Practice

8. Graph the line $-2y = 9$.

Calculator Connections

A viewing window of a graphing calculator shows a portion of a rectangular coordinate system. The *standard viewing window* for most calculators shows both the x- and y-axes between -10 and 10. Furthermore, the scale defined by the tick marks on both axes is usually set to 1.

The standard viewing window.



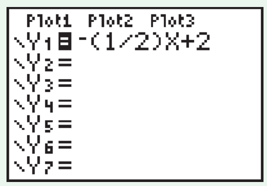
Linear equations can be analyzed with a graphing calculator.

- It is important to isolate the y-variable in the equation. Then enter the equation in the calculator. For example, to enter the equation from Example 4, we have:

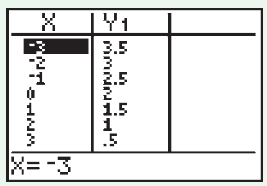
$$2x + 4y = 8 \longrightarrow 4y = -2x + 8$$

$$\frac{4y}{4} = \frac{-2x}{4} + \frac{8}{4}$$

$$y = -\frac{1}{2}x + 2$$

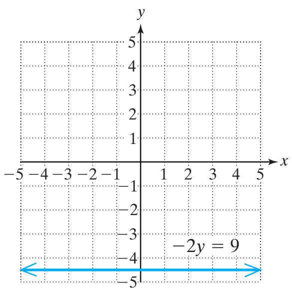


- A *Table* feature can be used to find many solutions to an equation. Several solutions to $y = -\frac{1}{2}x + 2$ are shown here.

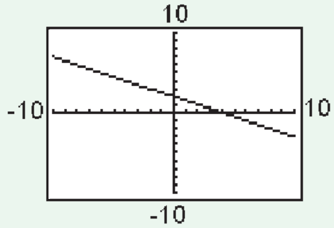


Skill Practice Answers

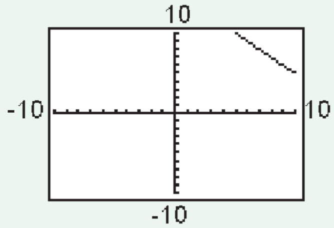
8.



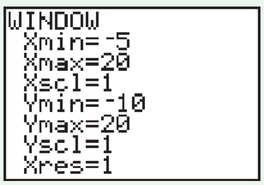
- A *Graph* feature can be used to graph a line.

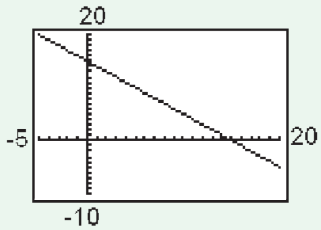


Sometimes the standard viewing window does not provide an adequate display for the graph of an equation. For example, in the standard viewing window, the graph of $y = -x + 15$ is visible only in a small portion of the upper right corner.



To see the x- and y-intercepts of this line, we can change the viewing window to accommodate larger values of x and y. Most calculators have a *Range* or *Window* feature that enables the user to change the minimum and maximum x- and y-values. In this case, we changed the values of x to range between -5 and 20, and the values of y to range between -10 and 20.





Section 2.2 Practice Exercises

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Study Skills Exercises

1. A good technique for studying for a test is to choose four problems from each section of the chapter and write each of them along with the directions on a 3×5 card. On the back, put the page number where you found that problem. Then shuffle the cards and test yourself on the procedure to solve each problem. For any that you do not know how to solve, look at the page number and do several of that type. Write which four problems you would choose for this section.

2. Define the key terms.

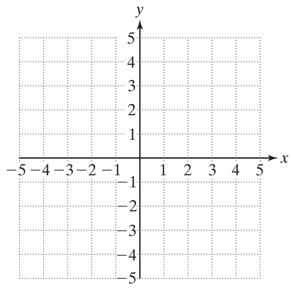
a. **Linear equation in two variables** **b.** ***x*-Intercept** **c.** ***y*-Intercept**

d. **Vertical line** **e.** **Horizontal line**

Review Exercises

3. Plot each point on a rectangular coordinate system, and identify the quadrant or axis where it is located.

a. $A(2, -3)$ **b.** $B(-1, -1)$ **c.** $C(4, 2)$ **d.** $D(0, -4)$



For Exercises 4–6, find the midpoint of the line segment between the given points. Check your answer by graphing the line segment and midpoint.

4. $(-3, 1)$ and $(-15, -1)$ **5.** $(7, 8)$ and $(-4, 1)$ **6.** $(-2, 10)$ and $(-2, 0)$

Concept 1: Linear Equations in Two Variables

For Exercises 7–10, determine if the ordered pair is a solution to the linear equation.

7. $2x - 3y = 9$

a. $(0, -3)$ **b.** $(-6, 1)$ **c.** $\left(1, -\frac{7}{3}\right)$

8. $-5x - 2y = 6$

a. $(0, 3)$ **b.** $\left(-\frac{6}{5}, 0\right)$ **c.** $(-2, 2)$

9. $x = \frac{1}{3}y + 1$

a. $(-1, 0)$ **b.** $(2, 3)$ **c.** $(-6, 1)$

10. $y = -\frac{3}{2}x - 4$

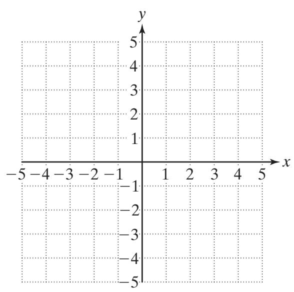
a. $(0, -4)$ **b.** $(2, -7)$ **c.** $(-4, -2)$

Concept 2: Graphing Linear Equations in Two Variables

For Exercises 11–14, complete the table. Then graph the line defined by the points.

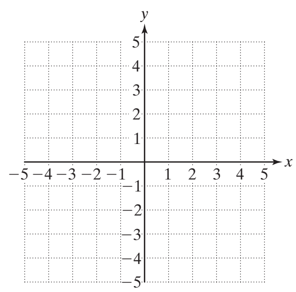
11. $3x - 2y = 4$

x	y
0	
	4
-1	



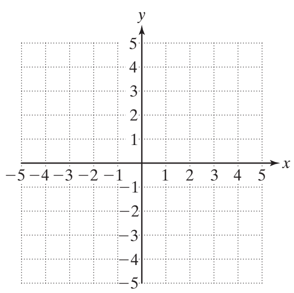
12. $4x + 3y = 6$

x	y
	2
3	
	-1



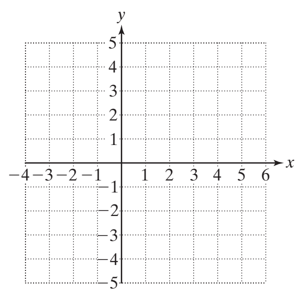
13. $y = -\frac{1}{5}x$

x	y
0	
5	
-5	



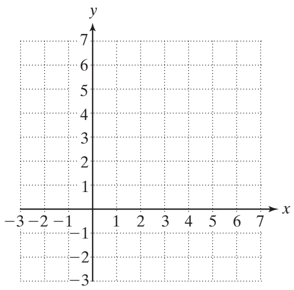
14. $y = \frac{1}{3}x$

x	y
0	
3	
6	

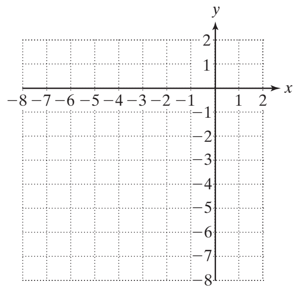


In Exercises 15–28, graph the linear equation.

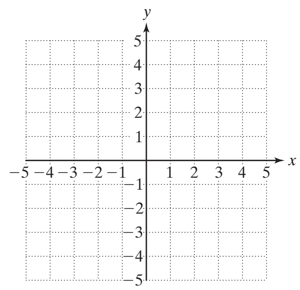
15. $x + y = 5$



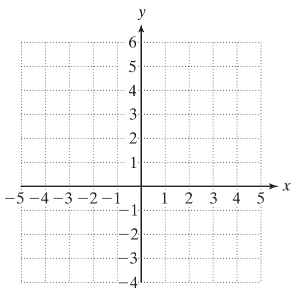
16. $x + y = -8$



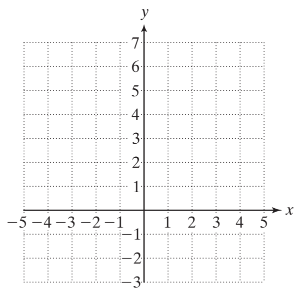
17. $3x - 4y = 12$



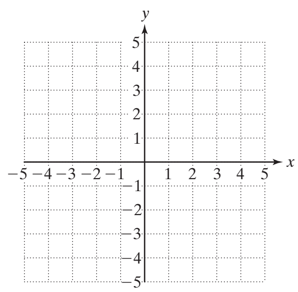
18. $5x + 3y = 15$



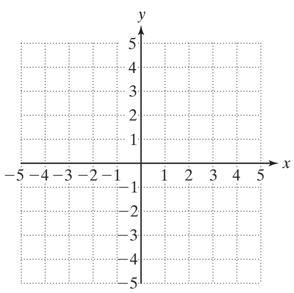
19. $y = -3x + 5$



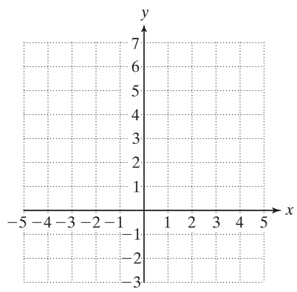
20. $y = -2x + 2$



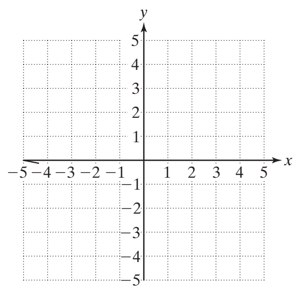
21. $y = \frac{2}{5}x - 1$



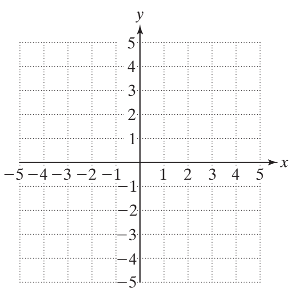
22. $y = \frac{5}{3}x + 1$



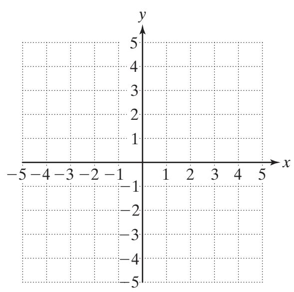
23. $x = -5y - 5$



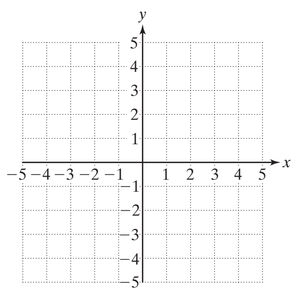
24. $x = 4y + 2$



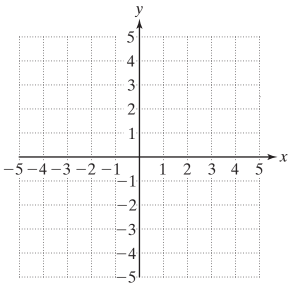
25. $3y = 4x - 12$



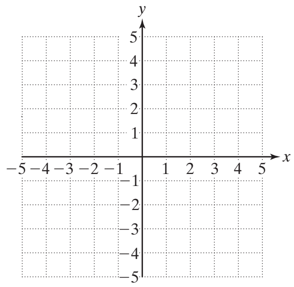
26. $2y = -3x + 2$



27. $x = 2y$



28. $x = -3y$



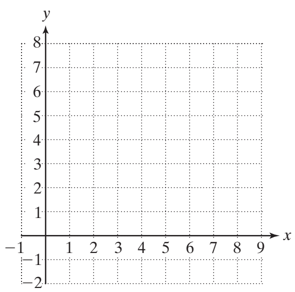
Concept 3: *x*-Intercepts and *y*-Intercepts

29. Given a linear equation, how do you find an x-intercept? How do you find a y-intercept?

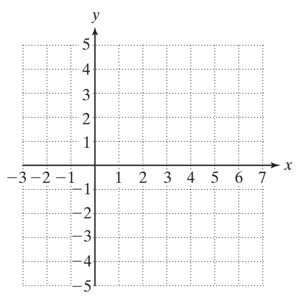
30. Can the point $(4, -1)$ be an x- or y-intercept? Why or why not?

For Exercises 31–42, **a.** find the x-intercept, **b.** find the y-intercept, and **c.** graph the line.

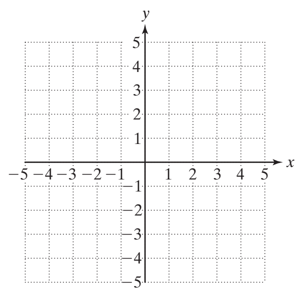
31. $2x + 3y = 18$



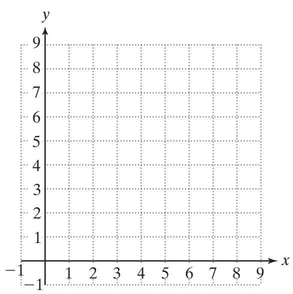
32. $2x - 5y = 10$



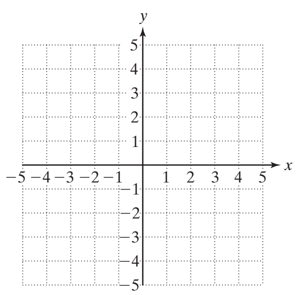
33. $x - 2y = 4$



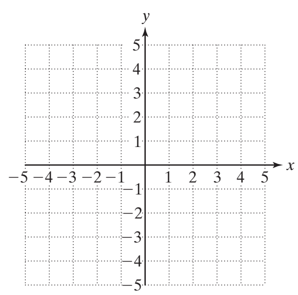
34. $x + y = 8$



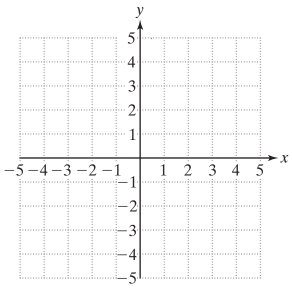
35. $5x = 3y$



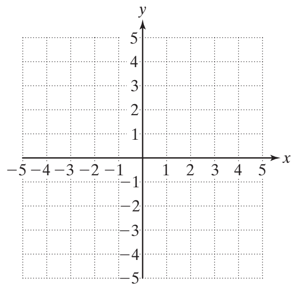
36. $3y = -5x$



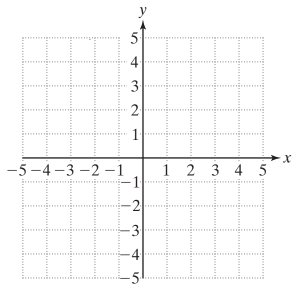
37. $y = 2x + 4$



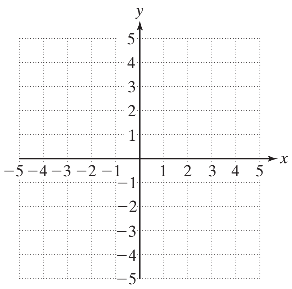
38. $y = -3x - 1$



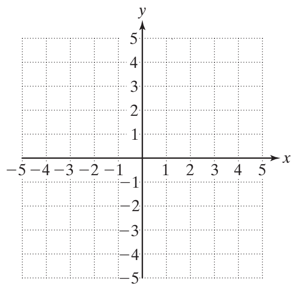
39. $y = -\frac{4}{3}x + 2$



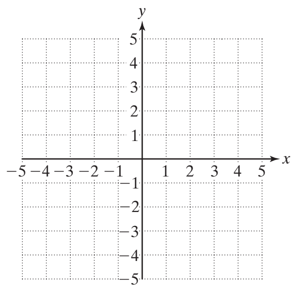
40. $y = -\frac{2}{5}x - 1$



41. $x = \frac{1}{4}y$



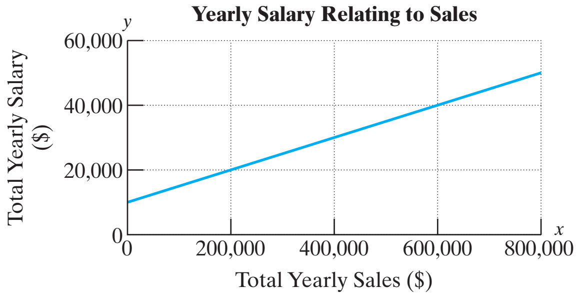
42. $x = \frac{2}{3}y$



43. A salesperson makes a base salary of \$10,000 a year plus a 5% commission on the total sales for the year. The yearly salary can be expressed as a linear equation as

$$y = 10{,}000 + 0.05x$$

where y represents the yearly salary and x represents the total yearly sales.



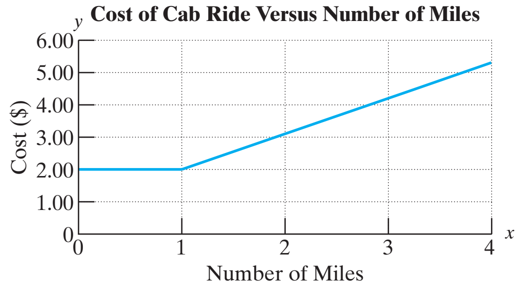
a. What is the salesperson's salary for a year in which his sales total \$500,000?

b. What is the salesperson's salary for a year in which his sales total \$300,000?

c. What does the y-intercept mean in the context of this problem?

d. Why is it unreasonable to use negative values for x in this equation?

44. A taxi company in Miami charges \$2.00 for any distance up to the first mile and \$1.10 for every mile thereafter. The cost of a cab ride can be modeled graphically.



a. Explain why the first part of the model is represented by a horizontal line.

b. What does the y-intercept mean in the context of this problem?

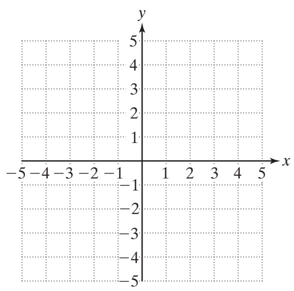
c. Explain why the line representing the cost of traveling more than 1 mi is not horizontal.

d. How much would it cost to take a cab $3\frac{1}{2}$ mi?

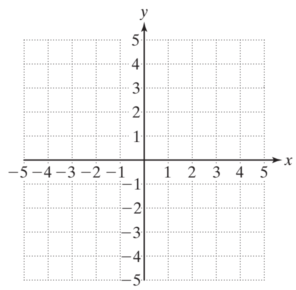
Concept 4: Horizontal and Vertical Lines

For Exercises 45–52, identify the line as either vertical or horizontal, and graph the line.

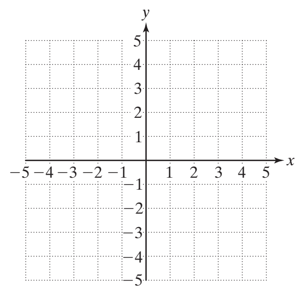
45. $y = -1$



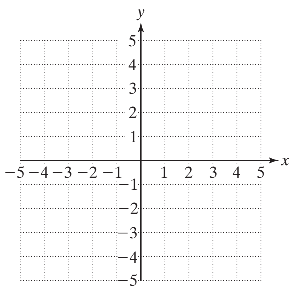
46. $y = 3$



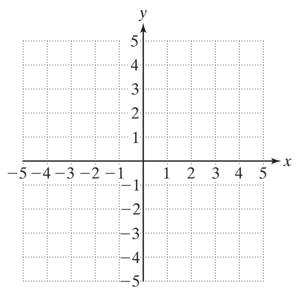
47. $x = 2$



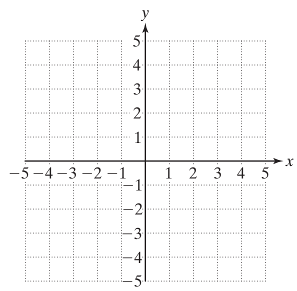
48. $x = -5$



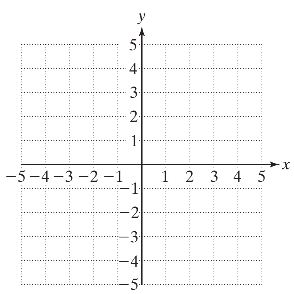
49. $2x + 6 = 5$



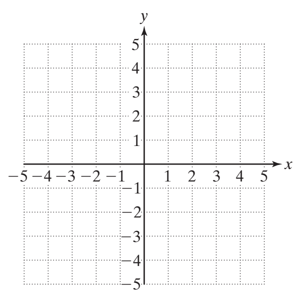
50. $-3x = 12$



51. $-2y + 1 = 9$



52. $-5y = -10$



Expanding Your Skills

For Exercises 53–55, find the x- and y-intercepts.

53. $\frac{x}{2} + \frac{y}{3} = 1$

54. $\frac{x}{7} + \frac{y}{4} = 1$

55. $\frac{x}{a} + \frac{y}{b} = 1$

Graphing Calculator Exercises

For Exercises 56–59, solve the equation for y. Use a graphing calculator to graph the equation on the standard viewing window.

56. $2x - 3y = 7$

57. $4x + 2y = -2$

58. $3y = 9$

59. $2y + 10 = 0$

For Exercises 60–63, use a graphing calculator to graph the lines on the suggested viewing window.

60. $y = -\frac{1}{2}x - 10$

$-30 \le x \le 10$
$-15 \le y \le 5$

61. $y = -\frac{1}{3}x + 12$

$-10 \le x \le 40$
$-10 \le y \le 20$

62. $-2x + 4y = 1$

$-1 \le x \le 1$
$-1 \le y \le 1$

63. $5y = 4x - 1$

$-0.5 \le x \le 0.5$
$-0.5 \le y \le 0.5$

For Exercises 64–65, graph the lines in parts (a)–(c) on the same viewing window. Compare the graphs. Are the lines exactly the same?

64. **a.** $y = x + 3$

b. $y = x + 3.1$

c. $y = x + 2.9$

65. **a.** $y = 2x + 1$

b. $y = 1.9x + 1$

c. $y = 2.1x + 1$

Section 2.3 Slope of a Line

Concepts

1. Introduction to the Slope of a Line
2. The Slope Formula
3. Parallel and Perpendicular Lines
4. Applications and Interpretation of Slope

1. Introduction to the Slope of a Line

In Section 2.2, we learned how to graph a linear equation and to identify its x- and y-intercepts. In this section, we learn about another important feature of a line called the *slope* of a line. Geometrically, slope measures the "steepness" of a line.

Figure 2-14 shows a set of stairs with a wheelchair ramp to the side. Notice that the stairs are steeper than the ramp.

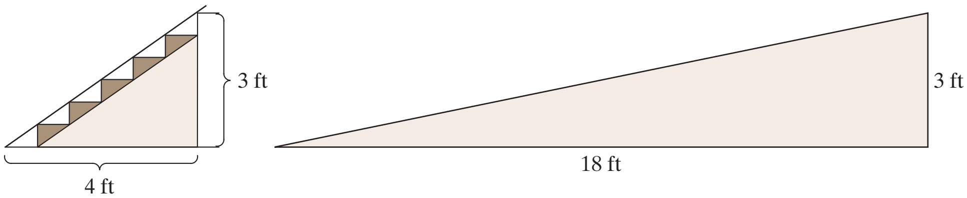
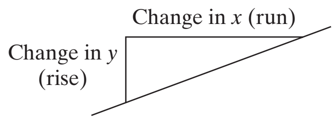


Figure 2-14

To measure the slope of a line quantitatively, consider two points on the line. The slope is the ratio of the vertical change between the two points to the horizontal change. That is, the slope is the ratio of the change in y to the change in x. As a memory device, we might think of the slope of a line as "rise over run."

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$



To move from point A to point B on the stairs, rise 3 ft and move to the right 4 ft (Figure 2-15).

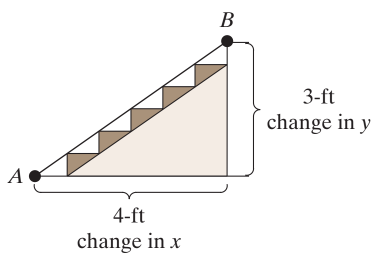


Figure 2-15

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{3 \text{ ft}}{4 \text{ ft}} = \frac{3}{4}$$

To move from point A to point B on the wheelchair ramp, rise 3 ft and move to the right 18 ft (Figure 2-16).

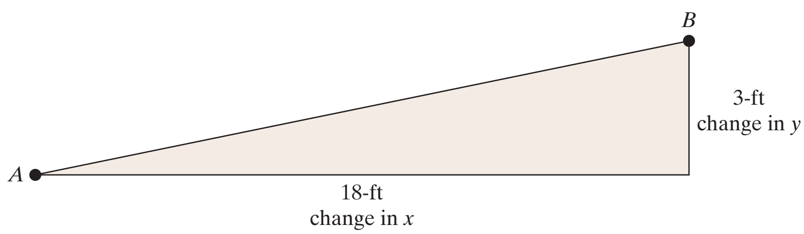


Figure 2-16

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{3 \text{ ft}}{18 \text{ ft}} = \frac{1}{6}$$

The slope of the stairs is $\frac{3}{4}$ which is greater than the slope of the ramp, which is $\frac{1}{6}$.

Example 1 Finding the Slope in an Application

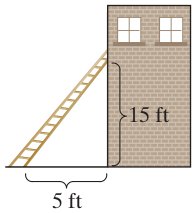
Find the slope of the ladder against the wall.

Solution:

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{15 \text{ ft}}{5 \text{ ft}}$$

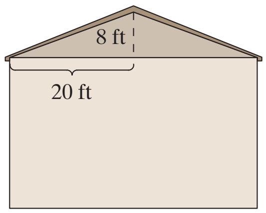
$$= \frac{3}{1} \text{ or } 3$$



The slope is $\frac{3}{1}$ which indicates that a person climbs 3 ft for every 1 ft traveled horizontally.

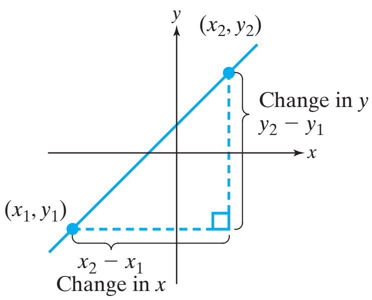
Skill Practice

1. Find the slope of the roof.



2. The Slope Formula

The slope of a line may be found by using *any* two points on the line—call these points (x_1, y_1) and (x_2, y_2). The change in y between the points can be found by taking the difference of the y-values: $y_2 - y_1$. The change in x can be found by taking the difference of the x-values in the same order: $x_2 - x_1$.



The slope of a line is often symbolized by the letter m and is given by the following formula.

Definition of the Slope of a Line

The **slope** of a line passing through the distinct points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{provided} \qquad x_2 - x_1 \neq 0$$

Skill Practice Answers

1. $\frac{2}{5}$

Example 2 Finding the Slope of a Line Through Two Points

Find the slope of the line passing through the points $(1, -1)$ and $(7, 2)$.

Solution:

To use the slope formula, first label the coordinates of each point, and then substitute their values into the slope formula.

$(1, -1)$ and $(7, 2)$

(x_1, y_1) (x_2, y_2) Label the points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-1)}{7 - 1} \quad \text{Apply the slope formula.}$$

$$= \frac{3}{6} \quad \text{Simplify.}$$

$$= \frac{1}{2}$$

The slope of the line can be verified from the graph (Figure 2-17).

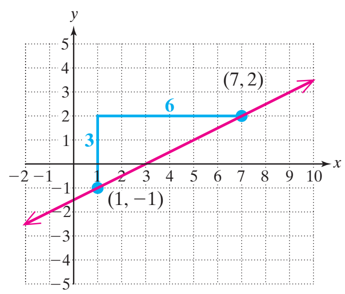


Figure 2-17

TIP: The slope formula does not depend on which point is labeled (x_1, y_1) and which point is labeled (x_2, y_2). For example, reversing the order in which the points are labeled in Example 2 results in the same slope:

$(1, -1)$ and $(7, 2)$

(x_2, y_2) (x_1, y_1)

then $m = \dfrac{-1 - 2}{1 - 7} = \dfrac{-3}{-6} = \dfrac{1}{2}$

Skill Practice

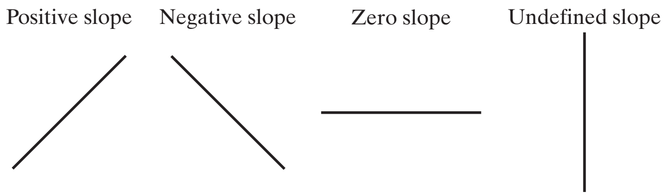
2. Find the slope of the line that passes through the points $(-4, 5)$ and $(6, 8)$.

Skill Practice Answers

2. $\frac{3}{10}$

When you apply the slope formula, you will see that the slope of a line may be positive, negative, zero, or undefined.

- Lines that "increase," or "rise," from left to right have a *positive slope*.
- Lines that "decrease," or "fall," from left to right have a *negative slope*.
- Horizontal lines have a *zero slope*.
- Vertical lines have an *undefined slope*.



Example 3

Finding the Slope of a Line Between Two Points

Find the slope of the line passing through the points $(3, -4)$ and $(-5, -1)$.

Solution:

$(3, -4)$ and $(-5, -1)$

(x_1, y_1) (x_2, y_2) Label points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-4)}{-5 - 3}$$ Apply the slope formula.

$$= \frac{3}{-8} = -\frac{3}{8}$$ Simplify.

The two points can be graphed to verify that $-\frac{3}{8}$ is the correct slope (Figure 2-18).

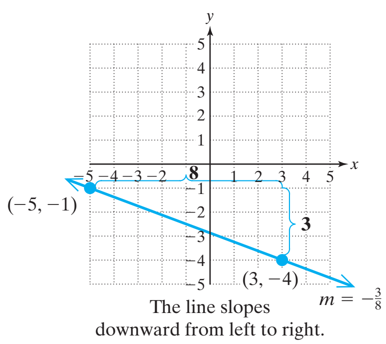


Figure 2-18

Skill Practice

3. Find the slope of the line that passes through the given points.

(1, –8) and (–5, –4)

Skill Practice Answers

3. $-\frac{2}{3}$

Example 4 Finding the Slope of a Line Between Two Points

a. Find the slope of the line passing through the points $(-3, 4)$ and $(-3, -2)$.

b. Find the slope of the line passing through the points $(0, 2)$ and $(-4, 2)$.

Solution:

a. $(-3, 4)$ and $(-3, -2)$

(x_1, y_1) (x_2, y_2) Label points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{-3 - (-3)} \quad \text{Apply slope formula.}$$

$$= \frac{-6}{-3 + 3}$$

$$= \frac{-6}{0} \quad \text{Undefined}$$

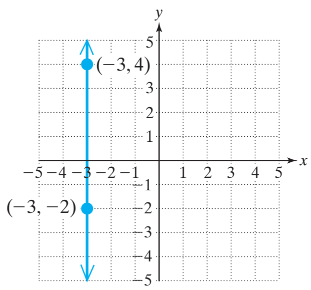


Figure 2-19

The slope is undefined. The points define a vertical line (Figure 2-19).

b. $(0, 2)$ and $(-4, 2)$

(x_1, y_1) and (x_2, y_2) Label the points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{-4 - 0} \quad \text{Apply the slope formula.}$$

$$= \frac{0}{-4}$$

$$= 0 \quad \text{Simplify.}$$

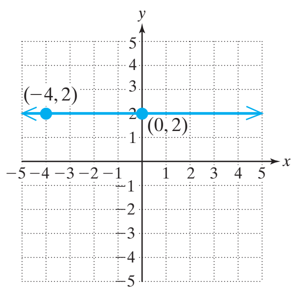


Figure 2-20

The slope is zero. The line through the two points is a horizontal line (Figure 2-20).

Skill Practice Find the slope of the line that passes through the given points.

4. $(5, -2)$ and $(5, 5)$ **5.** $(1, 6)$ and $(-7, 6)$

3. Parallel and Perpendicular Lines

Lines in the same plane that do not intersect are *parallel.* Nonvertical parallel lines have the same slope and different y-intercepts (Figure 2-21).

Lines that intersect at a right angle are *perpendicular.* If two lines are perpendicular, then the slope of one line is the opposite of the reciprocal of the slope of the other (provided neither line is vertical) (Figure 2-22).

Slopes of Parallel Lines

If m_1 and m_2 represent the slopes of two parallel (nonvertical) lines, then

$$m_1 = m_2$$

See Figure 2-21.

Skill Practice Answers

4. Undefined **5.** 0

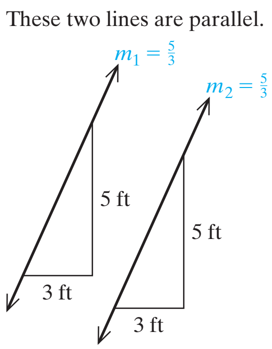


Figure 2-21

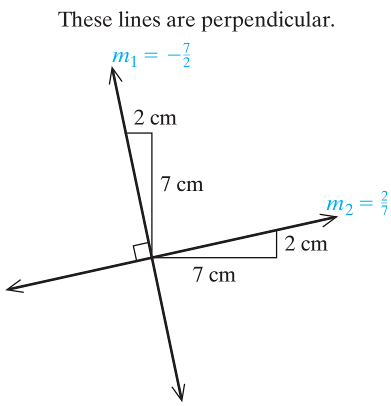


Figure 2-22

Slopes of Perpendicular Lines

If $m_1 \neq 0$ and $m_2 \neq 0$ represent the slopes of two perpendicular lines, then

$$m_1 = -\frac{1}{m_2} \text{ or equivalently, } m_1 \cdot m_2 = -1.$$

See Figure 2-22.

Example 5 Determining the Slope of Parallel and Perpendicular Lines

Suppose a given line has a slope of -5.

a. Find the slope of a line parallel to the given line.

b. Find the slope of a line perpendicular to the given line.

Solution:

a. The slope of a line parallel to the given line is $m = -5$ (same slope).

b. The slope of a line perpendicular to the given line is $m = \frac{1}{5}$ (the opposite of the reciprocal of -5).

Skill Practice

6. The slope of line L_1 is $-\frac{4}{3}$.

a. Find the slope of a line parallel to L_1.

b. Find the slope of a line perpendicular to L_1.

Example 6 Determining Whether Two Lines Are Parallel, Perpendicular, or Neither

Two points are given from each of two lines: L_1 and L_2. Without graphing the points, determine if the lines are parallel, perpendicular, or neither.

L_1: $(2, -3)$ and $(4, 1)$

L_2: $(5, -6)$ and $(-3, -2)$

Skill Practice Answers

6a. $-\frac{4}{3}$ **b.** $\frac{3}{4}$

Solution:

First determine the slope of each line. Then compare the values of the slopes to determine if the lines are parallel or perpendicular.

For line 1:

L_1: (2, −3) and (4, 1)

(x_1, y_1) (x_2, y_2)

$$m = \frac{1 - (-3)}{4 - 2} = \frac{4}{2} = 2$$

For line 2:

L_2: (5, −6) and (−3, −2)

(x_1, y_1) (x_2, y_2) Label the points.

$$m = \frac{-2 - (-6)}{-3 - (5)} = \frac{4}{-8} = -\frac{1}{2}$$

Apply the slope formula.

The slope of L_1 is 2. The slope of L_2 is $-\frac{1}{2}$. The slope of L_1 is the opposite of the reciprocal of L_2. By comparing the slopes, the lines must be perpendicular.

Skill Practice

7. Two points are given for lines L_1 and L_2. Determine if the lines are parallel, perpendicular, or neither.

L_1: (4, −1) and (−3, 6)

L_2: (−1, 3) and (2, 0)

4. Applications and Interpretation of Slope

Example 7 Interpreting the Slope of a Line in an Application

The number of males 20 years old or older who were employed full time in the United States varied linearly from 1970 to 2005. Approximately 43.0 million males 20 years old or older were employed full time in 1970. By 2005, this number grew to 65.4 million (Figure 2-23).

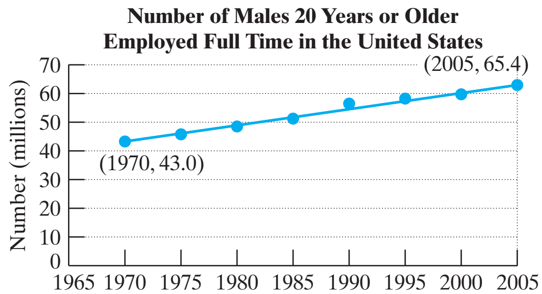


Figure 2-23

Source: Current population survey.

a. Find the slope of the line, using the points (1970, 43.0) and (2005, 65.4).

b. Interpret the meaning of the slope in the context of this problem.

Skill Practice Answers

7. Parallel

Solution:

a. (1970, 43.0) and (2005, 65.4)
(x_1, y_1) (x_2, y_2) Label the points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{65.4 - 43.0}{2005 - 1970}$$ Apply the slope formula.

$$m = \frac{22.4}{35} \quad \text{or} \quad m = 0.64$$

b. The slope is approximately 0.64, meaning that the full-time workforce has increased by approximately 0.64 million men (or 640,000 men) per year between 1970 and 2005.

Skill Practice The number of people per square mile in Alaska was 0.96 in 1990. This number increased to 1.17 in 2005.

8a. Find the slope of the line that represents the population growth of Alaska. Use the points (1990, 0.96) and (2005, 1.17).

b. Interpret the meaning of the slope in the context of this problem.

Skill Practice Answers

8a. 0.014
b. The population increased by 0.014 person per square mile per year.

Section 2.3 Practice Exercises

Boost *your* GRADE at mathzone.com!

MathZone

- Practice Problems
- Self-Tests
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- e-Professors
- Videos

Study Skills Exercises

1. Go to the online service called MathZone that accompanies this text (www.mathzone.com). Name two features that this online service offers that can help you in this course.

2. Define the key term **slope.**

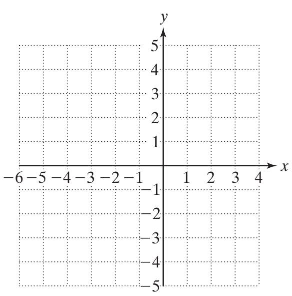
Review Exercises

3. Find the missing coordinate so that the ordered pairs are solutions to the equation $\frac{1}{2}x + y = 4$.

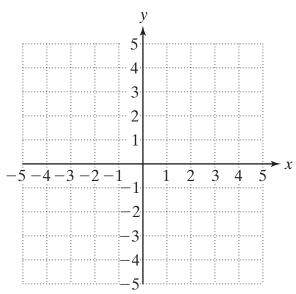
a. (0,) **b.** (, 0) **c.** (−4,)

For Exercises 4–7, find the x- and y-intercepts (if possible) for each equation, and sketch the graph.

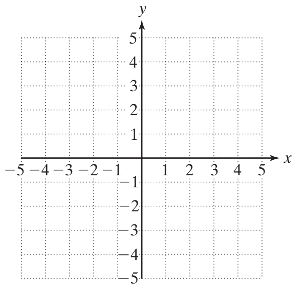
4. $2x + 8 = 0$



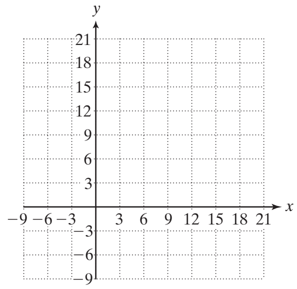
5. $4 - 2y = 0$



6. $2x - 2y - 6 = 0$

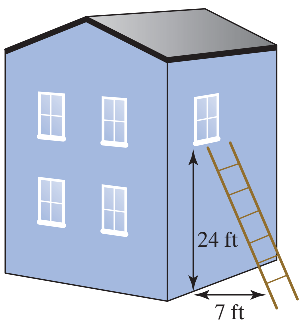


7. $x + \frac{1}{3}y = 6$

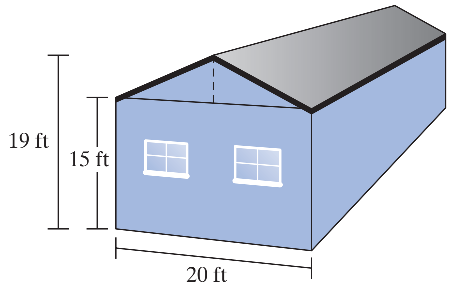


Concept 1: Introduction to the Slope of a Line

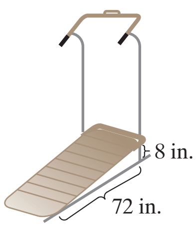
8. A 25-ft ladder is leaning against a house, as shown in the diagram. Find the slope of the ladder.



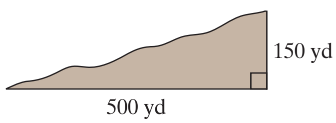
9. Find the pitch (slope) of the roof in the figure.



10. Find the slope of the treadmill.

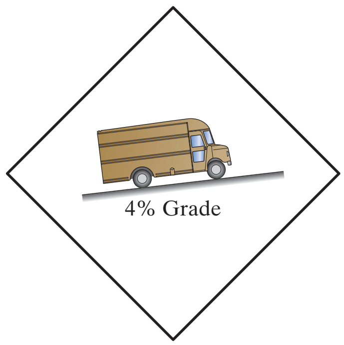


11. Find the average slope of the hill.



12. The road sign shown in the figure indicates the percent grade of a hill. This gives the slope of the road as the change in elevation per 100 horizontal ft. Given a 4% grade, write this as a slope in fractional form.

13. If a plane gains 1000 ft in altitude over a distance of 12,000 horizontal ft, what is the slope? Explain what this value means in the context of the problem.



Concept 2: The Slope Formula

For Exercises 14–29, use the slope formula to determine the slope of the line containing the two points.

14. $(6, 0)$ and $(0, -3)$

15. $(-5, 0)$ and $(0, -4)$

16. $(-2, 3)$ and $(1, -2)$

17. $(4, 5)$ and $(-1, 0)$

18. $(-2, 5)$ and $(-7, 1)$

19. $(4, -2)$ and $(3, -1)$

20. $(0.3, -1.1)$ and $(-0.1, -0.8)$

21. $(0.4, -0.2)$ and $(0.3, -0.1)$

22. $(2, 3)$ and $(2, 7)$

23. $(-1, 5)$ and $(-1, 0)$

24. $(5, -1)$ and $(-3, -1)$

25. $(-8, 4)$ and $(1, 4)$

26. $(-4.6, 4.1)$ and $(0, 6.4)$

27. $(1.1, 4)$ and $(-3.2, -0.3)$

28. $\left(\frac{3}{2}, \frac{4}{3}\right)$ and $\left(\frac{7}{2}, 1\right)$

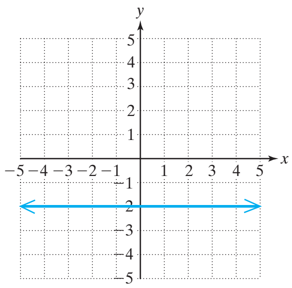
29. $\left(\frac{2}{3}, -\frac{1}{2}\right)$ and $\left(-\frac{1}{6}, -\frac{3}{2}\right)$

30. Explain how to use the graph of a line to determine whether the slope of a line is positive, negative, zero, or undefined.

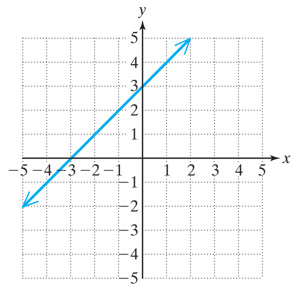
31. If the slope of a line is $\frac{4}{3}$, how many units of change in y will be produced by 6 units of change in x?

For Exercises 32–37, estimate the slope of the line from its graph.

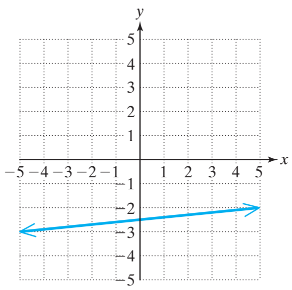
32.



33.



34.



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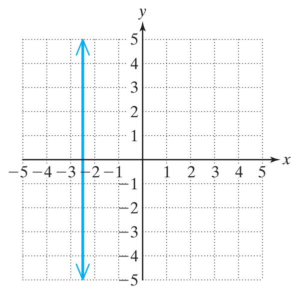
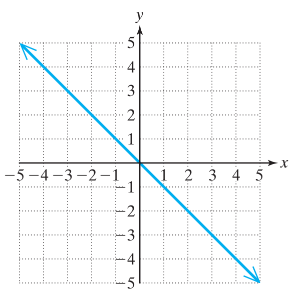
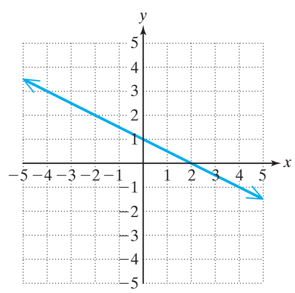


36.



37.



Concept 3: Parallel and Perpendicular Lines

38. Can the slopes of two perpendicular lines both be positive? Explain your answer.

For Exercises 39–44, the slope of a line is given.

a. Find the slope of a line parallel to the given line.

b. Find the slope of a line perpendicular to the given line.

39. $m = 5$

40. $m = 3$

41. $m = -\frac{4}{7}$

42. $m = -\frac{2}{11}$

43. $m = 0$

44. m is undefined.

In Exercises 45–52, two points are given from each of two lines L_1 and L_2. Without graphing the points, determine if the lines are perpendicular, parallel, or neither.

45. L_1: $(2, 5)$ and $(4, 9)$
L_2: $(-1, 4)$ and $(3, 2)$

46. L_1: $(-3, -5)$ and $(-1, 2)$
L_2: $(0, 4)$ and $(7, 2)$

47. L_1: $(4, -2)$ and $(3, -1)$
L_2: $(-5, -1)$ and $(-10, -16)$

48. L_1: $(0, 0)$ and $(2, 3)$
L_2: $(-2, 5)$ and $(0, -2)$

49. L_1: $(5, 3)$ and $(5, 9)$
L_2: $(4, 2)$ and $(0, 2)$

50. L_1: $(3, 5)$ and $(2, 5)$
L_2: $(2, 4)$ and $(0, 4)$

51. L_1: $(-3, -2)$ and $(2, 3)$
L_2: $(-4, 1)$ and $(0, 5)$

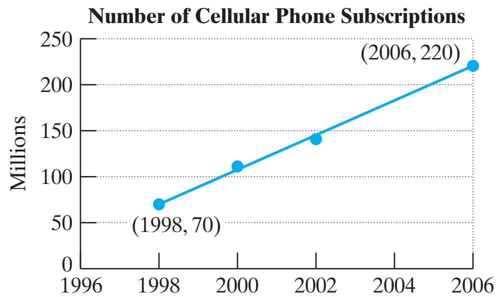
52. L_1: $(7, 1)$ and $(0, 0)$
L_2: $(-10, -8)$ and $(4, -6)$

Concept 4: Applications and Interpretation of Slope

53. The graph shows the number of cellular phone subscriptions (in millions) purchased in the United States for selected years.

a. Use the coordinates of the given points to find the slope of the line, and express the answer in decimal form.

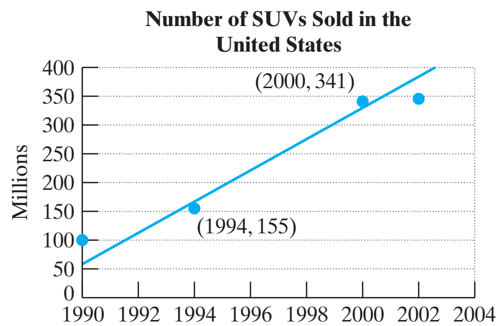
b. Interpret the meaning of the slope in the context of this problem.



54. The number of SUVs (in millions) sold in the United States grew approximately linearly between 1990 and 2002.

a. Find the slope of the line defined by the two given points.

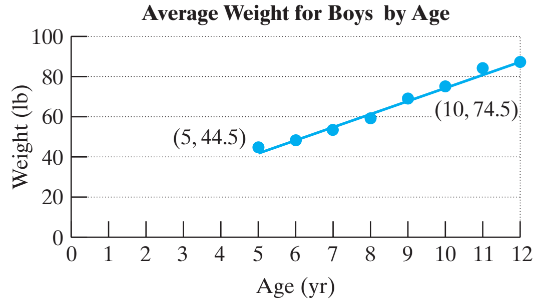
b. Interpret the meaning of the slope in the context of this problem.



55. The data in the graph show the average weight for boys based on age.

a. Use the coordinates of the given points to find the slope of the line.

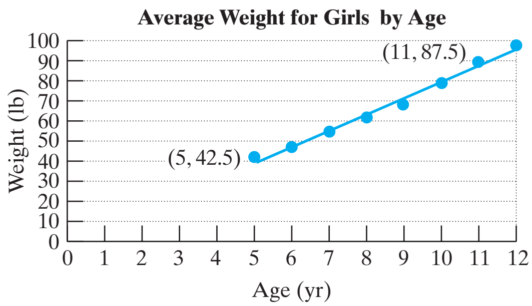
b. Interpret the meaning of the slope in the context of this problem.



56. The data in the graph show the average weight for girls based on age.

a. Use the coordinates of the given points to find the slope of the line, and write the answer in decimal form.

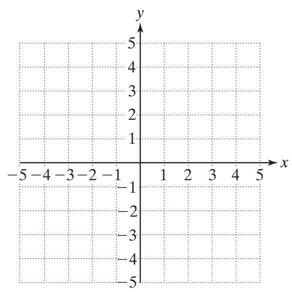
b. Interpret the meaning of the slope in the context of this problem.



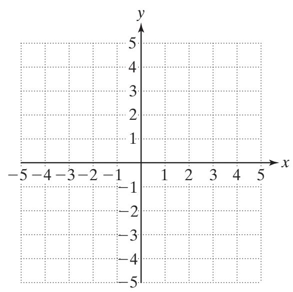
Expanding Your Skills

For Exercises 57–62, given a point P on a line and the slope m of the line, find a second point on the line (answers may vary). *Hint:* Graph the line to help you find the second point.

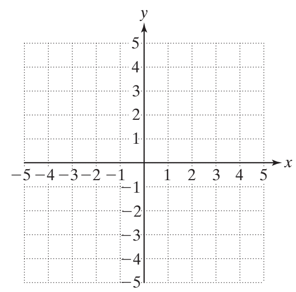
57. $P(0, 0)$ and $m = 2$



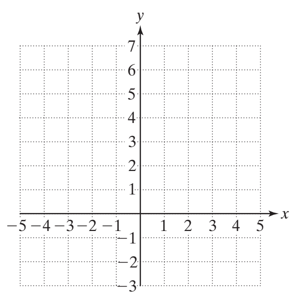
58. $P(-2, 1)$ and $m = -\frac{1}{3}$



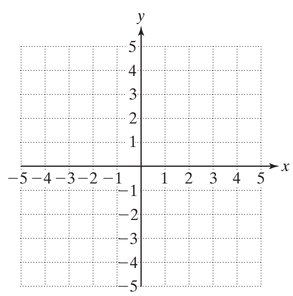
59. $P(2, -3)$ and m is undefined



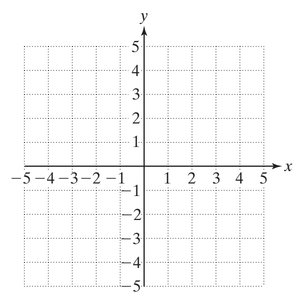
60. $P(-2, 4)$ and $m = 0$



61. $P(-1, 2)$ and $m = -\frac{2}{3}$



62. $P(-1, -4)$ and $m = \frac{4}{5}$



Section 2.4 Equations of a Line

Concepts

1. Slope-Intercept Form
2. The Point-Slope Formula
3. Different Forms of Linear Equations

1. Slope-Intercept Form

In Section 2.2, we learned that an equation of the form $Ax + By = C$ (where A and B are not both zero) represents a line in a rectangular coordinate system. An equation of a line written in this way is said to be in **standard form.** In this section, we will learn a new form, called the **slope-intercept form,** which is useful in determining the slope and y-intercept of a line.

Let $(0, b)$ represent the y-intercept of a line. Let (x, y) represent any other point on the line. Then the slope of the line through the two points is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{y - b}{x - 0} \qquad \text{Apply the slope formula.}$$

$$m = \frac{y - b}{x} \qquad \text{Simplify.}$$

$$m \cdot x = \left(\frac{y - b}{\cancel{x}}\right)\cancel{x} \qquad \text{Clear fractions.}$$

$$mx = y - b \qquad \text{Simplify.}$$

$$mx + b = y \quad \text{or} \quad y = mx + b \qquad \text{Solve for } y\text{: slope-intercept form.}$$

Slope-Intercept Form of a Line

$y = mx + b$ is the slope-intercept form of a line.

m is the slope and the point $(0, b)$ is the y-intercept.

The equation $y = -4x + 7$ is written in slope-intercept form. By inspection, we can see that the slope of the line is -4 and the y-intercept is $(0, 7)$.

Example 1 Finding the Slope and y-Intercept of a Line

Given the line $3x + 4y = 4$, write the equation of the line in slope-intercept form, then find the slope and y-intercept.

Solution:

Write the equation in slope-intercept form, $y = mx + b$, by solving for y.

$$3x + 4y = 4$$

$$4y = -3x + 4$$

$$\frac{4y}{4} = \frac{-3x}{4} + \frac{4}{4}$$

$$y = -\frac{3}{4}x + 1 \qquad \text{The slope is } -\frac{3}{4} \text{ and the } y\text{-intercept is } (0, 1).$$

Skill Practice

1. Write the equation in slope-intercept form. Determine the slope and the y-intercept.

$$2x - 4y = 3$$

Skill Practice Answers

1. $y = \frac{1}{2}x - \frac{3}{4}$

Slope: $\frac{1}{2}$; y-intercept: $\left(0, -\frac{3}{4}\right)$

The slope-intercept form is a useful tool to graph a line. The y-intercept is a known point on the line, and the slope indicates the "direction" of the line and can be used to find a second point. Using slope-intercept form to graph a line is demonstrated in Example 2.

Example 2 Graphing a Line by Using the Slope and *y*-Intercept

Graph the line $y = -\frac{3}{4}x + 1$ by using the slope and y-intercept.

Solution:

First plot the y-intercept (0, 1). The slope $m = -\frac{3}{4}$ can be written as

$$m = \frac{-3}{4}$$

The change in y is -3.

The change in x is 4.

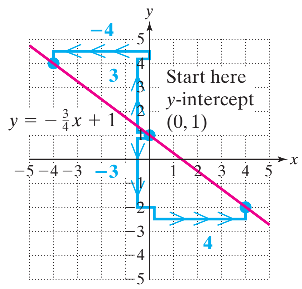


Figure 2-24

To find a second point on the line, start at the y-intercept and move *down* 3 *units* and to the *right* 4 *units*. Then draw the line through the two points (Figure 2-24).

Similarly, the slope can be written as

$$m = \frac{3}{-4}$$

The change in y is 3.

The change in x is -4.

To find a second point on the line, start at the y-intercept and move *up* 3 *units* and to the *left* 4 *units*. Then draw the line through the two points (see Figure 2-24).

Skill Practice

2. Graph the line $y = \frac{1}{5}x - 2$ by using the slope and y-intercept.

TIP: To graph a line using the y-intercept and slope, always begin by plotting a point at the y-intercept (not at the origin).

Two lines are parallel if they have the same slope and different y-intercepts. Two lines are perpendicular if the slope of one line is the opposite of the reciprocal of the slope of the other line. Otherwise, the lines are neither parallel nor perpendicular.

Example 3 Determining if Two Lines Are Parallel, Perpendicular, or Neither

Given the pair of linear equations, determine if the lines are parallel, perpendicular, or neither parallel nor perpendicular.

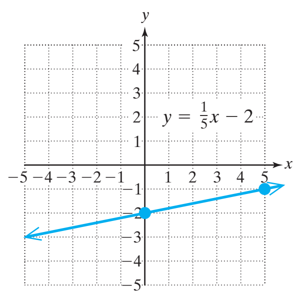
a. L_1: $y = -2x + 7$
L_2: $y = -2x - 1$

b. L_1: $2y = -3x + 2$
L_2: $-4x + 6y = -12$

c. L_1: $x + y = 6$
L_2: $y = 6$

Skill Practice Answers

2.



Solution:

a. The equations are written in slope-intercept form.

L_1: $y = -2x + 7$ The slope is -2 and the y-intercept is $(0, 7)$.

L_2: $y = -2x - 1$ The slope is -2 and the y-intercept is $(0, -1)$.

Because the slopes are the same and the y-intercepts are different, the lines are parallel.

b. Write each equation in slope-intercept form by solving for y.

L_1: $2y = -3x + 2$

$$\frac{2y}{2} = \frac{-3x}{2} + \frac{2}{2} \quad \text{Divide by 2.}$$

$$y = -\frac{3}{2}x + 1$$

L_2: $-4x + 6y = -12$

$$6y = 4x - 12 \quad \text{Add } 4x \text{ to both sides.}$$

$$\frac{6y}{6} = \frac{4}{6}x - \frac{12}{6} \quad \text{Divide by 6.}$$

$$y = \frac{2}{3}x - 2$$

The slope of L_1 is $-\frac{3}{2}$. The slope of L_2 is $\frac{2}{3}$.

The value $-\frac{3}{2}$ is the opposite of the reciprocal of $\frac{2}{3}$. Therefore, the lines are perpendicular.

c. L_1: $x + y = 6$ is equivalent to $y = -x + 6$. The slope is -1.

L_2: $y = 6$ is a horizontal line, and the slope is 0.

The slopes are not the same. Therefore, the lines are not parallel. The slope of one line is not the opposite of the reciprocal of the other slope. Therefore, the lines are not perpendicular. The lines are neither parallel nor perpendicular.

Skill Practice Given the pair of equations, determine if the lines are parallel, perpendicular, or neither.

3. $y = -\frac{3}{4}x + 1$
$y = \frac{4}{3}x + 3$

4. $3x + y = 4$
$6x = 6 - 2y$

5. $x - y = 7$
$x = 1$

Example 4 Using Slope-Intercept Form to Find an Equation of a Line

Use slope-intercept form to find an equation of a line with slope -3 and passing through the point $(1, -4)$.

Solution:

To find an equation of a line in slope-intercept form, $y = mx + b$, it is necessary to find the slope, m, and the y-intercept, b. The slope is given in the problem as $m = -3$. Therefore, the slope-intercept form becomes

$$y = mx + b$$

$$\downarrow$$

$$y = -3x + b$$

Skill Practice Answers

3. Perpendicular **4.** Parallel
5. Neither

Furthermore, because the point $(1, -4)$ is on the line, it is a solution to the equation. Therefore, if we substitute $(1, -4)$ for x and y in the equation, we can solve for b.

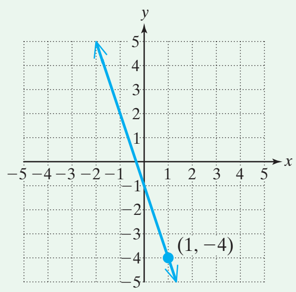
$$-4 = -3(1) + b$$
$$-4 = -3 + b$$
$$-1 = b$$

Thus, the slope-intercept form is $y = -3x - 1$.

Skill Practice

6. Use slope-intercept form to find an equation of a line with slope 2 and passing through $(-3, -5)$.

TIP: We can check the answer to Example 4, by graphing the line. Notice that the line appears to pass through $(1, -4)$ as desired.



2. The Point-Slope Formula

In Example 4, we used the slope-intercept form of a line to construct an equation of a line given its slope and a known point on the line. Here we provide another tool called the *point-slope formula* that (as its name suggests) can accomplish the same result.

Suppose a line passes through a given point (x_1, y_1) and has slope m. If (x, y) is any other point on the line, then

$$m = \frac{y - y_1}{x - x_1} \qquad \text{Slope formula}$$

$$m(x - x_1) = \frac{y - y_1}{x - x_1}(x - x_1) \qquad \text{Clear fractions.}$$

$$m(x - x_1) = y - y_1$$

or

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope formula}$$

The Point-Slope Formula

The **point-slope formula** is given by

$$y - y_1 = m(x - x_1)$$

where m is the slope of the line and (x_1, y_1) is a known point on the line.

Skill Practice Answers

6. $y = 2x + 1$

The point-slope formula is used specifically to find an equation of a line when a point on the line is known and the slope is known. To illustrate the point-slope formula, we will repeat the problem from Example 4.

Example 5 Using the Point-Slope Formula to Find an Equation of a Line

Use the point-slope formula to find an equation of the line having a slope of -3 and passing through the point $(1, -4)$. Write the answer in slope-intercept form.

Solution:

$m = -3$ and $(x_1, y_1) = (1, -4)$

$y - y_1 = m(x - x_1)$

$y - (-4) = -3(x - 1)$ Apply the point-slope formula.

$y + 4 = -3(x - 1)$ Simplify.

To write the answer in slope-intercept form, clear parentheses and solve for y.

$y + 4 = -3x + 3$ Clear parentheses.

$y = -3x - 1$ Solve for y. The answer is written in slope-intercept form. Notice that this is the same equation as in Example 4.

Skill Practice

7. Use the point-slope formula to write an equation for a line passing through the point $(-2, -6)$ and with a slope of -5. Write the answer in slope-intercept form.

Example 6 Finding an Equation of a Line Given Two Points

Find an equation of the line passing through the points $(5, -1)$ and $(3, 1)$. Write the answer in slope-intercept form.

Solution:

The slope formula can be used to compute the slope of the line between two points. Once the slope is known, the point-slope formula can be used to find an equation of the line.

First find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{3 - 5} = \frac{2}{-2} = -1 \qquad \text{Hence, } m = -1.$$

Next, apply the point-slope formula.

$y - y_1 = m(x - x_1)$

$y - 1 = -1(x - 3)$ Substitute $m = -1$ and use *either* point for (x_1, y_1). We will use $(3, 1)$ for (x_1, y_1).

$y - 1 = -x + 3$ Clear parentheses.

$y = -x + 4$ Solve for y. The final answer is in slope-intercept form.

TIP: In Example 6, the point $(3, 1)$ was used for (x_1, y_1) in the point-slope formula. However, either point could have been used. Using the point $(5, -1)$ for (x_1, y_1) produces the same final equation:

$y - (-1) = -1(x - 5)$

$y + 1 = -x + 5$

$y = -x + 4$

Skill Practice Answers

7. $y = -5x - 16$

Skill Practice

8. Use the point-slope formula to write an equation of the line that passes through the points $(-5, 2)$ and $(-1, -1)$. Write the answer in slope-intercept form.

Example 7 Finding an Equation of a Line Parallel to Another Line

Find an equation of the line passing through the point $(-2, -3)$ and parallel to the line $4x + y = 8$. Write the answer in slope-intercept form.

Solution:

To find an equation of a line, we must know a point on the line and the slope. The known point is $(-2, -3)$. Because the line is parallel to $4x + y = 8$, the two lines must have the same slope. Writing the equation $4x + y = 8$ in slope-intercept form, we have $y = -4x + 8$. Therefore, the slope of both lines must be -4.

We must now find an equation of the line passing through $(-2, -3)$ having a slope of -4.

$y - y_1 = m(x - x_1)$	Apply the point-slope formula.
$y - (-3) = -4[x - (-2)]$	Substitute $m = -4$ and $(-2, -3)$ for (x_1, y_1).
$y + 3 = -4(x + 2)$	
$y + 3 = -4x - 8$	Clear parentheses.
$y = -4x - 11$	Write the answer in slope-intercept form.

Skill Practice

9. Find an equation of a line containing $(4, -1)$ and parallel to $2x = y - 7$. Write the answer in slope-intercept form.

We can verify the answer to Example 7 by graphing both lines. We see that the line $y = -4x - 11$ passes through the point $(-2, -3)$ and is parallel to the line $y = -4x + 8$. See Figure 2-25.

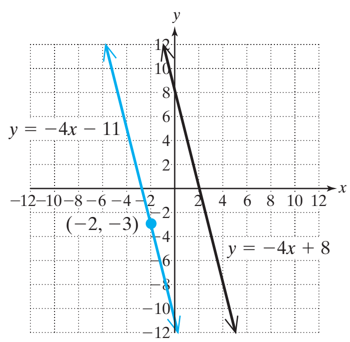


Figure 2-25

Skill Practice Answers

8. $y = -\frac{3}{4}x - \frac{7}{4}$

9. $y = 2x - 9$

Example 8 **Finding an Equation of a Line Perpendicular to Another Line**

Find an equation of the line passing through the point (4, 3) and perpendicular to the line $2x + 3y = 3$. Write the answer in slope-intercept form.

Solution:

The slope of the given line can be found from its slope-intercept form.

$$2x + 3y = 3$$

$$3y = -2x + 3 \qquad \text{Solve for } y.$$

$$\frac{3y}{3} = \frac{-2x}{3} + \frac{3}{3}$$

$$y = -\frac{2}{3}x + 1 \qquad \text{The slope is } -\frac{2}{3}.$$

The slope of a line *perpendicular* to this line must be the opposite of the reciprocal of $-\frac{2}{3}$; hence, $m = \frac{3}{2}$. Using $m = \frac{3}{2}$ and the known point (4, 3), we can apply the point-slope formula to find an equation of the line.

$$y - y_1 = m(x - x_1) \qquad \text{Apply the point-slope formula.}$$

$$y - 3 = \frac{3}{2}(x - 4) \qquad \text{Substitute } m = \tfrac{3}{2} \text{ and } (4, 3) \text{ for } (x_1, y_1).$$

$$y - 3 = \frac{3}{2}x - 6 \qquad \text{Clear parentheses.}$$

$$y = \frac{3}{2}x - 3 \qquad \text{Solve for } y.$$

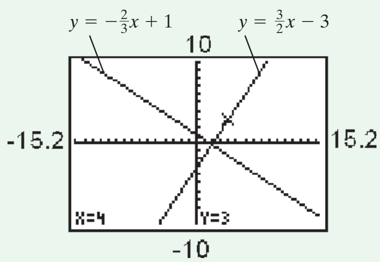
Skill Practice

10. Find an equation of the line passing through the point $(1, -6)$ and perpendicular to the line $x + 2y = 8$. Write the answer in slope-intercept form.

Calculator Connections

From Example 8, the line $y = \frac{3}{2}x - 3$ should be perpendicular to the line $y = -\frac{2}{3}x + 1$ and should pass through the point (4, 3).

Note: In this example, we are using a *square window option*, which sets the scale to display distances on the x- and y-axes as equal units of measure.



Skill Practice Answers

10. $y = 2x - 8$

3. Different Forms of Linear Equations

A linear equation can be written in several different forms, as summarized in Table 2-2.

Table 2-2

Form	Example	Comments
Standard Form $Ax + By = C$	$2x + 3y = 6$	A and B must not *both* be zero.
Horizontal Line $y = k$ (k is constant)	$y = 3$	The slope is zero, and the y-intercept is $(0, k)$.
Vertical Line $x = k$ (k is constant)	$x = -2$	The slope is undefined and the x-intercept is $(k, 0)$.
Slope-Intercept Form $y = mx + b$ Slope is m. y-Intercept is $(0, b)$.	$y = -2x + 5$ Slope $= -2$ y-Intercept is $(0, 5)$.	Solving a linear equation for y results in slope-intercept form. The coefficient of the x-term is the slope, and the constant defines the location of the y-intercept.
Point-Slope Formula $y - y_1 = m(x - x_1)$ Slope is m and (x_1, y_1) is a point on the line.	$m = -2$ $(x_1, y_1) = (3, 1)$ $y - 1 = -2(x - 3)$	This formula is typically used to build an equation of a line when a point on the line is known and the slope is known.

Although it is important to understand and apply slope-intercept form and the point-slope formula, they are not necessarily applicable to all problems. Example 9 illustrates how a little ingenuity may lead to a simple solution.

Example 9 Finding an Equation of a Line

Find an equation of the line passing through the point $(-4, 1)$ and perpendicular to the x-axis.

Solution:

Any line perpendicular to the x-axis must be *vertical*. Recall that all vertical lines can be written in the form $x = k$, where k is constant. A quick sketch can help find the value of the constant (Figure 2-26).

Because the line must pass through a point whose x-coordinate is -4, the equation of the line is $x = -4$.

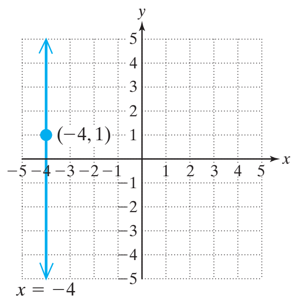


Figure 2-26

Skill Practice

11. Write an equation of the line through the point (20, 50) and having a slope of 0.

Skill Practice Answers

11. $y = 50$

Section 2.4 Practice Exercises

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Study Skills Exercises

1. For this chapter, find the page numbers for the Chapter Review Exercises, the Chapter Test, and the Cumulative Review Exercises.

Chapter Review Exercises, page(s)________________

Chapter Test, page(s)________________

Cumulative Review Exercises, page(s)________________

Compare these features and state the advantages of each.

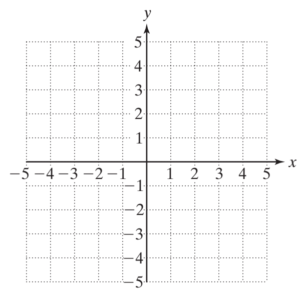
2. Define the key terms.

a. Standard form **b. Slope-intercept form** **c. Point-slope formula**

Review Exercises

3. Given $\frac{x}{2} + \frac{y}{3} = 1$

a. Find the x-intercept. **b.** Find the y-intercept. **c.** Sketch the graph.



4. Using slopes, how do you determine whether two lines are parallel?

5. Using the slopes of two lines, how do you determine whether the lines are perpendicular?

6. Write the formula to find the slope of a line given two points (x_1, y_1) and (x_2, y_2).

7. Given the two points $(-1, -2)$ and $(2, 4)$,

a. Find the slope of the line containing the two points.

b. Find the slope of a line parallel to the line containing the points.

c. Find the slope of a line perpendicular to the line containing the points.

Concept 1: Slope-Intercept Form

For Exercises 8–17, determine the slope and the y-intercept of the line.

8. $-3x + y = 2$ **9.** $-7x - y = -5$ **10.** $17x + y = 0$ **11.** $x + y = 0$

12. $18 = 2y$ **13.** $-7 = \frac{1}{2}y$ **14.** $8x + 12y = 9$ **15.** $-9x + 10y = -4$

16. $y = 0.625x - 1.2$ **17.** $y = -2.5x + 1.8$

In Exercises 18–23, match the equation with the correct graph.

18. $y = \frac{3}{2}x - 2$

19. $y = -x + 3$

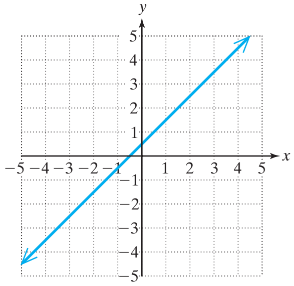
20. $y = \frac{13}{4}$

21. $y = x + \frac{1}{2}$

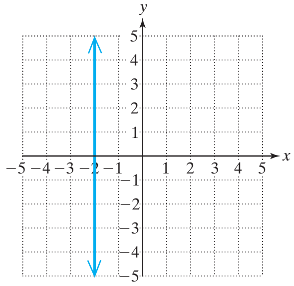
22. $x = -2$

23. $y = -\frac{1}{2}x + 2$

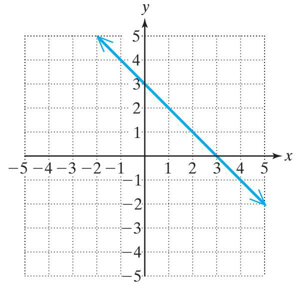
a.



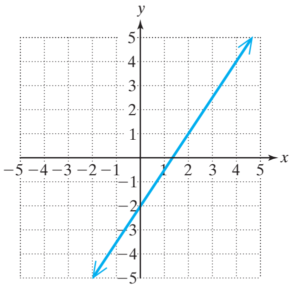
b.



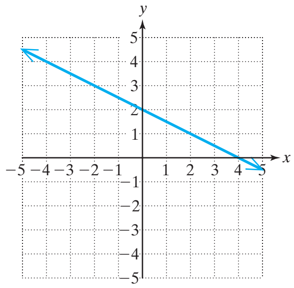
c.



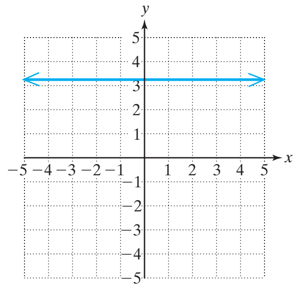
d.



e.

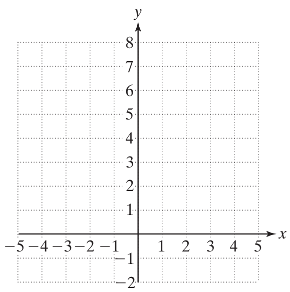


f.

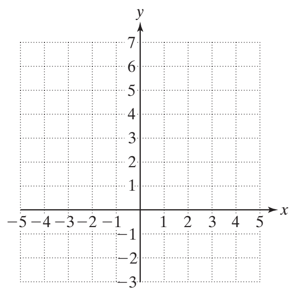


For Exercises 24–31, write the equations in slope-intercept form (if possible). Then graph each line, using the slope and y-intercept.

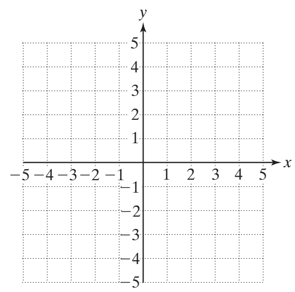
24. $y - 2 = 4x$



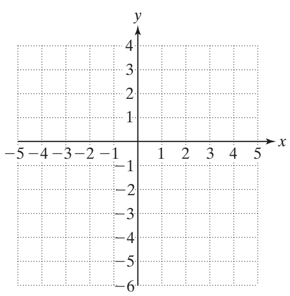
25. $3x = 5 - y$



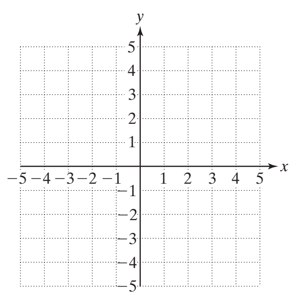
26. $3x + 2y = 6$



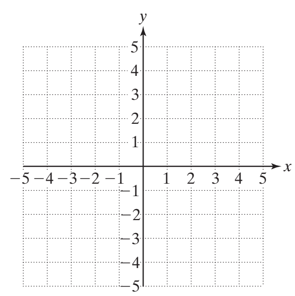
27. $x - 2y = 8$



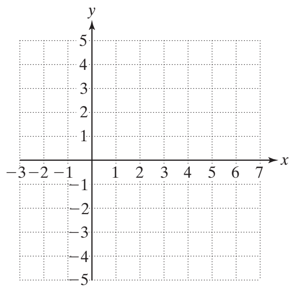
28. $-5x = -3y - 6$



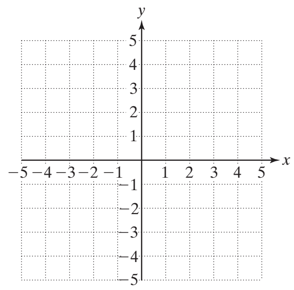
29. $-x = 6y - 2$



30. $2x - 5y = 0$



31. $3x - y = 0$



32. Given the standard form of a linear equation $Ax + By = C$, $B \neq 0$, solve for y and write the equation in slope-intercept form. What is the slope of the line? What is the y-intercept?

33. Use the result of Exercise 32 to determine the slope and y-intercept of the line $3x + 7y = 9$.

For Exercises 34–39, determine if the lines are parallel, perpendicular, or neither.

34. $-3y = 5x - 1$
$6x = 10y - 12$

35. $x = 6y - 3$
$3x + \frac{1}{2}y = 0$

36. $3x - 4y = 12$
$\frac{1}{2}x - \frac{2}{3}y = 1$

37. $4.8x = 1.2y + 3.6$
$y - 1 = 4x$

38. $3y = 5x + 6$
$5x + 3y = 9$

39. $-y = 3x - 2$
$-6x + 2y = 6$

For Exercises 40–51, use the slope-intercept form of a line to find an equation of the line having the given slope and passing through the given point.

40. $m = 2, (4, -3)$

41. $m = 3, (-1, 5)$

42. $m = -4, (-1, -2)$

43. $m = -2, (-4, -3)$

44. $m = \frac{3}{4}, (4, 0)$

45. $m = -\frac{4}{5}, (10, 0)$

46. $m = -\frac{2}{7}, (-3, 1)$

47. $m = \frac{3}{8}, (-2, 4)$

48. $m = 3, (0, 5)$

49. $m = -4, (0, 3)$

50. $m = 0, (1, 2)$

51. $m = 0, (1, 4)$

Concept 2: The Point-Slope Formula

For Exercises 52–79, write an equation of the line satisfying the given conditions. Write the answer in slope-intercept form or standard form.

52. The line passes through the point $(0, -2)$ and has a slope of 3.

53. The line passes through the point $(0, 5)$ and has a slope of $-\frac{1}{2}$.

54. The line passes through the point $(2, 7)$ and has a slope of 2.

55. The line passes through the point $(3, 10)$ and has a slope of -2.

56. The line passes through the point $(-2, -5)$ and has a slope of -3.

57. The line passes through the point $(-1, -6)$ and has a slope of 4.

58. The line passes through the point $(6, -3)$ and has a slope of $-\frac{4}{5}$.

59. The line passes through the point $(7, -2)$ and has a slope of $\frac{7}{2}$.

60. The line passes through $(0, 4)$ and $(3, 0)$.

61. The line passes through $(1, 1)$ and $(3, 7)$.

62. The line passes through $(6, 12)$ and $(4, 10)$.

63. The line passes through $(-2, -1)$ and $(3, -4)$.

64. The line passes through $(-5, 2)$ and $(-1, 2)$.

65. The line passes through $(-4, -1)$ and $(2, -1)$.

66. The line contains the point $(3, 2)$ and is parallel to a line with a slope of $-\frac{3}{4}$.

67. The line contains the point $(-1, 4)$ and is parallel to a line with a slope of $\frac{1}{2}$.

68. The line contains the point $(3, 2)$ and is perpendicular to a line with a slope of $-\frac{3}{4}$.

69. The line contains the point $(-2, 5)$ and is perpendicular to a line with a slope of $\frac{1}{2}$.

70. The line contains the point $(2, -5)$ and is parallel to $y = \frac{3}{4}x + \frac{7}{4}$.

71. The line contains the point $(-6, -1)$ and is parallel to $y = -\frac{2}{3}x - 4$.

72. The line contains the point $(-8, -1)$ and is parallel to $x + 5y = 8$.

73. The line contains the point $(4, -2)$ and is parallel to $3x - 4y = 8$.

74. The line contains the point $(4, 0)$ and is parallel to the line defined by $3x = 2y$.

75. The line contains the point $(-3, 0)$ and is parallel to the line defined by $-5x = 6y$.

76. The line is perpendicular to the line defined by $3y + 2x = 21$ and passes through the point $(2, 4)$.

77. The line is perpendicular to $7y - x = -21$ and passes through the point $(-14, 8)$.

78. The line is perpendicular to $\frac{1}{2}y = x$ and passes through $(-3, 5)$.

79. The line is perpendicular to $-\frac{1}{4}y = x$ and passes through $(-1, -5)$.

Concept 3: Different Forms of Linear Equations

For Exercises 80–87, write an equation of the line satisfying the given conditions.

80. The line passes through $(2, -3)$ and has a zero slope.

81. The line contains the point $(\frac{5}{2}, 0)$ and has an undefined slope.

82. The line contains the point $(2, -3)$ and has an undefined slope.

83. The line contains the point $(\frac{5}{2}, 0)$ and has a zero slope.

84. The line is parallel to the x-axis and passes through $(4, 5)$.

85. The line is perpendicular to the x-axis and passes through $(4, 5)$.

86. The line is parallel to the line $x = 4$ and passes through $(5, 1)$.

87. The line is parallel to the line $y = -2$ and passes through $(-3, 4)$.

Expanding Your Skills

88. Is the equation $x = -2$ in slope-intercept form? Identify the slope and y-intercept.

89. Is the equation $x = 1$ in slope-intercept form? Identify the slope and y-intercept.

90. Is the equation $y = 3$ in slope-intercept form? Identify the slope and the y-intercept.

91. Is the equation $y = -5$ in slope-intercept form? Identify the slope and the y-intercept.

Graphing Calculator Exercises

92. Use a graphing calculator to graph the lines on the same viewing window. Then explain how the lines are related.

$$y_1 = \frac{1}{2}x + 4$$

$$y_2 = \frac{1}{2}x - 2$$

93. Use a graphing calculator to graph the lines on the same viewing window. Then explain how the lines are related.

$$y_1 = -\frac{1}{3}x + 5$$

$$y_2 = -\frac{1}{3}x - 3$$

94. Use a graphing calculator to graph the lines on the same viewing window. Then explain how the lines are related.

$$y_1 = x - 2$$

$$y_2 = 2x - 2$$

$$y_3 = 3x - 2$$

95. Use a graphing calculator to graph the lines on the same viewing window. Then explain how the lines are related.

$$y_1 = -2x + 1$$

$$y_2 = -3x + 1$$

$$y_3 = -4x + 1$$

96. Use a graphing calculator to graph the lines on a square viewing window. Then explain how the lines are related.

$$y_1 = 4x - 1$$

$$y_2 = -\frac{1}{4}x - 1$$

97. Use a graphing calculator to graph the lines on a square viewing window. Then explain how the lines are related.

$$y_1 = \frac{1}{2}x - 3$$

$$y_2 = -2x - 3$$

98. Use a graphing calculator to graph the equation from Exercise 60. Use an *Eval* feature to verify that the line passes through the points (0, 4) and (3, 0).

99. Use a graphing calculator to graph the equation from Exercise 61. Use an *Eval* feature to verify that the line passes through the points (1, 1) and (3, 7).

Applications of Linear Equations and Graphing

Section 2.5

Concepts

1. Writing a Linear Model

Algebra is a tool used to model events that occur in physical and biological sciences, sports, medicine, economics, business, and many other fields. The purpose of modeling is to represent a relationship between two or more variables with an algebraic equation.

For an equation written in slope-intercept form $y = mx + b$, the term mx is the *variable term*, and the term b is the *constant term*. The value of the term mx changes with the value of x (this is why the slope is called a rate of change). However, the term b remains constant regardless of the value of x. With these ideas in mind, a linear equation can be created if the rate of change and the constant are known.

Example 1 Finding a Linear Relationship

Buffalo, New York, had 2 ft (24 in.) of snow on the ground before a snowstorm. During the storm, snow fell at an average rate of $\frac{5}{8}$ in./hr.

a. Write a linear equation to compute the total snow depth y after x hours of the storm.

b. Graph the equation.

c. Use the equation to compute the depth of snow after 8 hr.

d. If the snow depth was 31.5 in. at the end of the storm, determine how long the storm lasted.

Solution:

a. The constant or base amount of snow before the storm began is 24 in. The variable amount is given by $\frac{5}{8}$ in. of snow per hour. If m is replaced by $\frac{5}{8}$ and b is replaced by 24, we have the linear equation

$$y = mx + b$$

$$y = \frac{5}{8}x + 24$$

b. The equation is in slope-intercept form, and the corresponding graph is shown in Figure 2-27.

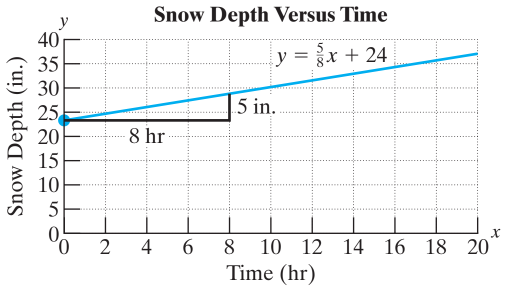


Figure 2-27

c. $y = \frac{5}{8}x + 24$

$y = \frac{5}{8}(8) + 24$ Substitute $x = 8$.

$y = 5 + 24$ Solve for y.

$y = 29$ in.

The snow depth was 29 in. after 8 hr. The corresponding ordered pair is (8, 29) and can be confirmed from the graph.

d. $y = \frac{5}{8}x + 24$

$31.5 = \frac{5}{8}x + 24$ Substitute $y = 31.5$.

$8(31.5) = 8\left(\frac{5}{8}x + 24\right)$ Multiply by 8 to clear fractions.

$252 = 5x + 192$ Clear parentheses.

$60 = 5x$ Solve for x.

$12 = x$

The storm lasted for 12 hr. The corresponding ordered pair is (12, 31.5) and can be confirmed from the graph.

Skill Practice

1. When Joe graduated from college, he had \$1000 in his savings account. When he began working, he decided he would add \$120 per month to his savings account.

a. Write a linear equation to compute the amount of money y in Joe's account after x months of saving.

b. Use the equation to compute the amount of money in Joe's account after 6 months.

c. Joe needs \$3160 for a down payment for a car. How long will it take for Joe's account to reach this amount?

2. Interpreting a Linear Model

Example 2 Interpreting a Linear Model

In 1938, President Franklin D. Roosevelt signed a bill enacting the Fair Labor Standards Act of 1938 (FLSA). In its final form, the act banned oppressive child labor and set the minimum hourly wage at 25 cents and the maximum workweek at 44 hr. Over the years, the minimum hourly wage has been increased by the government to meet the rising cost of living.

The minimum hourly wage y (in dollars per hour) in the United States between 1960 and 2005 can be approximated by the equation

$$y = 0.10x + 0.82 \qquad x \geq 0$$

Skill Practice Answers

1a. $y = 120x + 1000$
b. \$1720 **c.** 18 months

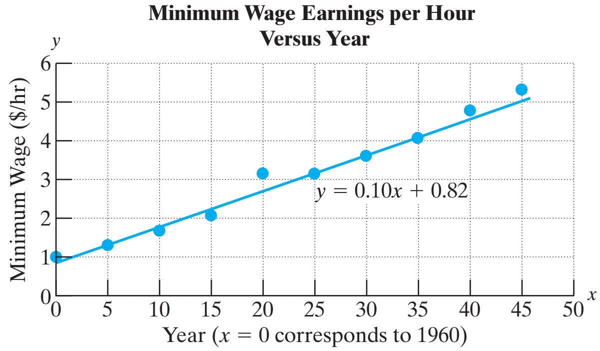


Figure 2-28

where x represents the number of years since 1960 ($x = 0$ corresponds to 1960, $x = 1$ corresponds to 1961, and so on) (Figure 2-28).

a. Find the slope of the line and interpret the meaning of the slope in the context of this problem.

b. Find the y-intercept of the line and interpret the meaning of the y-intercept in the context of this problem.

c. Use the linear equation to approximate the minimum wage in 1985.

d. Use the linear equation to predict the minimum wage in the year 2010.

Solution:

a. The equation $y = 0.10x + 0.82$ is written in slope-intercept form. The slope is 0.10 and indicates that minimum hourly wage rose an average of \$0.10 per year between 1960 and 2005.

b. The y-intercept is $(0, 0.82)$. The y-intercept indicates that the minimum wage in the year 1960 ($x = 0$) was approximately \$0.82 per hour. (The actual value of minimum wage in 1960 was \$1.00 per hour.)

c. The year 1985 is 25 years after the year 1960. Substitute $x = 25$ into the linear equation.

$$y = 0.10x + 0.82$$

$$y = 0.10(25) + 0.82 \qquad \text{Substitute } x = 25.$$

$$y = 2.50 + 0.82$$

$$y = 3.32$$

According to the linear model, the minimum wage in 1985 was approximately \$3.32 per hour. (The actual minimum wage in 1985 was \$3.35 per hour.)

d. The year 2010 is 50 years after the year 1960. Substitute $x = 50$ into the linear equation.

$$y = 0.10x + 0.82$$

$$y = 0.10(50) + 0.82 \qquad \text{Substitute } x = 50.$$

$$y = 5.82$$

According to the linear model, minimum wage in 2010 will be approximately \$5.82 per hour provided the linear trend continues. (How does this compare with the current value for minimum wage?)

Skill Practice

2. The cost of long-distance service with a certain phone company is given by the equation $y = 0.12x + 6.95$, where y represents the monthly cost in dollars and x represents the number of minutes of long distance.

a. Find the slope of the line, and interpret the meaning of the slope in the context of this problem.

b. Find the y-intercept and interpret the meaning of the y-intercept in the context of this problem.

c. Use the equation to determine the cost of using 45 min of long-distance service in a month.

3. Finding a Linear Model from Observed Data Points

Graphing a set of data points offers a visual method to determine whether the points follow a linear pattern. If a linear trend exists, we say that there is a linear correlation between the two variables. The better the points "line up," the stronger the correlation.*

When two variables are correlated, it is often desirable to find a mathematical equation (or *model*) to describe the relationship between the variables.

Example 3 Writing a Linear Model from Observed Data

Figure 2-29 represents the winning gold medal times for the women's 100-m freestyle swimming event for selected summer Olympics. Let y represent the winning time in seconds and let x represent the number of years since 1900 ($x = 0$ corresponds to 1900, $x = 1$ corresponds to 1901, and so on).

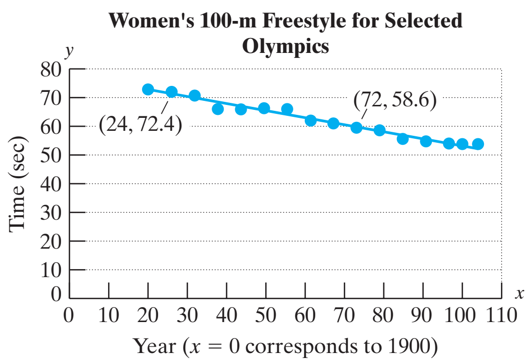


Figure 2-29

In 1924, the winning time was 72.4 sec. This corresponds to the ordered pair (24, 72.4). In 1972, the winning time was 58.6 sec, yielding the ordered pair (72, 58.6).

a. Use these ordered pairs to find a linear equation to model the winning time versus the year.

b. What is the slope of the line, and what does it mean in the context of this problem?

Skill Practice Answers

2a. The slope is 0.12. This means that the monthly cost increases by 12 cents per minute.

b. The y-intercept is (0, 6.95). The cost of the long-distance service is \$6.95 if 0 min is used.

c. \$12.35

*The strength of a linear correlation can be measured mathematically by using techniques often covered in statistics courses.

c. Use the linear equation to approximate the winning time for the 1964 Olympics.

d. Would it be practical to use the linear model to predict the winning time in the year 2048?

Solution:

a. The slope formula can be used to compute the slope of the line between the two points. (Round the slope to 2 decimal places.)

$$(24, 72.4) \quad \text{and} \quad (72, 58.6)$$
$$(x_1, y_1) \quad \text{and} \quad (x_2, y_2)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{58.6 - 72.4}{72 - 24} = -0.2875 \qquad \text{Hence, } m \approx -0.29.$$

$$y - y_1 = m(x - x_1) \qquad \text{Apply the point-slope formula, using } m = -0.29 \text{ and the point } (24, 72.4).$$

$$y - 72.4 = -0.29(x - 24)$$

$$y - 72.4 = -0.29x + 6.96 \qquad \text{Clear parentheses.}$$

$$y = -0.29x + 6.96 + 72.4 \qquad \text{Solve for } y.$$

$$y = -0.29x + 79.36 \qquad \text{The answer is in slope-intercept form.}$$

b. The slope is -0.29 and indicates that the winning time in the women's 100-m Olympic freestyle event has *decreased* on average by 0.29 sec/yr during this period.

c. The year 1964 is 64 years after the year 1900. Substitute $x = 64$ into the linear model.

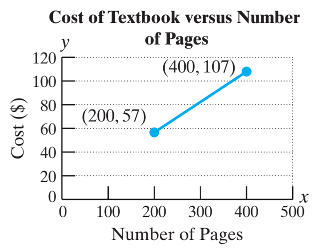
$$y = -0.29x + 79.36$$
$$y = -0.29(64) + 79.36 \qquad \text{Substitute } x = 64.$$
$$y = -18.56 + 79.36$$
$$y = 60.8$$

According to the linear model, the winning time in 1964 was approximately 60.8 sec. (The actual winning time in 1964 was set by Dawn Fraser from Australia in 59.5 sec. The linear equation can only be used to *approximate* the winning time.)

d. It would not be practical to use the linear model $y = -0.29x + 79.36$ to predict the winning time in the year 2048. There is no guarantee that the linear trend will continue beyond the last observed data point in 2004. In fact, the linear trend cannot continue indefinitely; otherwise, the swimmers' times would eventually be negative. The potential for error increases for predictions made beyond the last observed data value.

Skill Practice

3. The figure shows data relating the cost of college textbooks in dollars to the number of pages in the book. Let y represent the cost of the book, and let x represent the number of pages.



a. Use the ordered pairs indicated in the figure to write a linear equation to model the cost of textbooks versus the number of pages.

b.

Skill Practice Answers

3a. $y = 0.25x + 7$
b. $97

Section 2.5 Practice Exercises

Boost *your* GRADE at mathzone.com!

MathZone

- Practice Problems
- Self-Tests
- NetTutor
- e-Professors
- Videos

Study Skills Exercise

1. On test day, take a look at any formulas or important points that you had to memorize before you enter the classroom. Then when you sit down to take your test, write these formulas on the test or on scrap paper. This is called a memory dump. Write down the formulas from Chapter 2.

Review Exercises

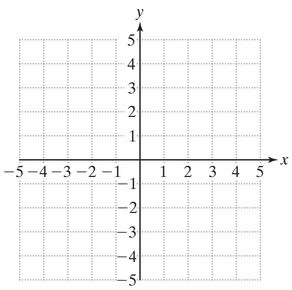
For Exercises 2–5,

a. Find the slope (if possible) of the line passing through the two points.

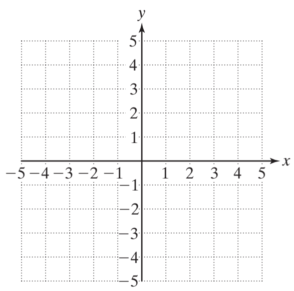
b. Find an equation of the line passing through the two points. Write the answer in slope-intercept form (if possible) and in standard form.

c. Graph the line by using the slope and y-intercept. Verify that the line passes through the two given points.

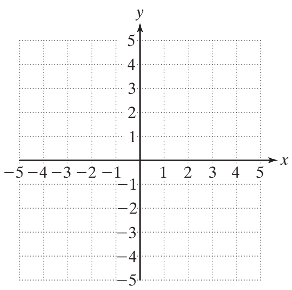
2. $(-3, 0)$ and $(3, -2)$



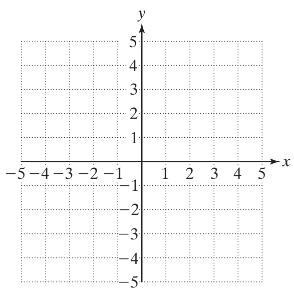
3. $(1, -1)$ and $(3, -5)$



4. $(-4, 3)$ and $(-2, 3)$



5. $(-2, 4)$ and $(-2, 0)$

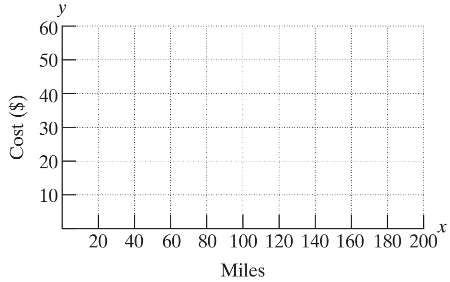


Concept 1: Writing a Linear Model

6. A car rental company charges a flat fee of \$19.95 plus \$0.20 per mile.

a. Write an equation that expresses the cost y of renting a car if the car is driven for x miles.

b. Graph the equation.



c. What is the *y*-intercept and what does it mean in the context of this problem?

d. Using the equation from part (a), find the cost of driving the rental car 50, 100, and 200 mi.

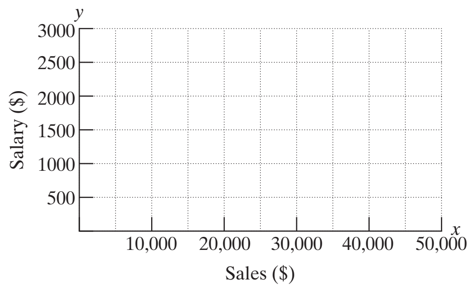
e. Find the total cost of driving the rental car 100 mi if the sales tax is 6%.

f. Is it reasonable to use negative values for *x* in the equation? Why or why not?

7. Alex is a sales representative and earns a base salary of \$1000 per month plus a 4% commission on his sales for the month.

a. Write a linear equation that expresses Alex's monthly salary *y* in terms of his sales *x*.

b. Graph the equation.



c. What is the *y*-intercept and what does it represent in the context of this problem?

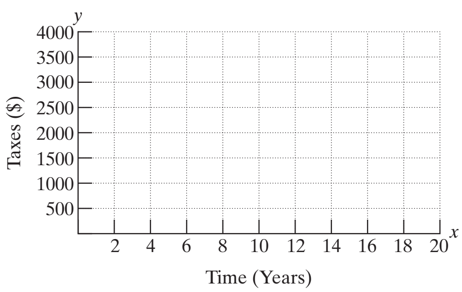
d. What is the slope of the line and what does it represent in the context of this problem?

e. How much will Alex make if his sales for a given month are \$30,000?

8. Ava recently purchased a home in Crescent Beach, Florida. Her property taxes for the first year are \$2742. Ava estimates that her taxes will increase at a rate of \$52 per year.

a. Write an equation to compute Ava's yearly property taxes. Let *y* be the amount she pays in taxes, and let *x* be the time in years.

b. Graph the line.



c. What is the slope of this line? What does the slope of the line represent in the context of this problem?

d. What is the *y*-intercept? What does the *y*-intercept represent in the context of this problem?

e. What will Ava's yearly property tax be in 10 years? In 15 years?

9. Luigi Luna has started a chain of Italian restaurants called Luna Italiano. He has 19 restaurants in various locations in the northeast United States and Canada. He plans to open five new restaurants per year.

a. Write a linear equation to express the number of restaurants, y, Luigi opens in terms of the time in years, x.

b. How many restaurants will he have in 4 years?

c. How many years will it take him to have 100 restaurants?

Concept 2: Interpreting a Linear Model

10. Sound travels at approximately one-fifth of a mile per second. Therefore, for every 5-sec difference between seeing lightning and hearing thunder, we can estimate that a storm is approximately 1 mi away. Let y represent the distance (in miles) that a storm is from an observer. Let x represent the difference in time between seeing lightning and hearing thunder. Then the distance of the storm can be approximated by the equation $y = 0.2x$, where $x \geq 0$.

a. Use the linear model provided to determine how far away a storm is for the following differences in time between seeing lightning and hearing thunder: 4 sec, 12 sec, and 16 sec.

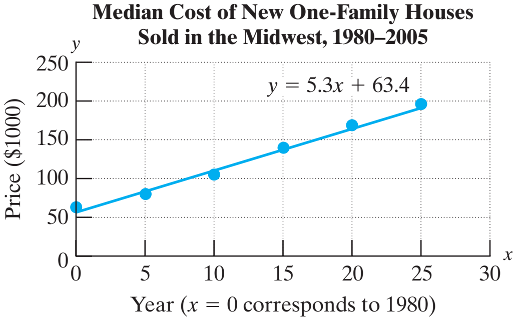
b. If a storm is 4.2 mi away, how many seconds will pass between seeing lightning and hearing thunder?

11. The force y (in pounds) required to stretch a particular spring x inches beyond its rest (or "equilibrium") position is given by the equation $y = 2.5x$, where $0 \leq x \leq 20$.

a. Use the equation to determine the amount of force necessary to stretch the spring 6 in. from its rest position. How much force is necessary to stretch the spring twice as far?

b. If 45 lb of force is exerted on the spring, how far will the spring be stretched?

12. The figure represents the median cost of new privately owned, one-family houses sold in the midwest from 1980 to 2005.



Source: U.S. Bureau of the Census and U.S. Department of Housing and Urban Development.

Let y represent the median cost of a new privately owned, one-family house sold in the midwest. Let x represent the year, where $x = 0$ corresponds to the year 1980, $x = 1$ represents 1981, and so on. Then the median cost of new privately owned, one-family houses sold in the midwest can be approximated by the equation $y = 5.3x + 63.4$, where $0 \leq x \leq 25$.

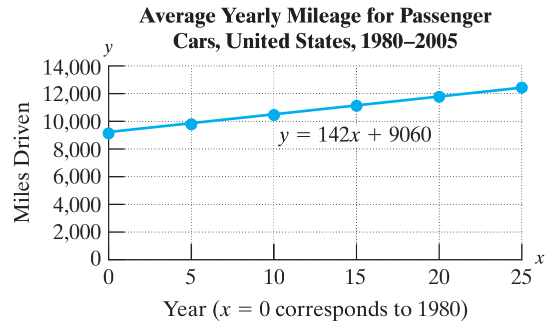
a. Use the linear equation to approximate the median cost of new privately owned, one-family houses in the midwest for the year 2005.

b. Use the linear equation to approximate the median cost for the year 1988, and compare it with the actual median cost of \$101,600.

c. What is the slope of the line and what does it mean in the context of this problem?

d. What is the y-intercept and what does it mean in the context of this problem?

13. Let y represent the average number of miles driven per year for passenger cars in the United States between 1980 and 2005. Let x represent the year where $x = 0$ corresponds to 1980, $x = 1$ corresponds to 1981, and so on. The average yearly mileage for passenger cars can be approximated by the equation $y = 142x + 9060$, where $0 \le x \le 25$.



a. Use the linear equation to approximate the average yearly mileage for passenger cars in the United States in the year 2005.

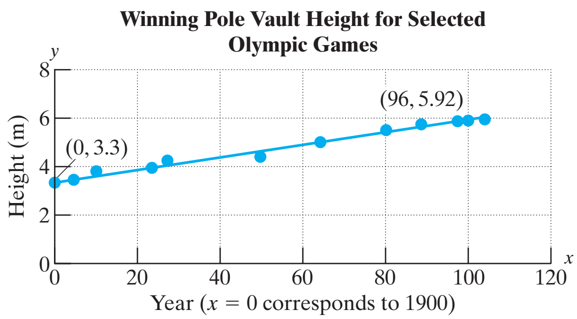
b. Use the linear equation to approximate the average mileage for the year 1985, and compare it with the actual value of 9700 mi.

c. What is the slope of the line and what does it mean in the context of this problem?

d. What is the y-intercept and what does it mean in the context of this problem?

Concept 3: Finding a Linear Model from Observed Data Points

14. The figure represents the winning heights for men's pole vault in selected Olympic games.



a. Let y represent the winning height. Let x represent the year, where $x = 0$ corresponds to the year 1900, $x = 4$ represents 1904, and so on. Use the ordered pairs given in the graph (0, 3.3) and (96, 5.92) to find a linear equation to estimate the winning pole vault height versus the year. (Round the slope to three decimal places.)

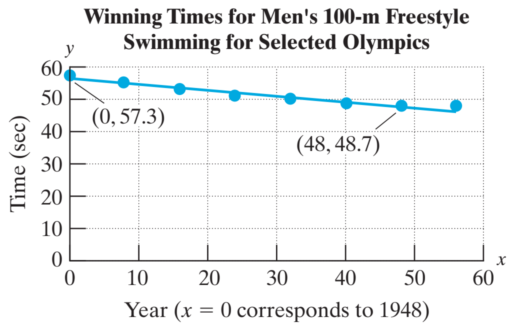
b. Use the linear equation from part (a) to approximate the winning vault for the 1920 Olympics.

c. Use the linear equation to approximate the winning vault for 1976.

d. The actual winning vault in 1920 was 4.09 m, and the actual winning vault in 1976 was 5.5 m. Are your answers from parts (b) and (c) different from these? Why?

e. What is the slope of the line? What does the slope of the line mean in the context of this problem?

15. The figure represents the winning time for the men's 100-m freestyle swimming event for selected Olympic games.



a. Let y represent the winning time. Let x represent the number of years since 1948 (where $x = 0$ corresponds to the year 1948, $x = 4$ represents 1952, and so on). Use the ordered pairs given in the graph (0, 57.3) and (48, 48.7) to find a linear equation to estimate the winning time for the men's 100-m freestyle versus the year. (Round the slope to 2 decimal places.)

b. Use the linear equation from part (a) to approximate the winning 100-m time for the year 1972, and compare it with the actual winning time of 51.2 sec.

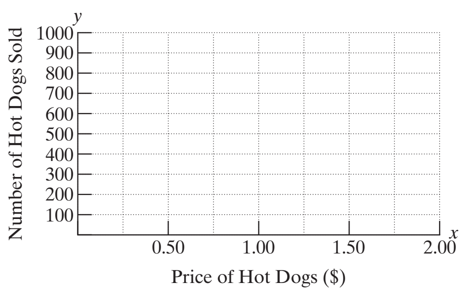
c. Use the linear equation to approximate the winning time for the year 1988.

d. What is the slope of the line and what does it mean in the context of this problem?

e. Interpret the meaning of the x-intercept of this line in the context of this problem. Explain why the men's swimming times will never "reach" the x-intercept. Do you think this linear trend will continue for the next 50 years, or will the men's swimming times begin to "level off" at some time in the future? Explain your answer.

16. At a high school football game in Miami, hot dogs were sold for \$1.00 each. At the end of the night, it was determined that 650 hot dogs were sold. The following week, the price of hot dogs was raised to \$1.50, and this resulted in fewer sales. Only 475 hot dogs were sold.

a. Make a graph with the price of hot dogs on the x-axis and the corresponding sales on the y-axis. Graph the points (1.00, 650) and (1.50, 475), using suitable scaling on the x- and y-axes.

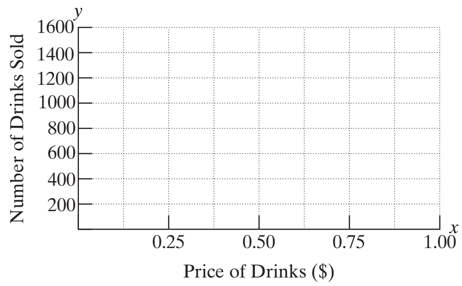


b. Find an equation of the line through the given points. Write the equation in slope-intercept form.

c. Use the equation from part (b) to predict the number of hot dogs that would sell if the price were changed to \$1.70 per hot dog.

17. At a high school football game, soft drinks were sold for \$0.50 each. At the end of the night, it was determined that 1020 drinks were sold. The following week, the price of drinks was raised to \$0.75, and this resulted in fewer sales. Only 820 drinks were sold.

a. Make a graph with the price of drinks on the x-axis and the corresponding sales per night on the y-axis. Graph the points (0.50, 1020) and (0.75, 820), using suitable scaling on the x- and y-axes.



b. Find an equation of the line through the given points. Write the equation in slope-intercept form.

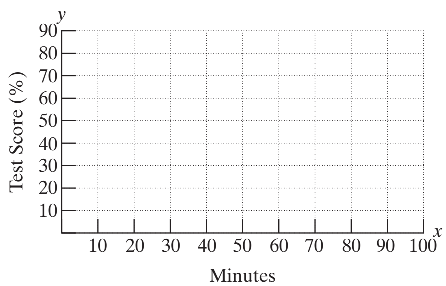
c. Use the equation from part (b) to predict the number of drinks that would sell if the price were changed to \$0.85 per drink.

Expanding Your Skills

18. Loraine is enrolled in an algebra class that meets 5 days per week. Her instructor gives a test every Friday. Loraine has a study plan and keeps a portfolio with notes, homework, test corrections, and vocabulary. She also records the amount of time per day that she studies and does homework. The following data represent the amount of time she studied per day and her weekly test grades.

Time Studied per Day (min) x	Weekly Test Grade (percent) y
60	69
70	74
80	79
90	84
100	89

a. Graph the points on a rectangular coordinate system. Do the data points appear to follow a linear trend?



b. Find a linear equation that relates Loraine's weekly test score y to the amount of time she studied per day x. (*Hint:* Pick two ordered pairs from the observed data, and find an equation of the line through the points.)

c. How many minutes should Loraine study per day in order to score at least 90% on her weekly examination? Would the equation used to determine the time Loraine needs to study to get 90% work for other students? Why or why not?

d. If Loraine is only able to spend $\frac{1}{2}$ hr/day studying her math, predict her test score for that week.

Points are *collinear* if they lie on the same line. For Exercises 19–22, use the slope formula to determine if the points are collinear.

19. $(3, -4)\ (0, -5)\ (9, -2)$

20. $(4, 3)\ (-4, -1)\ (2, 2)$

21. $(0, 2)\ (-2, 12)\ (-1, 6)$

22. $(-2, -2)\ (0, -3)\ (-4, -1)$

Graphing Calculator Exercises

23. Use a Table feature to confirm your answers to Exercise 11.

24. Use a Table feature to confirm your answers to Exercise 10(a).

25. Graph the line $y = -800x + 1420$ on the viewing window defined by $0 \le x \le 1$ and $0 \le y \le 1600$. Use the Trace key to support your answer to Exercise 17 by showing that the line passes through the points (0.50, 1020) and (0.75, 820).

26. Graph the line $y = -350x + 1000$ on the viewing window defined by $0 \le x \le 2$ and $0 \le y \le 1000$. Use the Trace key to support your answer to Exercise 16 by showing that the line passes through the points (1.00, 650) and (1.50, 475).

Chapter 2 SUMMARY

Section 2.1 The Rectangular Coordinate System and Midpoint Formula

Key Concepts

Graphical representation of numerical data is often helpful to study problems in real-world applications.

A **rectangular coordinate system** is made up of a horizontal line called the ***x*-axis** and a vertical line called the ***y*-axis.** The point where the lines meet is the **origin.** The four regions of the plane are called **quadrants.**

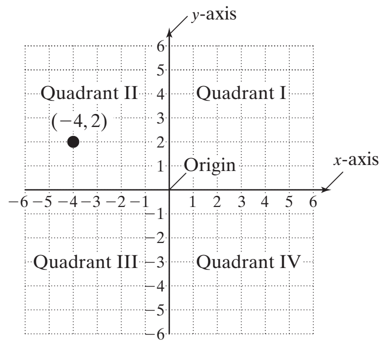
The point (x, y) is an **ordered pair.** The first element in the ordered pair is the point's horizontal position from the origin. The second element in the ordered pair is the point's vertical position from the origin.

The midpoint between two points is found by using the formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Examples

Example 1



Example 2

Find the midpoint between $(-3, 1)$ and $(5, 7)$.

$$\left(\frac{-3 + 5}{2}, \frac{1 + 7}{2}\right) = (1, 4)$$

Section 2.2 Linear Equations in Two Variables

Key Concepts

A **linear equation in two variables** can be written in the form $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

The graph of a linear equation in two variables is a line and can be represented in the rectangular coordinate system.

An ***x*-intercept** is a point $(a, 0)$ where a graph intersects the x-axis. To find an x-intercept, substitute 0 for y in the equation and solve for x.

A ***y*-intercept** is a point $(0, b)$ where a graph intersects the y-axis. To find a y-intercept, substitute 0 for x in the equation and solve for y.

A **vertical line** can be written in the form $x = k$.
A **horizontal line** can be written in the form $y = k$.

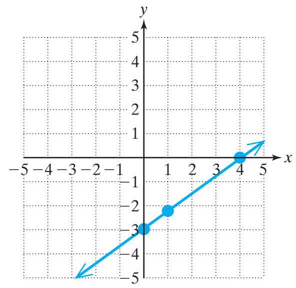
Examples

Example 1

$3x - 4y = 12$

Complete a table of ordered pairs.

x	y
0	-3
4	0
1	$-\frac{9}{4}$



Example 2

Given the equation, $2x + 3y = 8$

x-intercept: $2x + 3(0) = 8$

$2x = 8$

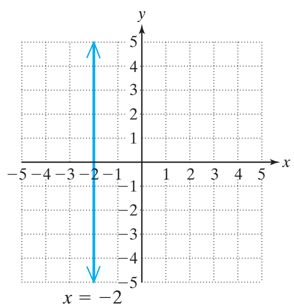
$x = 4$ $(4, 0)$

y-intercept: $2(0) + 3y = 8$

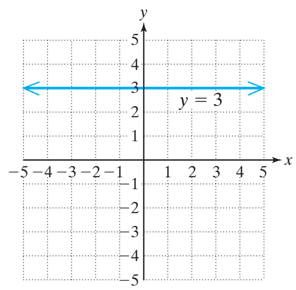
$3y = 8$

$y = \frac{8}{3}$ $\left(0, \frac{8}{3}\right)$

Example 3



Example 4



Section 2.3 Slope of a Line

Key Concepts

The **slope** of a line m between two distinct points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_2 - x_1 \neq 0$$

The slope of a line may be positive, negative, zero, or undefined.

Two parallel (nonvertical) lines have the same slope: $m_1 = m_2$.

Two lines are perpendicular if the slope of one line is the opposite of the reciprocal of the slope of the other line:

$$m_1 = \frac{-1}{m_2} \text{ or equivalently, } m_1 m_2 = -1.$$

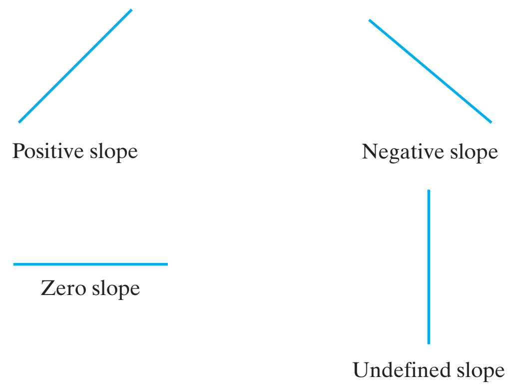
Examples

Example 1

The slope of the line between $(1, -3)$ and $(-3, 7)$ is

$$m = \frac{7 - (-3)}{-3 - 1} = \frac{10}{-4} = -\frac{5}{2}$$

Example 2



Example 3

The slopes of two lines are given. Determine whether the lines are parallel, perpendicular, or neither.

a. $m_1 = -7$ and $m_2 = -7$ Parallel

b. $m_1 = -\frac{1}{5}$ and $m_2 = 5$ Perpendicular

c. $m_1 = -\frac{3}{2}$ and $m_2 = -\frac{2}{3}$ Neither

Section 2.4 Equations of a Line

Key Concepts

Standard Form: $Ax + By = C$ (A and B are not both zero)

Horizontal line: $y = k$

Vertical line: $x = k$

Slope-intercept form: $y = mx + b$

Point-slope formula: $y - y_1 = m(x - x_1)$

Slope-intercept form is used to identify the slope and y-intercept of a line when the equation is given. Slope-intercept form can also be used to graph a line.

The point-slope formula can be used to construct an equation of a line, given a point and a slope.

Examples

Example 1

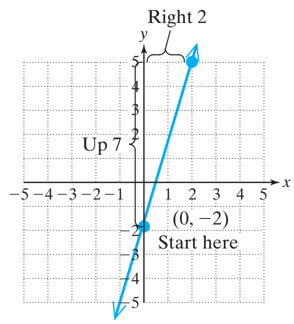
Find the slope and y-intercept. Then graph the equation.

$7x - 2y = 4$ Solve for y.

$-2y = -7x + 4$

$y = \frac{7}{2}x - 2$ Solve-intercept form

The slope is $\frac{7}{2}$; the y-intercept is $(0, -2)$.



Example 2

Find an equation of the line passing through the point $(2, -3)$ and having slope $m = -4$.

Using the point-slope formula gives

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -4(x - 2)$$
$$y + 3 = -4x + 8$$
$$y = -4x + 5$$

Section 2.5 Applications of Linear Equations and Graphing

Key Concepts

A linear model can often be constructed to describe data for a given situation.

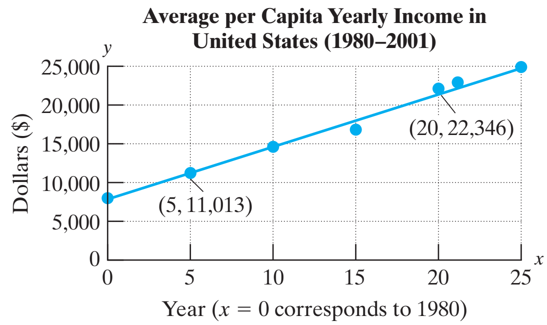
- Given two points from the data, use the point-slope formula to find an equation of the line.
- Interpret the meaning of the slope and y-intercept in the context of the problem.
- Use the equation to predict values.

Examples

Example 1

The graph shows the average per capita income in the United States for 1980–2005.

The year 1980 corresponds to $x = 0$ and income is measured in dollars.



Write an equation of the line, using the points (5, 11,013) and (20, 22,346).

$$\text{Slope: } \frac{22{,}346 - 11{,}013}{20 - 5} = \frac{11{,}333}{15} \approx 756$$

$$y - 11{,}013 = 756(x - 5)$$

$$y - 11{,}013 = 756x - 3780$$

$$y = 756x + 7233$$

The slope $m \approx 756$ indicates that the average income has increased by \$756 per year.

The y-intercept (0, 7233) means that the average income in 1980 ($x = 0$) was \$7233.

Predict the average income for 2010 ($x = 30$).

$y = 756(30) + 7233$

$y = 29{,}913$

According to this model, the average income in 2010 will be approximately \$29,913.

Chapter 2 Review Exercises

Section 2.1

1. Label the following on the diagram:

a. Origin

b. x-Axis

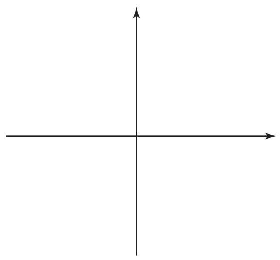
c. y-Axis

d. Quadrant I

e. Quadrant II

f. Quadrant III

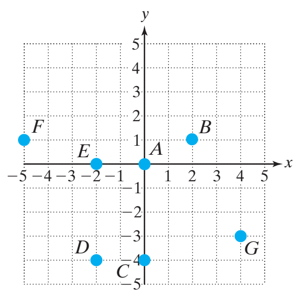
g. Quadrant IV



2. Find the midpoint of the line segment between the two points $(-13, 12)$ and $(4, -18)$.

3. Find the midpoint of the line segment between the two points $(1.2, -3.7)$ and $(-4.1, -8.3)$.

4. Determine the coordinates of the points labeled in the graph.

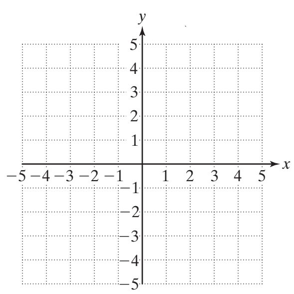


Section 2.2

For Exercises 5–7, complete the table and graph the line defined by the points.

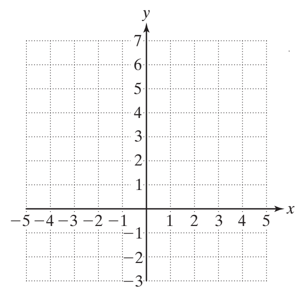
5. $3x - 2y = -6$

x	y
0	
	0
1	



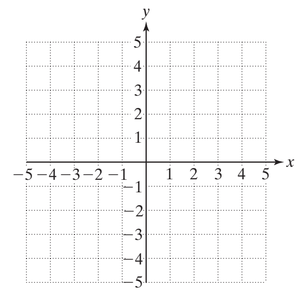
6. $2y - 3 = 10$

x	y
0	
5	
−4	



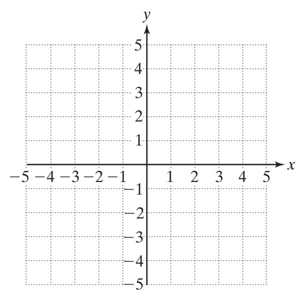
7. $6 - x = 2$

x	y
	0
	1
	−2

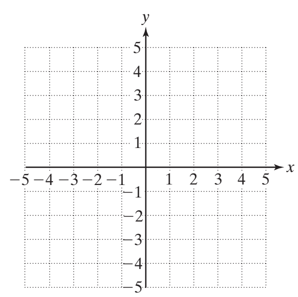


For Exercises 8–11, graph the lines. In each case find at least three points and identify the x- and y-intercepts (if possible).

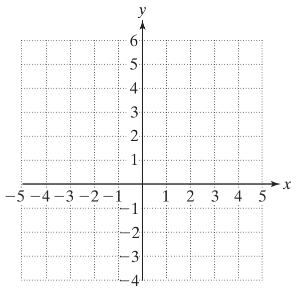
8. $2x = 3y - 6$



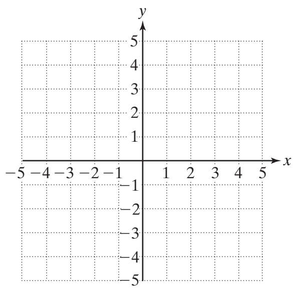
9. $5x - 2y = 0$



10. $2y = 6$



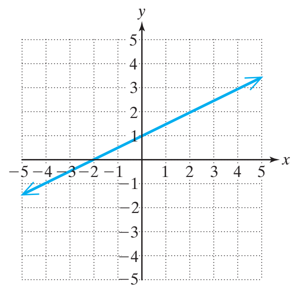
11. $-3x = 6$



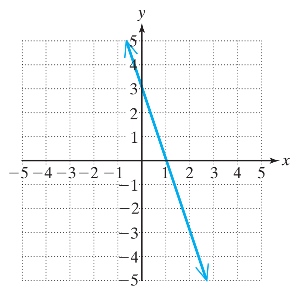
Section 2.3

12. Find the slope of the line.

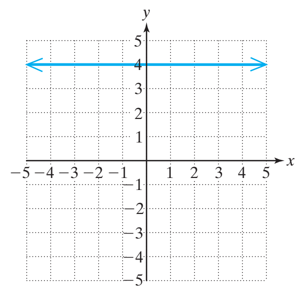
a.



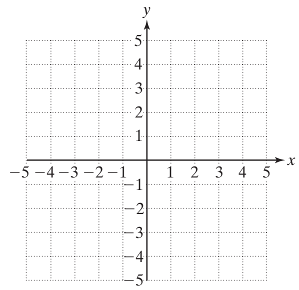
b.



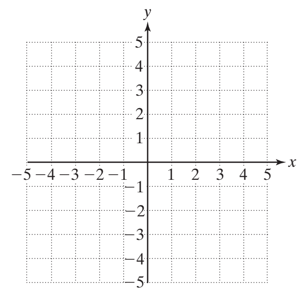
c.



13. Draw a line with slope 2 (answers may vary).



14. Draw a line with slope $-\frac{3}{4}$ (answers may vary).



For Exercises 15–18, find the slope of the line that passes through each pair of points.

15. $(2, 6), (-1, 0)$

16. $(7, 2), (-3, -5)$

17. $(8, 2), (3, 2)$

18. $\left(-4, \frac{1}{2}\right), (-4, 1)$

19. Two points for each of two lines are given. Determine if the lines are parallel, perpendicular, or neither.

L_1: $(4, -6)$ and $(3, -2)$

L_2: $(3, -1)$ and $(7, 0)$

For Exercises 20–22, the slopes of two lines are given. Based on the slopes, are the lines parallel, perpendicular, or neither?

20. $m_1 = -\frac{1}{3}, m_2 = 3$

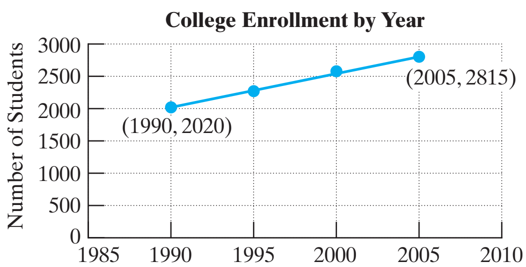
21. $m_1 = \frac{5}{4}, m_2 = \frac{4}{5}$

22. $m_1 = 7, m_2 = 7$

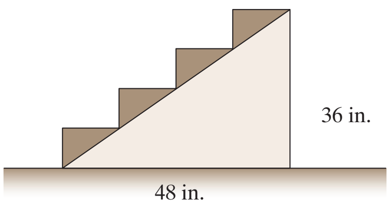
23. The graph indicates that the enrollment for a small college has been increasing linearly between 1990 and 2005.

a. Use the two data points to find the slope of the line.

b. Interpret the meaning of the slope in the context of this problem.



24. Find the slope of the stairway pictured here.



Section 2.4

25. Write an equation for each of the following.

a. Horizontal line

b. Point-slope formula

c. Standard form

d. Vertical line

e. Slope-intercept form

For Exercises 26–30, write your answer in slope-intercept form or in standard form.

26. Write an equation of the line that has slope $\frac{1}{9}$ and y-intercept $(0, 6)$.

27. Write an equation of the line that has slope $-\frac{2}{3}$ and x-intercept $(3, 0)$.

28. Write an equation of the line that passes through the points $(-8, -1)$ and $(-5, 9)$.

29. Write an equation of the line that passes through the point $(6, -2)$ and is perpendicular to the line $y = -\frac{1}{3}x + 2$.

30. Write an equation of the line that passes through the point $(0, -3)$ and is parallel to the line $4x + 3y = -1$.

31. For each of the given conditions, find an equation of the line

a. Passing through the point $(-3, -2)$ and parallel to the x-axis.

b. Passing through the point $(-3, -2)$ and parallel to the y-axis.

c. Passing through the point $(-3, -2)$ and having an undefined slope.

d. Passing through the point $(-3, -2)$ and having a zero slope.

32. Are any of the lines in Exercise 31 the same?

Section 2.5

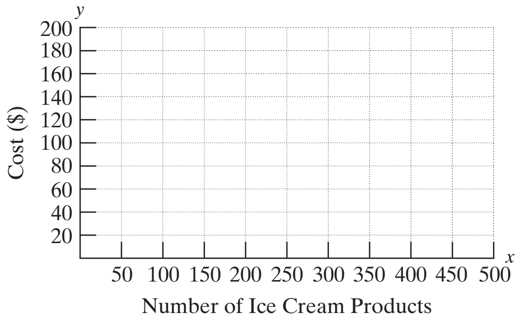
33. Keosha loves the beach and decides to spend the summer selling various ice cream products on the beach. From her accounting course, she knows that her total cost is calculated as

Total cost = fixed cost + variable cost

She estimates that her fixed cost for the summer season is \$20 per day. She also knows that each ice cream product costs her \$0.25 from her distributor.

a. Write a relationship for the daily cost y in terms of the number of ice cream products sold per day x.

b. Graph the equation from part (a) by letting the horizontal axis represent the number of ice cream products sold per day and letting the vertical axis represent the daily cost.



c. What does the y-intercept represent in the context of this problem?

d. What is her cost if she sells 450 ice cream products?

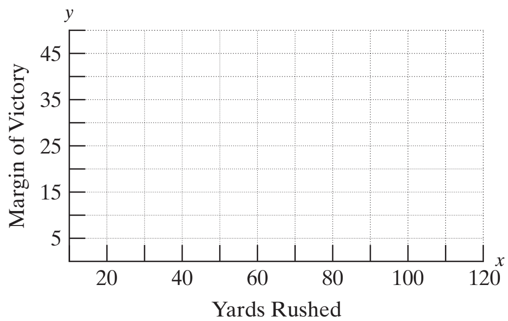
e. What is the slope of the line?

f. What does the slope of the line represent in the context of this problem?

34. The margin of victory for a certain college football team seems to be linearly related to the number of rushing yards gained by the star running back. The table shows the statistics.

Yards Rushed	Margin of Victory
100	20
60	10
120	24
50	7

a. Graph the data to determine if a linear trend exists. Let x represent the number of yards rushed by the star running back and y represent the points in the margin of victory.

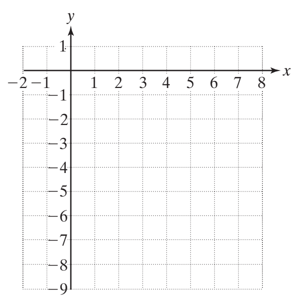


b. Find an equation for the line through the points (50, 7) and (100, 20).

c. Based on the equation, what would be the result of the football game if the star running back did not play?

Chapter 2 Test

1. Given the equation $x - \frac{2}{3}y = 6$, complete the ordered pairs and graph the corresponding points. (0,) (, 0) (, −3)



2. Determine whether the following statements are true or false and explain your answer.

a. The product of the x- and y-coordinates is positive only for points in quadrant I.

b. The quotient of the x- and y-coordinates is negative only for points in quadrant IV.

c. The point (−2, −3) is in quadrant III.

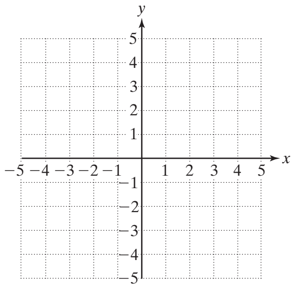
d. The point (0, 0) lies on the x-axis.

3. Find the midpoint of the line segment between the points (21, −15) and (5, 32).

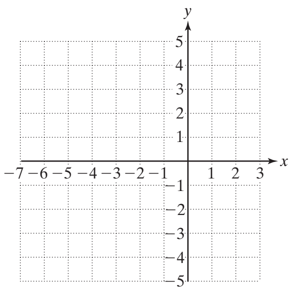
4. Explain the process for finding the x- and y-intercepts.

For Exercises 5–8, identify the x- and y-intercepts (if possible) and graph the line.

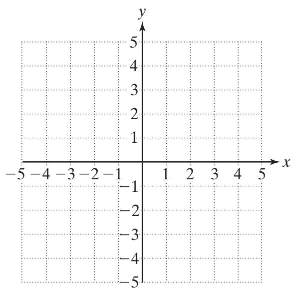
5. $6x - 8y = 24$



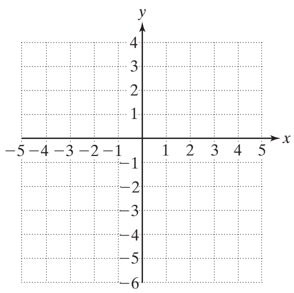
6. $x = -4$



7. $3x = 5y$



8. $2y = -6$



9. Find the slope of the line, given the following information:

a. The line passes through the points $(7, -3)$ and $(-1, -8)$.

b. The line is given by $6x - 5y = 1$.

10. Describe the relationship of the slopes of

a. Two parallel lines

b. Two perpendicular lines

11. The slope of a line is -7.

a. Find the slope of a line parallel to the given line.

b. Find the slope of a line perpendicular to the given line.

12. Two points are given for each of two lines. Determine if the lines are parallel, perpendicular, or neither.

L_1: $(4, -4)$ and $(1, -6)$

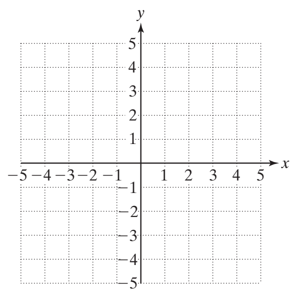
L_2: $(-2, 0)$ and $(0, 3)$

13. Given the equation $-3x + 4y = 4$,

a. Write the line in slope-intercept form.

b. Determine the slope and y-intercept.

c. Graph the line, using the slope and y-intercept.



14. Determine if the lines are parallel, perpendicular, or neither.

a. $y = -x + 4$
$y = x - 3$

b. $9x - 3y = 1$
$15x - 5y = 10$

c. $3y = 6$
$x = 0.5$

d. $5x - 3y = 9$
$3x - 5y = 10$

15. Write an equation that represents a line subject to the following conditions. (Answers may vary.)

a. A line that does not pass through the origin and has a positive slope

b. A line with an undefined slope

c. A line perpendicular to the y-axis. What is the slope of such a line?

d. A slanted line that passes through the origin and has a negative slope

16. Write an equation of the line that passes through the point $(8, -\frac{1}{2})$ with slope -2. Write the answer in slope-intercept form.

17. Write an equation of the line containing the points $(2, -3)$ and $(4, 0)$.

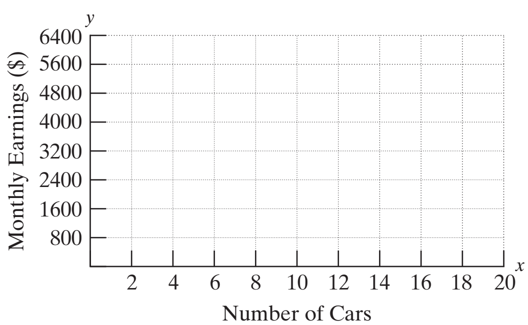
18. Write an equation of a line containing $(4, -3)$ and parallel to $6x - 3y = 1$.

19. Write an equation of the line that passes through the point $(-10, -3)$ and is perpendicular to $3x + y = 7$. Write the answer in slope-intercept form.

20. Jack sells used cars. He is paid \$800 per month plus \$300 commission for each automobile he sells.

a. Write an equation that represents Jack's monthly earnings y in terms of the number of automobiles he sells x.

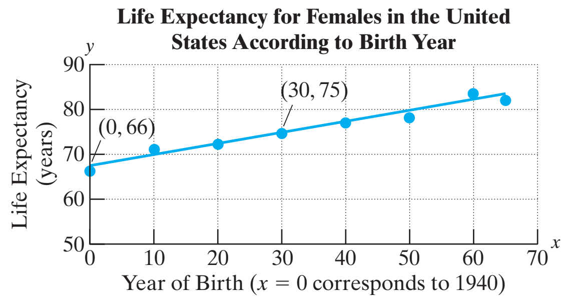
b. Graph the linear equation you found in part (a).



c. What does the y-intercept mean in the context of this problem?

d. How much will Jack earn in a month if he sells 17 automobiles?

21. The following graph represents the life expectancy for females in the United States born from 1940 through 2005.



Source: National Center for Health Statistics

a. Determine the y-intercept from the graph. What does the y-intercept represent in the context of this problem?

b. Using the two points $(0, 66)$ and $(30, 75)$, determine the slope of the line. What does the slope of the line represent in the context of this problem?

c. Use the y-intercept and the slope found in parts (a) and (b) to write an equation of the line by letting x represent the year of birth and y represent the corresponding life expectancy.

d. Using the linear equation from part (c), approximate the life expectancy for women born in the United States in 1994. How does your answer compare with the reported life expectancy of 79 years?

Chapters 1–2 Cumulative Review Exercises

1. Simplify the expression.

$$\frac{5 - 2^3 \div 4 + 7}{-1 - 3(4 - 1)}$$

2. Simplify the expression $3 + \sqrt{25} - 8(\sqrt{9}) \div 6$.

3. Solve the equation for z.

$$z - (3 + 2z) + 5 = -2z - 5$$

4. Solve the equation for b.

$$\frac{2b - 3}{6} - \frac{b + 1}{4} = -2$$

5. A bike rider pedals 10 mph to the top of a mountain and 15 mph on the way down. The total time for the round trip is 10 hr. Find the distance to the top of the hill.

6. The formula for the volume of a right circular cylinder is $V = \pi r^2 h$.

a. Solve for h.

b. Find h if a soda can contains 355 cm^3 (which is approximately 12 oz) of soda and the diameter is 6.6 cm. Round the answer to 1 decimal place.

7. Solve the inequalities. Write your answers in interval notation.

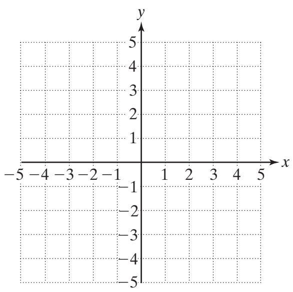
a. $-5x - 4 \le -2(x - 1)$ **b.** $-x + 4 > 1$

8. Find the slope of the line that passes through the points $(4, -5)$ and $(-6, -3)$.

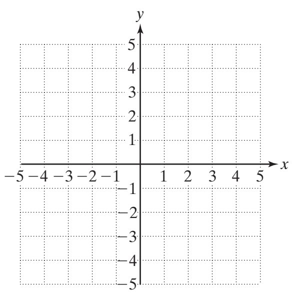
9. Find the midpoint of the line segment with endpoints $(-2, -3)$ and $(0, 15)$.

For Exercises 10–11, **a.** find the x- and y-intercept, **b.** find the slope, and **c.** graph the line.

10. $3x - 5y = 10$



11. $2y + 4 = 10$



12. Find an equation for the vertical line that passes through the point $(7, 11)$.

13. Find an equation for the horizontal line that passes through the point $(19, 20)$.

14. Find an equation of the line passing through $(1, -4)$ and parallel to $2x + y = 6$. Write the answer in slope-intercept form.

15. Find an equation of the line passing through $(1, -4)$ and perpendicular to $y = \frac{1}{4}x - 2$. Write the answer in slope-intercept form.

16. At the movies, Laquita paid for drinks and popcorn for herself and her two children. She spent twice as much on popcorn as on drinks. If her total bill came to \$17.94, how much did she spend on drinks and how much did she spend on popcorn?

17. Three raffle tickets are represented by three consecutive integers. If the sum of the ticket numbers is 1776, find the three numbers.

18. A chemist mixes a 20% salt solution with a 50% salt solution to get 25 L of a 38% salt solution. How much of the 20% solution and how much of the 50% solution did she use?

19. The yearly rainfall for Seattle, Washington, is 0.7 in. less than twice the yearly rainfall for Los Angeles, California. If the total yearly rainfall for the two cities is 50 in., how much rain does each city get per year?

20. Simplify. $4[-3x - 5(y - 2x) + 3] - 7(6y + x)$