UNIT 7 (Extended) FURTHER ALGEBRA

Recommended Prior Knowledge
It is strongly recommended that candidates have a thorough knowledge and understanding of the algebraic topics covered in Unit 1 and Unit 6. The Unit also requires candidates to be competent in manipulating fractions.

Context
This unit draws on basic concepts of algebra to establish a deeper understanding.

Outline
The topics in this unit may be studied sequentially. There is some element of choice, however, and Centres may wish to teach topics in a different order. Work on functions, transforming complex formulae, solving simultaneous and quadratic equations, is completed together with solving simple problems in linear programming. With all sections it is expected that candidates will be set questions of varying difficulty to complete for themselves.

Learning Outcomes		Suggested Teaching Activities	Resources
20 21	Construct and transform more complicated formulae. Manipulate directed numbers; use brackets and extract common factors. Expand products of algebraic expressions; factorise where possible expressions of the form ax + bx + kay + kby, $a^2x^2 - b^2y^2; a^2 + 2ab + b^2; ax^2 + bx + c$; manipulate algebraic fractions, e.g. $\frac{x}{3} + \frac{x-4}{2}; \frac{2x}{3} - \frac{3(x-5)}{2};$ $\frac{3a}{4} x \frac{5ab}{3}; \frac{3a}{4} - \frac{9a}{10}; \frac{1}{x-2} - \frac{2}{x-3};$ factorise and simplify expressions such as $\frac{x^2-2x}{x^2-5x+6}$.	**Revise :** transforming simple formulae (use examples similar to those used in Unit 1). Use straightforward examples (with both positive and negative numbers) to illustrate expanding brackets. Extend this technique to multiplying two brackets together - use a 2x2 grid to help understanding. Use straightforward examples (with both positive and negative numbers) to illustrate factorising simple expressions. Extend this technique to factorising quadratic expressions, including spotting expressions which are the difference of two squares. Transform complex formulae, e.g. $x^2 + y^2 = r^2$, $s = ut + ½at^2$, expressions involving square roots, etc. Use examples to illustrate how to simplify algebraic fractions - build on the work with fractions in Unit 1. Transform formulae involving algebraic fractions, e.g. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.	 Information and worksheets on many aspects of algebra at http://www.algebrahelp.com/worksheets.htm Factorising quadratic expressions at http://www.bbc.co.uk/schools/gcsebitesize/maths/algebraih/index.shtml
20 24	Construct and transform more complicated equations Solve simple linear equations in one unknown; solve simultaneous linear equations in two unknowns.	**Revise** simpler equations from Unit 1. **Revise** how to solve linear equations (including expressions with brackets) from Unit 1. Use straightforward examples to illustrate how to solve simultaneous equations by elimination, by substitution and by graphical means.	

	Solve quadratic equations by factorisation and *either* by use of the formula *or* by completing the square.	Use straightforward examples to illustrate how to solve quadratic equations by factorisation, by using the quadratic formula and by completing the square (real solutions only). Construct equations from information given and then solve them to find the unknown quantity. This could involve the solution of linear, simultaneous or quadratic equations.	
24 25	Solve simple linear inequalities. Represent inequalities graphically and use this representation in the solution of simple linear programming problems (the conventions of using broken lines for strict inequalities and shading unwanted regions will be expected).	Use straightforward examples to illustrate how to solve simple linear inequalities. Start by showing that multiplying or dividing an expression by a negative number reverses the inequality sign. Use straightforward examples to illustrate how to solve linear programming problems by graphical means. Construct inequalities from constraints given and show that a number of possible solutions to a problem exist, indicated by the unshaded region on a graph.	Information about inequalities and graphs at http://www.projectgcse.co.uk/maths/inequalities.htm
22	Use function notation, e.g. $f(x) = 3x - 5$, $f: x \rightarrow 3x - 5$ to describe simple functions, and the notation $f^{-1}(x)$ to describe their inverses; form composite functions as defined by $gf(x) = g(f(x))$.	Define $f(x)$ to be a rule applied to values of x. Evaluate simple functions for specific values, describing the functions using $f(x)$ notation and mapping notation. Introduce the inverse function as an operation which 'undoes' the effect of a function. Evaluate simple inverse functions for specific values, describing the functions using $f^{-1}(x)$ notation and mapping notation. Using linear and/or quadratic functions, $f(x)$ and $g(x)$, form composite functions, $gf(x)$, and evaluate them for specific values of x.	
18	Construct tables of values for functions of the form ax + b, $\pm x^2 + ax + b$, a/x $(x \neq 0)$ where a and b are integral constants; draw and interpret such graphs; solve linear and quadratic equations approximately by graphical methods. Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant and n = -2, -1, 0, 1, 2, 3 and simple sums of not more than three of these and for functions of the form a^x where a is a positive integer; estimate gradients of curves by drawing tangents; solve associated equations approximately by graphical methods.	Draw quadratic functions from a table of values. Show how the solutions to a quadratic equation may be approximated using a graph. Extend this work to show how the solution(s) to pairs of equations (e.g. $y = x^2 - 2x - 3$ and $y = x$) can be estimated using a graph. **Class activity:** Computer packages such as Omnigraph or Derive are useful here. Draw functions of the form $\frac{a}{x^2}; \frac{a}{x}; ax^3; a^x$ where a is a constant, from tables of values. Recognise common types of function from their graphs, e.g. parabola, hyperbola, quadratic, cubic, exponential. Use straightforward examples to find the gradient at a point on a curve. Extend this to find the equation of the tangent at a point on a curve.	